

BUSINESS ARITHMETIC



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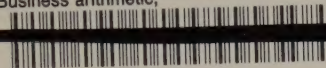
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BUSINESS ARITHMETIC

BY

ashington
GEORGE W. MINER

FAYETTE H. ELWELL

PROFESSOR OF ACCOUNTING, UNIVERSITY OF WISCONSIN

AND

FRANK C. TOUTON

PROFESSOR OF EDUCATION, UNIVERSITY OF SOUTHERN CALIFORNIA

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PREFACE

The study of a well-planned course in business arithmetic should develop in the student the ability to handle the fundamental operations with numbers rapidly and accurately, and at the same time should develop an understanding of business situations which will enable the student to solve problems of the type that are of everyday occurrence in modern business. It is evident that to achieve this aim there must be a nice balance between what constitutes the arithmetic part of the course and the part that sets forth the principles and situations which the student will meet upon entering the business world. To this point the authors of this text have given much attention.

It is not intended that this book shall furnish a course in business practice, but, in the selection of the business situations which illustrate the different applications of the arithmetic principles, the best of up-to-date practice has been chosen and the problem material has been largely adapted from actual transactions as recorded upon the books of various individuals, merchants, banks, and corporations. Some topics always found in business arithmetics of earlier days have been excluded, since the commercial world of today no longer has need of them.

In the development of the individual topics, the authors have taken particular care that the introduction to the topic shall be simple and in language which is not too technical. Completely solved examples are given in the introduction to each new topic, and particular emphasis is placed upon a solution which shall conform to up-to-date business procedure. The illustrations of the various business forms and records have been chosen with a view to showing their principles and uses, and to stimulate the student to a neat and orderly arrangement of his own work.

Throughout, especial care has been taken to have the presentation of the topics accord with the best pedagogical principles and to make a text that shall be readily teachable. To this end frequent drills, speed tests, and review exercises are given to develop skill in the handling of numbers. Numerous easy oral exercises are presented in the introduction of new topics, and the problems are carefully graded as to difficulty. It is not, of course, practical to cover in detail the whole field of commercial activity, but the authors have endeavored to stimulate class discussion and the gathering of more detailed information relative to the several topics under discussion.

A knowledge of the way in which modern business is transacted is a real asset to every young man or woman, whatever line of work may be followed later. It is the authors' hope that, while this text is organized about the solution of business problems, it may also serve to spread valuable information about up-to-date business methods. A glance at the Contents will indicate the wide range of topics treated. A general knowledge of these phases of commercial activity on the part of all well-informed members of a community would be of inestimable value.

The authors desire to acknowledge their indebtedness to numerous business executives who have made many helpful suggestions concerning the parts of the book which deal with business procedure. In particular, acknowledgment is made to the Northwestern Mutual Life Insurance Co., of Milwaukee, Wis., for assistance in the preparation of the chapter on life insurance.

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BUSINESS ARITHMETIC

CHAPTER I

FUNDAMENTAL PROCESSES

Fundamental processes. The students who use this book already know how to add, subtract, multiply, and divide with whole numbers, decimals, and common fractions, and they have already learned the common units of our tables of measures. However, before these processes are applied to the type of problems that arise in business, it is essential that a review of these topics be undertaken so as to give that efficiency in the rapid handling of numbers which is an important factor in success in the business world.

ADDITION

Addition. The process of combining two or more numbers and expressing the total as one number is called **addition**.

A **unit** is a single thing. A number is either a unit or a collection of units. Only numbers the units of which are not named, or the units of which have the same name, can be added.

For example, $5 + 2 = 7$, and $5 \text{ lb.} + 2 \text{ lb.} = 7 \text{ lb.}$; but 5 lb. and 2 ft. cannot be added because their units do not have the same name.

The ability to combine numbers rapidly and accurately is a necessity in all business practice, and efficiency in the making of the combinations depends on that quick perception which enables one to add numbers as readily as he comprehends printed words.

ORAL EXERCISES

1. The following forty-five groups are all the possible combinations of two figures each. Practice adding until the sums can be named in $\frac{1}{2}$ min.:

1	4	2	4	7	6	2	3	2	1	4	2	2	1	1
7	3	5	6	7	3	4	3	3	5	1	2	1	3	1
<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>
3	4	7	1	5	6	6	8	9	4	1	5	8	9	9
5	5	2	8	6	6	9	6	1	4	6	5	8	9	8
<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>
7	9	3	2	4	5	7	2	6	7	9	4	5	3	8
9	5	8	9	9	8	4	6	7	8	3	8	7	7	2
<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>

Practice adding the following groups until the sums in each exercise can be given in less than 20 sec.:

2.	6	7	8	7	4	1	5	9	6	4	0	3	8	2	5
	1	4	2	4	7	6	2	3	2	1	4	2	2	1	1
	7	3	5	6	7	3	4	3	3	5	1	2	1	3	1
	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>
3.	3	2	1	5	4	0	9	7	6	8	3	5	2	4	6
	3	4	7	1	5	6	6	8	9	4	1	5	8	9	9
	5	5	2	8	6	6	9	6	1	4	6	5	8	9	8
	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>
4.	2	3	5	1	4	6	0	9	8	7	3	4	7	6	2
	7	9	3	2	4	5	7	2	6	7	9	4	5	3	8
	9	5	8	9	9	8	4	6	7	8	3	8	7	7	2
	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>
5.	4	7	3	2	5	9	6	1	8	5	2	6	9	4	5
	3	5	8	7	0	1	4	9	7	6	3	4	2	8	7
	8	5	4	1	9	7	8	0	6	9	8	9	6	5	3
	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>

Addition by grouping numbers. In the following exercises the space between the groups in columns 1, 2, and 3 suggests adding the sum of each group instead of adding each individual number.

Thus, in column 1, Ex. 1, begin at the bottom and say 10, 26, 36.

ORAL EXERCISES

1. Practice adding until the fifteen totals can be named in approximately 1 min.:

[illegible]

In the following columns, two or three numbers should be added as a group when their sum is a whole number of tens. Practice adding these columns:

2.	2	9	8	8	7	9	9	8	8	9	4	5	7	9	9
	9	7	5	6	5	8	4	6	7	9	8	9	7	7	9
	9	4	7	6	8	3	7	6	5	2	8	6	6	4	2
	8	5	6	4	9	7	2	1	3	6	8	4	0	3	2
	7	8	6	7	0	3	9	0	4	5	2	7	1	5	4
				6	8	3	0	6	5	2	8	9	6	4	2
	5	4	6	3	1	2	8	9	5	2	6	3	5	7	1
	5	6	4	7	9	8	2	1	5	8	4	7	5	3	2

The fifteen sums in each of Exs. 2-5 should be named in less than 2 min.

3.	2	5	6	8	4	9	3	7	1	6	4	5	5	3	7
	8	3	4	0	1	2	6	3	4	9	3	0	7	2	4
	3	6	3	1	8	2	3	5	4	1	8	9	6	8	3
	2	2	4	9	1	1	7	6	3	2	4	6	4	5	6
	6	5	3	2	8	4	0	9	7	5	6	2	3	2	2
	4	2	4	5	0	8	2	1	0	4	8	1	9	5	4
	2	3	6	5	9	4	3	7	8	1	0	3	2	4	1
	6	5	9	2	1	6	5	4	2	9	7	8	6	7	9
	2	5	4	1	3	7	0	8	6	9	1	4	6	3	8
	7	6	7	9	5	2	8	6	5	3	7	6	2	5	2

4.	3	9	2	2	7	4	5	6	0	8	1	4	3	6	5
	5	8	9	6	4	3	8	5	3	4	9	2	2	4	1
	5	7	3	4	2	6	2	3	8	9	0	7	6	4	4
	2	5	4	8	4	7	3	6	9	1	9	7	0	5	2
	3	2	2	8	5	4	3	8	7	9	1	0	6	2	8
	7	3	5	1	0	9	4	2	2	6	8	9	2	1	6
	6	5	2	3	9	2	0	6	2	3	1	1	7	9	4
	3	2	4	8	9	3	5	0	5	7	2	8	1	4	5
	1	1	2	2	3	3	4	4	5	4	5	6	6	6	5
	4	7	9	8	1	5	6	4	3	0	3	5	3	7	8
	<u>—</u>	<u>—</u>	<u>—</u>	<u>—</u>	<u>—</u>	<u>—</u>	<u>—</u>	<u>—</u>	<u>—</u>	<u>—</u>	<u>—</u>	<u>—</u>	<u>—</u>	<u>—</u>	<u>—</u>

5.	5	4	3	8	2	5	4	7	6	3	3	2	2	1	0
	8	0	7	2	3	4	6	6	5	9	8	7	9	4	6
	7	5	8	7	4	1	0	5	9	2	1	4	5	6	7
	4	2	7	6	3	5	1	2	2	8	7	6	0	5	6
	9	8	6	7	5	6	8	8	9	5	4	1	9	6	8
	0	4	5	2	2	1	7	4	3	3	4	4	1	1	2
	7	7	7	8	9	8	7	9	4	6	6	4	6	6	5
	2	3	4	2	6	3	4	3	5	0	2	1	2	3	3
	4	5	9	6	5	8	1	2	0	4	8	3	2	5	4
	6	5	4	0	1	1	2	8	9	5	6	6	7	8	6
	2	4	3	6	0	5	8	1	6	1	3	2	4	2	8
	<u>—</u>	<u>—</u>	<u>—</u>	<u>—</u>	<u>—</u>	<u>—</u>	<u>—</u>	<u>—</u>	<u>—</u>	<u>—</u>	<u>—</u>	<u>—</u>	<u>—</u>	<u>—</u>	<u>—</u>

6. State results only, and practice adding the following until all the sums can be named in approximately 1 min. :

6	9	5	1	4	8	5	5	6	2	7	4	9
<u>46</u>	<u>27</u>	<u>32</u>	<u>61</u>	<u>22</u>	<u>52</u>	<u>13</u>	<u>31</u>	<u>34</u>	<u>81</u>	<u>17</u>	<u>24</u>	<u>36</u>

8	9	4	3	9	9	5	5	7	9	3	8	6
<u>16</u>	<u>12</u>	<u>33</u>	<u>52</u>	<u>64</u>	<u>18</u>	<u>25</u>	<u>16</u>	<u>62</u>	<u>45</u>	<u>71</u>	<u>78</u>	<u>35</u>

4	8	9	7	7	8	9	2	8	7	8	6	6
<u>11</u>	<u>29</u>	<u>37</u>	<u>44</u>	<u>35</u>	<u>21</u>	<u>33</u>	<u>42</u>	<u>93</u>	<u>21</u>	<u>25</u>	<u>26</u>	<u>31</u>

3	6	9	7	8	7	8	6	5	4	9	1	7
<u>53</u>	<u>42</u>	<u>21</u>	<u>43</u>	<u>14</u>	<u>16</u>	<u>34</u>	<u>77</u>	<u>89</u>	<u>92</u>	<u>15</u>	<u>23</u>	<u>54</u>

Adding two-figure numbers. In adding two numbers of two figures each it is often easier to begin at the left. The student should state only the results of the additions.

For example, in adding 37 to 74, say only 107, 111.

ORAL EXERCISES

1. Add the following, beginning at the left:

<u>74</u>	<u>46</u>	<u>38</u>	<u>79</u>	<u>18</u>	<u>54</u>	<u>23</u>	<u>92</u>	<u>84</u>	<u>73</u>	<u>29</u>	<u>28</u>	<u>34</u>
<u>37</u>	<u>52</u>	<u>76</u>	<u>28</u>	<u>46</u>	<u>75</u>	<u>88</u>	<u>19</u>	<u>27</u>	<u>38</u>	<u>49</u>	<u>57</u>	<u>46</u>
<u>37</u>	<u>65</u>	<u>57</u>	<u>89</u>	<u>78</u>	<u>34</u>	<u>23</u>	<u>64</u>	<u>36</u>	<u>59</u>	<u>68</u>	<u>87</u>	<u>19</u>
<u>13</u>	<u>28</u>	<u>64</u>	<u>23</u>	<u>44</u>	<u>57</u>	<u>69</u>	<u>38</u>	<u>55</u>	<u>44</u>	<u>32</u>	<u>37</u>	<u>77</u>
<u>58</u>	<u>76</u>	<u>79</u>	<u>42</u>	<u>39</u>	<u>82</u>	<u>87</u>	<u>68</u>	<u>18</u>	<u>73</u>	<u>65</u>	<u>46</u>	<u>58</u>
<u>91</u>	<u>71</u>	<u>43</u>	<u>35</u>	<u>53</u>	<u>68</u>	<u>35</u>	<u>51</u>	<u>73</u>	<u>44</u>	<u>67</u>	<u>36</u>	<u>50</u>
<u>41</u>	<u>53</u>	<u>92</u>	<u>63</u>	<u>84</u>	<u>35</u>	<u>33</u>	<u>34</u>	<u>62</u>	<u>90</u>	<u>28</u>	<u>77</u>	<u>46</u>
<u>72</u>	<u>38</u>	<u>74</u>	<u>55</u>	<u>61</u>	<u>83</u>	<u>35</u>	<u>58</u>	<u>17</u>	<u>14</u>	<u>42</u>	<u>16</u>	<u>27</u>

2. State results only, and practice adding the following until all the sums can be given in approximately 2 min.:

<u>62</u>	<u>32</u>	<u>24</u>	<u>23</u>	<u>34</u>	<u>43</u>	<u>16</u>	<u>25</u>	<u>15</u>	<u>42</u>	<u>26</u>	<u>22</u>	<u>41</u>
<u>12</u>	<u>22</u>	<u>23</u>	<u>24</u>	<u>53</u>	<u>33</u>	<u>13</u>	<u>43</u>	<u>24</u>	<u>33</u>	<u>42</u>	<u>13</u>	<u>27</u>
<u>24</u>	<u>25</u>	<u>23</u>	<u>22</u>	<u>37</u>	<u>53</u>	<u>72</u>	<u>80</u>	<u>43</u>	<u>60</u>	<u>41</u>	<u>72</u>	<u>26</u>
<u>30</u>	<u>41</u>	<u>15</u>	<u>37</u>	<u>22</u>	<u>36</u>	<u>54</u>	<u>29</u>	<u>36</u>	<u>52</u>	<u>28</u>	<u>47</u>	<u>33</u>
<u>22</u>	<u>34</u>	<u>24</u>	<u>35</u>	<u>47</u>	<u>35</u>	<u>56</u>	<u>44</u>	<u>31</u>	<u>43</u>	<u>32</u>	<u>41</u>	<u>53</u>
<u>17</u>	<u>23</u>	<u>62</u>	<u>14</u>	<u>21</u>	<u>96</u>	<u>42</u>	<u>74</u>	<u>18</u>	<u>25</u>	<u>17</u>	<u>36</u>	<u>42</u>
<u>35</u>	<u>56</u>	<u>32</u>	<u>23</u>	<u>22</u>	<u>33</u>	<u>14</u>	<u>82</u>	<u>43</u>	<u>70</u>	<u>41</u>	<u>37</u>	<u>52</u>
<u>14</u>	<u>12</u>	<u>14</u>	<u>22</u>	<u>37</u>	<u>44</u>	<u>55</u>	<u>66</u>	<u>72</u>	<u>49</u>	<u>56</u>	<u>21</u>	<u>33</u>
<u>42</u>	<u>53</u>	<u>65</u>	<u>47</u>	<u>31</u>	<u>17</u>	<u>92</u>	<u>50</u>	<u>76</u>	<u>38</u>	<u>25</u>	<u>34</u>	<u>66</u>
<u>27</u>	<u>36</u>	<u>45</u>	<u>73</u>	<u>22</u>	<u>83</u>	<u>41</u>	<u>67</u>	<u>32</u>	<u>81</u>	<u>90</u>	<u>57</u>	<u>33</u>

3. Add 19 to each of the following numbers:

17	83	12	75	34
49	32	81	26	27
67	42	62	25	19
23	28	79	46	53
58	71	44	39	44
14	55	22	41	15

In adding 19, first add 20, and then from the sum subtract 1.

Addition by counting. Increased efficiency in addition may be gained by drill in counting, provided that the work is done rapidly.

ORAL EXERCISES

Count to approximately 100, as follows:

- | | |
|------------------------------|-------------------------------|
| 1. By 5's, beginning at 34. | 13. By 8's, beginning at 28. |
| 2. By 4's, beginning at 47. | 14. By 6's, beginning at 22. |
| 3. By 2's, beginning at 69. | 15. By 3's, beginning at 58. |
| 4. By 7's, beginning at 13. | 16. By 9's, beginning at 11. |
| 5. By 8's, beginning at 11. | 17. By 7's, beginning at 16. |
| 6. By 9's, beginning at 15. | 18. By 6's, beginning at 17. |
| 7. By 11's, beginning at 18. | 19. By 12's, beginning at 21. |
| 8. By 13's, beginning at 25. | 20. By 15's, beginning at 33. |
| 9. By 14's, beginning at 31. | 21. By 16's, beginning at 19. |
| 10. By 5's, beginning at 28. | 22. By 4's, beginning at 43. |
| 11. By 2's, beginning at 57. | 23. By 3's, beginning at 49. |
| 12. By 6's, beginning at 33. | 24. By 7's, beginning at 9. |

Checking results. If the student is to be efficient in the business world, he must be accurate. Accuracy is a habit that can be formed by constantly checking all the work that is done. There are several methods of checking results in addition, which will be taken up in the order of their importance.

Adding in reverse order. After a column is added by beginning at the bottom, the sum should be checked by doing the addition again, beginning at the top. If the results agree, the work is probably correct.

$$\begin{array}{r} 212 \\ 7654 \\ 4876 \\ 6441 \\ 7119 \\ \hline 26090 \end{array}$$

It is also very convenient, particularly when checking addition in a large column of figures, to write lightly in pencil at the head of each column the figure which has been carried from the column just added.

Check of 9's. Another check, known as **casting out the 9's**, is based on the principle that the **excess of 9's**, the remainder left after dividing any number by 9, is equal to the excess of 9's in the sum of its digits.

Thus, in 6235 the sum of the digits 6, 2, 3, 5 is 16; the excess of 16 over 9 is 7. Also, 6235 divided by 9 equals 692, with a remainder of 7.

The same result is obtained by eliminating all the 9's and such combinations as make 9, using each digit only once. For example, in 5963 there is one 9; also, 6 and 3 added make 9; rejecting these, there remains 5 only as the excess of 9's. Elimination of the 9's by inspection saves time.

ORAL EXERCISES

State the excess of 9's in each of the following numbers :

1. 246	691	3972	7186	23,745	37,645
2. 312	702	4074	8490	17,531	87,681
3. 236	863	7452	3875	73,541	59,459

The 9's applied to addition. In checking a result in addition by this method, first cast out the 9's in each number added, and then in the sum of these excess numbers. Then cast out the 9's in the sum which is to be verified. If the excess of 9's in the last two cases is the same, the work is probably correct.

EXAMPLE. Add 467, 713, and 642; and check by casting out the 9's.

<i>Solution.</i> 467	The excess of 9's in 467 is	8
713	The excess of 9's in 713 is	2
642	The excess of 9's in 642 is	3
<u>1822</u>	The sum of these excesses is	<u>13</u>

The excess of 9's in 13 is 4, and the excess of 9's in 1822 is also 4.

EXERCISES

It is not necessary to copy the figures in these exercises. A piece of paper should be placed at the foot of the columns and the sums written on it.

Add the following and check by (1) adding the columns in reverse order, and (2) casting out the 9's:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
67	51	24	81	49	21	197	291	116	215
86	25	61	95	80	49	923	149	949	730
52	78	64	74	29	16	176	329	556	726
28	71	82	43	18	27	912	166	223	418
38	59	25	72	47	84	108	344	543	142
71	32	79	28	42	98	845	207	732	269
85	48	31	82	51	28	644	158	148	159
15	23	56	18	29	66	266	321	432	281
47	37	22	55	59	48	327	775	332	762
20	67	18	21	31	62	428	423	239	357
<u>39</u>	<u>27</u>	<u>16</u>	<u>38</u>	<u>73</u>	<u>40</u>	<u>426</u>	<u>237</u>	<u>955</u>	<u>333</u>

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Casting out the 9's is not absolutely reliable in all cases, as it does not reveal an error made by the transposition of figures or by the omission of a 9.

11.	12.	13.	14.	15.	16.
\$156.25	\$237.08	\$144.15	\$654.35	\$423.50	\$356.75
28.45	73.20	9.23	8.75	17.65	1.75
176.62	210.63	54.92	65.24	125.23	43.62
8.25	19.17	256.74	188.12	25.60	172.20
32.54	171.65	212.26	145.34	200.01	159.06
245.30	11.67	1.84	24.13	187.32	15.75
8.95	113.22	28.96	171.12	45.08	214.47
564.23	39.41	175.33	27.14	9.10	26.54
14.56	8.76	23.42	164.92	908.50	368.07
8.65	543.42	176.04	28.55	33.46	74.06
135.75	74.23	65.08	4.56	9.11	7.53
498.75	122.31	426.07	765.53	277.60	534.22
43.64	78.95	4.12	9.24	21.56	25.69
<u>204.53</u>	<u>173.20</u>	<u>63.54</u>	<u>174.55</u>	<u>513.41</u>	<u>638.12</u>

Check of 11's. Another method of verifying results is known as **casting out the 11's**. In applying this method, first find the sum of the digits in the odd places, counting from the right, and subtract from it the sum of the digits in the even places. If the latter sum is greater than the former, add 11, or some multiple of 11, to the former and subtract as before. The remainder is called **the excess of 11's**.

For example, in 478 the excess is 5; that is, $8 + 4 - 7 = 5$. In 185 the excess is 9; that is, $5 + 1 + 11 - 8 = 9$. In 63,456 the excess is 8; that is, $6 + 4 + 6 - (5 + 3) = 8$.

In applying this method to the checking of addition, find the sum of the excesses of the 11's in the several numbers. Then cast out the 11's from the sum of these excesses, and if this excess of 11's is equal to the excess of 11's in the sum of the several numbers, the addition is correct.

EXAMPLE. Add 4203, 8735, 4263, and 3856; and check by casting out the 11's.

Solution

4203	$3 + 2 - (0 + 4) =$	1, the excess of 11's in 4203
8735	$5 + 7 - (3 + 8) =$	1, the excess of 11's in 8735
4263	$3 + 2 + 11 - (6 + 4) =$	6, the excess of 11's in 4263
3856	$6 + 8 - (5 + 3) =$	6, the excess of 11's in 3856
21057, the sum of the numbers	14, the sum of the excesses	

Cast out the 11's in 21,057; that is, $7 + 0 + 2 - (5 + 1) = 3$. The excess of 11's in 14 is also 3; hence the work is correct.

EXERCISE

1. Check Exs. 11-16 on page 8 by casting out the 11's.

Horizontal addition. On some business forms **horizontal addition**, or **addition by lines**, is necessary. It is used, for instance, in finding the total number of yards in several pieces of cloth, each piece perhaps containing a different number. In billing such goods it is customary to write the figures in a line on the bill, as shown below :

Jan.	7	6 pc. dress goods							
		34, 32, 31, 35, 36, 30	198						

The line is added horizontally from left to right, first the units and then the tens, and the sum is verified by adding in the opposite direction.

EXERCISES

Add each horizontal line and check:

1.	6	7	8	4	2	8	3	3	2	3		
2.	3	3	2	8	0	5	7	8	4	7		
3.	2	4	5	4	4	6	0	8	9	4		
4.	32	44	33	45	30	40	35	45	37	43		
5.	52	53	54	56	57	50	59	58	53	51	30	
6.	24	26	25	23	27	28	29	22	21	20	24	26
7.	33	32	36	38	37	35	30	31	39	34	32	38
8.	25	32	27	33	24	37	26	31	28	35	21	39
9.	42	45	46	40	41	42	43	48	49	43	44	47
10.	37	43	36	42	33	44	35	45	48	32	49	41
11.	54	53	56	52	51	50	59	53	58	57	51	53
12.	43	57	45	56	47	53	48	52	44	56	49	51

Addition by both lines and columns. On some business forms addition both by lines and by columns must be used. If the work is correct, the sum of the line totals equals the sum of the column totals; that is, the work checks itself.

EXAMPLE. In the following, find the line totals, the column totals, the sum of the line totals, and the sum of the column totals.

$$\begin{array}{r}
 \$24 + \$16 + \$26 = \$66 \\
 22 + 27 + 33 = 82 \\
 58 + 45 + 41 = 144 \\
 24 + 32 + 29 = 85 \\
 55 + 28 + 40 = 123 \\
 \hline
 \$183 + \$148 + \$169 = \$500
 \end{array}$$

Solution. Adding as directed, the sums in the black figures are obtained. The sum in the lower right-hand corner is both the sum of the line totals and the sum of the column totals.

EXERCISES

Copy and add:

1. $\$27 + \$43 + \$56 = ?$

$83 + 97 + 44 = ?$

$65 + 42 + 38 = ?$

$34 + 52 + 45 = ?$

$56 + 82 + 19 = ?$

$25 + 36 + 41 = ?$

$40 + 33 + 22 = ?$

$65 + 72 + 92 = ?$

$75 + 31 + 58 = ?$

$\frac{81}{?} + \frac{72}{?} + \frac{54}{?} = ?$

$\frac{?}{?} + \frac{?}{?} + \frac{?}{?} = ?$

2. $\$356 + \$288 + \$416 = ?$

$255 + 342 + 617 = ?$

$155 + 411 + 268 = ?$

$437 + 239 + 432 = ?$

$216 + 132 + 642 = ?$

$252 + 743 + 175 = ?$

$173 + 236 + 435 = ?$

$341 + 252 + 135 = ?$

$215 + 176 + 523 = ?$

$\frac{430}{?} + \frac{226}{?} + \frac{311}{?} = ?$

$\frac{?}{?} + \frac{?}{?} + \frac{?}{?} = ?$

3. Without copying, find the total sales, adding by lines and by columns:

DEPARTMENT	CASH SALES	ON-ACCOUNT SALES	TOTAL SALES
A	\$175.40	\$100.33	275 275.73
B	200.52	135.20	335.72
C	175.44	105.17	280.61
D	250.47	125.16	375.63
E	145.62	110.94	256.56
F	237.27	140.82	
G	115.07	103.08	
H	162.39	108.73	
I	139.28	112.04	
J	190.97	128.07	
K	218.76	248.96	
L	255.03	163.76	
M	173.42	113.20	
N	250.40	125.30	
O	<u>156.72</u>	<u>112.35</u>	
TOTALS			<u> </u>

On-account sales are those that are not paid for at the time of sale but by agreement, expressed or implied, are to be paid for at a future date.

4. Without copying, add by lines and by columns :

CLERK	CASH SALES	ON-ACCOUNT SALES	TOTAL SALES
1	\$53.50	\$71.40	
2	65.20	80.78	
3	58.49	79.46	
4	54.78	85.43	
5	57.93	68.46	
6	60.40	92.59	
7	80.98	117.40	
8	55.93	84.79	
9	98.53	125.20	
10	51.40	65.04	
11	70.07	116.89	
12	50.50	75.08	
13	65.31	87.42	
14	<u>92.34</u>	<u>121.45</u>	
TOTALS			

Dictation of numbers. In the dictation, or calling off, of numbers in a business office short methods are often used. The expressions, however, must be absolutely clear and definite so that no mistake is likely to be made.

For example, in calling an amount which represents cents only, the number should be read in a way that will not confuse it with dollars; thus, 84¢ may be read "cents eighty-four."

When one person calls a number to another, it is best for the second person to repeat the number in a slightly different form.

Thus \$104.22 might be called off as "one hundred four dollars and twenty-two cents," and then called back as "one-o-four-point-two-two."

There is no uniform method of calling numbers, but the following illustrations represent typical cases.

NUMBER	READ
7256	Seven-two-five-six, or seven thousand two hundred fifty-six.
\$843.70	Eight-four-three-point-seven-o, or eight hundred forty-three dollars and seventy cents.

ORAL EXERCISES

Call off each of the following numbers and let another member of the class call it back:

1.	2.	3.	4.	5.
\$4256.00	\$814.75	\$85,754.00	\$562.80	\$784.13
\$718.00	\$6.73	\$7,332.00	\$8429.50	\$1055.07
\$242.70	\$19.21	\$201,700.00	\$1756.40	\$40.10
\$75.82	\$1006.64	\$46,511.00	\$18,356.00	\$2001.01

As before, dictate and call back these telephone numbers:

6.	7.	8.	9.	10.
303	0742-J	2921-M	465-R	River 115-R
1144-W	1093-M	7500-W	101-R	Beach 192-M
5600	0028	1001-R	866	Walnut 133-W
173-27	410-92	319-12	606-X	River 832-R

Writing numbers from dictation. In business one is sometimes required to write numbers from dictation. To do this work rapidly and correctly requires care and much concentration of thought.

EXERCISES

Write the following from dictation and check with these columns:

1.	2.	3.	4.
\$4856.89	\$3909.01	\$8374.09	\$426.54
3420.15	6172.19	1756.84	76.19
6780.67	2453.21	8345.67	150.46
6477.28	8364.29	9765.39	.54
8135.98	5216.34	2694.10	128.53
3052.18	7348.70	1456.85	44.56
6435.26	2794.62	9649.10	23.95
2561.33	3748.08	9018.29	9.45
1543.28	1340.70	4835.15	981.34
<u>8590.73</u>	<u>2894.53</u>	<u>6784.92</u>	<u>39.41</u>

5. Add the columns which you have written and check.

Adding machines and other mechanical devices are now commonly used for many arithmetical operations. The rapidity and the accuracy of these machines when efficiently operated save much time in making computations.

ADDITION APPLIED TO BUSINESS PROBLEMS

Deposit ticket. Individuals or business houses deposit their money in banks for safety and convenience. At the right is shown a **deposit ticket**, which is made out and sent to the bank when money or checks are deposited. The general custom in listing checks is to use the name of the bank when it is located in the same city, and the name of the city or town for checks on outside banks.

A **check** is an order on a bank by a depositor for the payment of his money to himself or to a specified person or firm.

When the bank receives the deposit, a record of it is generally made in a pass book, which is returned to the depositor. Later in the day a record of the deposit is made on the books of the bank.

Deposit tickets are not always uniform in wording; thus, the term *currency* is often used in place of *bills*, and *specie* in place of *gold* and *silver*.

If a depositor does not have his pass book with him at the time the deposit is made, it is customary for him to make out duplicate deposit tickets. One ticket is retained by the bank; the other is stamped by the teller and returned to the depositor, the stamp showing that the deposit was received.

FIRST NATIONAL BANK		
NASHVILLE, TENN.		
DEPOSITED BY		
<i>Wm. B. Harmon & Co</i>		
<i>May 2,</i> 19 <i>—</i>		
List each check separately		
Bills	110	—
Gold	62	50
Silver	13	16
Check on		
<i>Union National Bank</i>	13	40
<i>Second National Bank</i>	106	35
<i>Columbus, O. Bank</i>	28	33
<i>Washington, D.C. Bank</i>	6	55
<i>New York, N.Y. Bank</i>	17	26
_____ Bank		
_____ Bank		
	357	55

DEPOSIT TICKET

EXERCISES

Prepare the following deposit tickets, following the model form on page 14:

1. *Student* makes the following deposit in the First National Bank: bills, \$215; gold, \$52.50; silver, \$9.25; checks, \$67.63, \$15.20, \$92.96, \$225.77, \$34.79, \$13.81, \$229.07, \$75.97, \$53.47, \$418, \$81.82, \$25, \$10.77, \$207.93, \$234.17, \$786.65, \$267.51, \$439.49.

2. James W. Masters makes the following deposit in the Second National Bank: bills, \$90; specie, \$81.78; checks, \$7.53, \$2.38, \$3.15, \$1.65, \$2.40, \$175, \$61.70, \$24, \$13, \$15.81, \$2.35, \$9.77, \$262.17, \$41.14, \$15.75, \$23.12, \$38.40, \$13.73.

3. Henry Ames & Co. make the following deposit in the Commercial Trust Co.: bills, \$340; specie, \$59.39; checks, \$163.60, \$104, \$78.90, \$208, \$498.32, \$15.88, \$105.60, \$433.33, \$298.45, \$534.77, \$660.59, \$1250, \$65, \$33, \$26.

4. John R. Taylor makes the following deposit in the Safe Deposit & Trust Co.: bills, \$157; gold, \$15; silver, \$33.25; checks, \$2.35, \$10.45, \$1.95, \$3, \$2.40, \$7.70, \$17.66, \$14.10, \$7.15, \$7.71, \$8.65, \$7.07, \$5.25, \$18.65, \$11.40, \$2.10, \$8.85, \$16.90, \$22.15, \$112.46.

Sales records. Much use is made of addition in keeping sales records where the amounts of the sales are classified by departments, by clerks, or in other ways. The forms in the following exercises illustrate typical sales records.

EXERCISES

1. The report on the following page shows one day's cash and on-account sales of a department store. Copy and add by lines and by columns, finding the following:

- a. Total sales for each department.
- b. Total cash sales for all departments.
- c. Total on-account sales for all departments.
- d. Total sales for all departments.

DAILY SALES SUMMARY						
May 11, 19—						
DEPARTMENT	CASH SALES		ON-ACCOUNT SALES		TOTAL SALES	
A	120	35	67	28		
B	132	46	90	26		
C	118	50	70	55		
D	162	38	83	42		
E	91	25	72	84		
F	155	33	100	75		
G	78	21	70	06		
H	109	54	73	77		
I	93	38	75	05		
J	126	32	84	11		
K	145	29	162	14		
L	169	35	103	64		
TOTALS						

The additions in Exs. 1 and 2 should be completed in approximately 10 min. each.

2. Copy and complete the following daily sales summary:

DAILY SALES SUMMARY						
May 23, 19—						
CLERK NO.	CASH SALES		ON-ACCOUNT SALES		TOTAL SALES	
1	79	52	95	38		
2	86	22	101	25		
3	78	56	98	33		
4	72	68	102	41		
5	88	93	89	45		
6	79	34	122	58		
7	112	26	153	38		
8	81	64	103	72		
9	131	13	162	35		
10	87	43	93	29		
11	95	57	147	85		
12	71	42	100	09		
TOTALS						

3. Without copying, complete the following summary:

WEEKLY SALES SUMMARY													
Week of Oct. 15, 19—													
SALES CLERK NO.	MON- DAY		TUES- DAY		WEDNES- DAY		THURS- DAY		FRI- DAY		SAT- URDAY		TOTAL
101	152	75	120	75	148	32	96	58	132	11	191	40	
102	126	90	134	37	139	56	112	95	121	14	162	35	
103	138	54	123	56	109	84	120	30	105	33	146	78	
104	88	75	91	42	101	86	104	70	95	35	128	76	
105	161	25	141	22	149	58	150	65	128	76	187	45	
106	68	28	76	54	83	26	74	36	70	90	105	66	
107	131	55	118	32	123	21	115	49	125	50	148	51	
108	152	12	138	40	146	22	133	16	135	25	176	24	
109	81	36	90	35	98	50	87	55	91	54	102	35	
110	128	56	139	25	102	48	120	62	123	45	133	31	
111	79	40	72	91	87	20	91	19	82	50	100	18	
112	145	41	159	60	142	16	139	17	141	16	162	09	
TOTALS													

The additions in Ex. 3 should be completed in approximately 25 min.

Business statistics. In all forms of business activity the use of statistics has greatly increased in recent years. In the preparation of such material addition plays the most prominent part, as is seen from the following exercises.

EXERCISES

1. The following columns show the receipts and shipments of wheat at Minneapolis during a period of five days in a recent year. Without copying, find the four totals:

<i>Receipts</i>		<i>Shipments</i>	
CARS	BUSHELS	CARS	BUSHELS
494	642,200	224	286,720
679	882,700	219	280,320
474	620,940	184	241,040
452	592,120	168	220,080
387	506,970	217	284,270

2. In the table below the two columns at the left show the number of bales of cotton and the value of the crop produced in each of the cotton-growing states in a recent year. Find the total number of bales and the total value of the crop:

	COTTON		COTTON SEED	
	Bales (500 lb.)	Value	Tons	Value
Alabama	713 236	\$124 103 000	316 000	\$23 020 000
Arizona	58 472	15 261 000	27 000	1 840 000
Arkansas	884 473	160 974 000	393 000	24 880 000
California	56 107	12 063 000	25 000	1 480 000
Florida	15 922	3 344 000	8 000	530 000
Georgia	1 659 529	297 056 000	736 000	55 260 000
Louisiana	297 681	52 094 000	132 000	8 660 000
Mississippi	960 086	180 166 000	427 000	28 100 000
Missouri	64 031	10 885 000	28 000	2 040 000
N. Carolina	830 203	146 232 000	368 000	27 340 000
Oklahoma	1 016 129	178 839 000	453 000	27 130 000
S. Carolina	1 426 146	254 567 000	633 000	47 460 000
Tennessee	310 044	51 932 000	138 000	9 210 000
Texas	3 098 967	542 319 000	1 379 000	82 640 000
Virginia	23 076	3 942 000	10 000	740 000
TOTALS				

The spacing in each of the above numbers indicates the periods into which the number is divided. (Commas may be used if they are preferred by the instructor.)

All such exercises should be done without copying, unless the student is specifically directed to copy the figures.

3. The two columns at the right in the table of Ex. 2 show the weight and value of the cotton seed produced in each of the cotton-growing states in a recent year. Find the total number of tons and the total value of the seed produced.

4. The schedule on the following page shows the amount and geographical distribution of traffic through the Panama Canal and the receipts in tolls for a recent year. Find the total of each column:

VESSELS FROM	NUMBER OF SHIPS	TOLLS RECEIVED	TONS OF CARGO
United States . .	1281	\$4,724,494	5,509,247
Europe	364	1,289,623	1,404,093
British Empire . .	867	3,357,357	3,305,681
Asia	126	596,830	778,606
South America . .	152	319,547	216,869
Miscellaneous . .	24	6,502	3,713
TOTALS			

5. In the following table the value of a variety of fruits produced in California in a recent year is given. Find the total value:

FRUIT	VALUE
Apples	\$9 605 000
Apricots	9 775 000
Figs	900 000
Grapes, table	16 000 000
Lemons	2 700 000
Olives	2 000 000
Oranges	41 456 000
Peaches	26 220 000
Pears	8 100 000
Plums	3 150 000
Prunes.	19 000 000
Raisins	55 800 000
Strawberries.	1 028 790
TOTAL	

A TIMED REVIEW

Without copying, add each horizontal line:

1.	26	32	34	25	28	31	33	24	29	35	37	39
2.	33	41	45	35	36	44	47	31	37	46	44	38
3.	42	53	44	56	47	58	43	55	49	59	48	50
4.	51	62	54	67	58	66	59	63	68	52	61	55

5.	26	37	45	53	67	65	44	28	33	52	41	35
6.	65	45	32	24	59	37	58	66	38	49	56	60
7.	71	59	63	72	66	54	65	75	78	52	73	57
8.	53	41	70	65	53	42	55	67	74	76	51	79
9.	43	56	44	68	53	32	28	75	41	63	40	75
10.	52	55	54	47	49	43	57	58	59	45	42	48
11.	35	44	53	62	71	55	61	56	65	73	52	41
12.	25	43	37	49	53	29	32	45	56	67	74	78
13.	78	35	67	22	45	33	37	49	58	44	37	30
14.	35	46	73	38	26	65	37	56	33	61	79	27
15.	47	33	26	74	35	45	29	61	36	45	37	42

The fifteen additions should be completed in less than 15 min.

Write the following from dictation, and then add each column:

16.	17.	18.
\$6124.92	\$4216.48	\$6170.50
395.14	29.51	516.42
76.23	398.62	34.26
9.15	6074.73	178.25
289.25	92.75	3819.47
6851.35	346.10	64.43
93.87	4.72	394.63
260.57	6475.29	47.12
346.73	742.08	8246.54
6382.05	4265.31	2.65
634.75	531.84	683.12
346.92	39.45	1569.84
5948.35	1307.58	58.76
65.14	417.63	543.72
1068.23	93.12	1.37
<u>93.24</u>	<u>172.48</u>	<u>212.93</u>

The three additions should be completed in less than 10 min.

19. Complete the following weekly sales summary:

WEEKLY SALES SUMMARY													
Week of June 7, 19—													
SALES CLERK NO.	MON- DAY		TUES- DAY		WEDNES- DAY		THURS- DAY		FRI- DAY		SAT- URDAY		TOTAL
61	102	50	79	84	95	78	64	32	89	76	128	40	
62	84	56	91	12	101	45	73	25	81	90	109	28	
63	93	20	81	84	72	56	80	65	69	85	98	75	
64	54	38	62	53	70	55	68	71	60	21	81	12	
65	112	67	94	42	98	75	101	10	93	73	122	35	
66	42	56	51	34	49	80	50	20	46	95	66	85	
67	89	57	78	56	82	48	75	44	85	32	99	26	
68	100	35	93	45	97	35	88	27	90	50	125	40	
69	54	24	60	70	63	75	57	65	61	58	72	35	
70	88	54	92	37	67	43	79	82	81	40	86	55	
71	53	20	47	63	58	13	61	12	65	32	68	49	
72	99	48	106	45	92	44	89	17	94	36	108	35	
73	76	11	69	45	82	14	81	08	85	33	97	20	
74	39	51	44	63	48	33	40	06	51	10	60	46	
75	104	35	94	44	80	17	93	15	100	55	98	40	
TOTALS													

The additions should be completed in approximately 30 min.

SUBTRACTION

Subtraction. The process of finding the difference between two numbers is called **subtraction**. Sometimes the larger number, the number from which to subtract, is called the **minuend**; the smaller number, the number to be subtracted, is called the **subtrahend**; and the difference is called the **remainder**.

The terms *minuend*, *subtrahend*, and *remainder* are not often used in business, but the pupil is already familiar with them from his earlier work in arithmetic.

The two following methods are commonly used in performing operations in subtraction:

1. Take the smaller number (the number to be subtracted) from the larger number (the number from which to subtract).

2. Think of the number which when added to the smaller number (the number to be subtracted) equals the larger number (the number from which to subtract).

Thus, in subtracting 8 from 19, think $8 + 11 = 19$. The difference between 19 and 8 is therefore 11.

This second method, which tends to substitute addition for subtraction, is one with which students should become familiar.

ORAL EXERCISES

In each of the following state the result rapidly, using the first method of subtraction:

$$\begin{array}{r} 1. \quad 8 \quad 7 \quad 9 \quad 6 \quad 4 \quad 5 \quad 8 \quad 9 \quad 6 \quad 4 \quad 3 \quad 7 \quad 9 \quad 5 \quad 6 \\ \quad \underline{3} \quad \underline{2} \quad \underline{5} \quad \underline{1} \quad \underline{4} \quad \underline{2} \quad \underline{6} \quad \underline{7} \quad \underline{3} \quad \underline{2} \quad \underline{1} \quad \underline{5} \quad \underline{4} \quad \underline{3} \quad \underline{2} \end{array}$$

$$\begin{array}{r} 2. \quad 7 \quad 9 \quad 8 \quad 4 \quad 7 \quad 5 \quad 4 \quad 6 \quad 2 \quad 9 \quad 4 \quad 8 \quad 5 \quad 6 \quad 3 \\ \quad \underline{2} \quad \underline{3} \quad \underline{5} \quad \underline{4} \quad \underline{6} \quad \underline{4} \quad \underline{2} \quad \underline{3} \quad \underline{2} \quad \underline{7} \quad \underline{1} \quad \underline{6} \quad \underline{3} \quad \underline{5} \quad \underline{2} \end{array}$$

$$\begin{array}{r} 3. \quad 11 \quad 15 \quad 18 \quad 17 \quad 16 \quad 13 \quad 14 \quad 12 \quad 16 \quad 18 \quad 15 \quad 19 \quad 13 \quad 12 \quad 17 \\ \quad \underline{1} \quad \underline{2} \quad \underline{4} \quad \underline{6} \quad \underline{5} \quad \underline{1} \quad \underline{4} \quad \underline{2} \quad \underline{3} \quad \underline{7} \quad \underline{3} \quad \underline{9} \quad \underline{2} \quad \underline{1} \quad \underline{6} \end{array}$$

$$\begin{array}{r} 4. \quad 14 \quad 17 \quad 16 \quad 11 \quad 19 \quad 18 \quad 15 \quad 12 \quad 13 \quad 16 \quad 19 \quad 15 \quad 14 \quad 18 \quad 17 \\ \quad \underline{6} \quad \underline{8} \quad \underline{7} \quad \underline{1} \quad \underline{5} \quad \underline{9} \quad \underline{6} \quad \underline{4} \quad \underline{2} \quad \underline{9} \quad \underline{8} \quad \underline{7} \quad \underline{6} \quad \underline{2} \quad \underline{7} \end{array}$$

$$\begin{array}{r} 5. \quad 12 \quad 14 \quad 16 \quad 18 \quad 11 \quad 13 \quad 15 \quad 17 \quad 19 \quad 16 \quad 17 \quad 15 \quad 14 \quad 18 \quad 13 \\ \quad \underline{5} \quad \underline{7} \quad \underline{9} \quad \underline{8} \quad \underline{4} \quad \underline{6} \quad \underline{7} \quad \underline{2} \quad \underline{4} \quad \underline{7} \quad \underline{6} \quad \underline{8} \quad \underline{9} \quad \underline{5} \quad \underline{6} \end{array}$$

$$\begin{array}{r} 6. \quad 47 \quad 38 \quad 41 \quad 28 \quad 23 \quad 32 \quad 55 \quad 61 \quad 37 \quad 43 \quad 54 \quad 49 \quad 72 \quad 67 \quad 76 \\ \quad \underline{8} \quad \underline{9} \quad \underline{7} \quad \underline{9} \quad \underline{5} \quad \underline{6} \quad \underline{7} \quad \underline{4} \quad \underline{8} \quad \underline{5} \quad \underline{7} \quad \underline{9} \quad \underline{6} \quad \underline{8} \quad \underline{7} \end{array}$$

$$\begin{array}{r} 7. \quad 82 \quad 30 \quad 57 \quad 60 \quad 90 \quad 10 \quad 47 \quad 69 \quad 71 \quad 80 \quad 40 \quad 33 \quad 49 \quad 50 \quad 70 \\ \quad \underline{15} \quad \underline{12} \quad \underline{29} \quad \underline{11} \quad \underline{24} \quad \underline{2} \quad \underline{28} \quad \underline{35} \quad \underline{42} \quad \underline{32} \quad \underline{23} \quad \underline{7} \quad \underline{35} \quad \underline{32} \quad \underline{43} \end{array}$$

When the figure in the units' place of the upper number is smaller than the one below, it is necessary to increase the upper figure by a ten taken from the tens' place. Thus, in subtracting 15 from 82, think 5 (units) from 12 (units) and 1 (ten) from 7 (tens). The difference is then 67. Proceed in a similar manner for cases in which this occurs in the tens' place, the hundreds' place, and so on.

$$\begin{array}{r} 8. \quad 38 \quad 45 \quad 52 \quad 37 \quad 39 \quad 24 \quad 33 \quad 22 \quad 30 \quad 26 \quad 34 \quad 41 \quad 56 \quad 64 \quad 73 \\ \quad \underline{15} \quad \underline{23} \quad \underline{26} \quad \underline{14} \quad \underline{25} \quad \underline{17} \quad \underline{16} \quad \underline{14} \quad \underline{21} \quad \underline{19} \quad \underline{23} \quad \underline{16} \quad \underline{28} \quad \underline{31} \quad \underline{25} \end{array}$$

$$\begin{array}{r} 9. \quad 64 \quad 35 \quad 77 \quad 58 \quad 51 \quad 35 \quad 74 \quad 52 \quad 73 \quad 46 \quad 59 \quad 74 \quad 83 \quad 91 \quad 63 \\ \quad \underline{25} \quad \underline{19} \quad \underline{34} \quad \underline{39} \quad \underline{22} \quad \underline{18} \quad \underline{26} \quad \underline{43} \quad \underline{27} \quad \underline{29} \quad \underline{27} \quad \underline{38} \quad \underline{37} \quad \underline{48} \quad \underline{57} \end{array}$$

$$\begin{array}{r} 10. \quad 45 \quad 28 \quad 55 \quad 44 \quad 76 \quad 48 \quad 50 \quad 37 \quad 80 \quad 91 \quad 73 \quad 29 \quad 40 \quad 65 \quad 75 \\ \quad \underline{13} \quad \underline{19} \quad \underline{18} \quad \underline{27} \quad \underline{36} \quad \underline{29} \quad \underline{31} \quad \underline{19} \quad \underline{53} \quad \underline{47} \quad \underline{35} \quad \underline{18} \quad \underline{22} \quad \underline{38} \quad \underline{47} \end{array}$$

11. State each difference in Exs. 1-10 by the second method of subtraction.

Check in subtraction. If the sum of the difference and the number subtracted is equal to the number from which the subtraction was made, the work is correct. This check is so simple and efficient that it is the only one needed in subtraction.

EXERCISES

Subtract and check:

$$\begin{array}{r} 1. \quad 288 \quad 533 \quad 331 \quad 275 \quad 174 \quad 337 \quad 439 \quad 358 \quad 274 \quad 431 \quad 283 \quad 476 \\ \quad \underline{9} \quad \underline{47} \quad \underline{155} \quad \underline{17} \quad \underline{8} \quad \underline{43} \quad \underline{9} \quad \underline{69} \quad \underline{127} \quad \underline{12} \quad \underline{94} \quad \underline{82} \end{array}$$

$$\begin{array}{r} 2. \quad 253 \quad 427 \quad 512 \quad 387 \quad 213 \quad 465 \quad 672 \quad 389 \quad 256 \quad 532 \quad 704 \quad 350 \\ \quad \underline{94} \quad \underline{82} \quad \underline{87} \quad \underline{113} \quad \underline{125} \quad \underline{87} \quad \underline{64} \quad \underline{126} \quad \underline{137} \quad \underline{338} \quad \underline{523} \quad \underline{176} \end{array}$$

$$\begin{array}{r} 3. \quad 406 \quad 398 \quad 355 \quad 277 \quad 483 \quad 612 \quad 589 \quad 411 \quad 235 \quad 336 \quad 249 \quad 422 \\ \quad \underline{187} \quad \underline{243} \quad \underline{177} \quad \underline{88} \quad \underline{45} \quad \underline{160} \quad \underline{205} \quad \underline{112} \quad \underline{150} \quad \underline{137} \quad \underline{82} \quad \underline{173} \end{array}$$

$$\begin{array}{r} 4. \quad 792 \quad 804 \quad 732 \quad 650 \quad 255 \quad 486 \quad 502 \quad 276 \quad 645 \quad 381 \quad 353 \quad 956 \\ \quad \underline{109} \quad \underline{216} \quad \underline{516} \quad \underline{337} \quad \underline{109} \quad \underline{290} \quad \underline{376} \quad \underline{128} \quad \underline{257} \quad \underline{194} \quad \underline{147} \quad \underline{207} \end{array}$$

5. The following table shows the production in tons of pig iron in the Pittsburgh district and in the whole United States during a five-year period. For each year, find the amount produced outside of the Pittsburgh district:

	1ST YEAR	2D YEAR	3D YEAR	4TH YEAR	5TH YEAR
United States	29,916,213	39,434,797	38,621,216	39,054,644	31,015,364
Pittsburgh district	5,970,407	7,239,913	6,226,601	6,357,660	5,719,842

6. On a certain day the total sales on the New York Stock Exchange were 640,051 shares of stock; on the corresponding day one year later they were 854,293 shares. What was the excess of sales on the later date?

7. The following table shows the value of American exports and imports in a recent year for four leading classes of goods. Find the excess of exports for each class:

	BREADSTUFFS	LEATHER AND WOOL	MANUFACTURES	MEAT AND DAIRY PRODUCTS
Exports	\$808,471,226	\$199,772,357	\$168,574,578	\$771,006,760
Imports	59,977,601	40,327,091	157,367,368	42,424,105

Making change. In purchases and sales **making change** is usually accomplished by counting the coins and bills as they are passed to the customer rather than by direct subtraction.

Thus, if a customer makes a purchase amounting to 88¢ and offers a two-dollar bill in payment, the change might be counted to him as follows: 88¢, 89¢, 90¢, \$1, \$2, the customer receiving two pennies, a dime, and a dollar.

It is customary to make change, if possible, by using the larger coins and bills. In giving out one dollar in change a dollar would be given rather than four quarters or ten dimes, unless the larger change was not at hand.

ORAL EXERCISES

1. Assuming that \$1 is offered by the customer in each case, count the change for a purchase which costs:

33¢ 50¢ 49¢ 34¢ 61¢ 25¢ 48¢ 12¢ 76¢ 22¢
47¢ 38¢ 39¢ 40¢ 88¢ 44¢ 11¢ 19¢ 35¢ 17¢

2. Assuming that \$2 is offered by the customer in each case, count the change for a purchase which costs:

42¢ 59¢ 14¢ 89¢ 13¢ 29¢ 78¢ 87¢ 72¢ 27¢
35¢ 63¢ 52¢ 75¢ 69¢ 84¢ 23¢ 55¢ 59¢ 99¢

3. From \$5 in each case, count the change for the sum of the two purchases:

55¢ 37¢ 49¢ 19¢ 91¢ 52¢ 11¢ 66¢ 35¢ 41¢
80¢ 48¢ 99¢ 63¢ 33¢ 47¢ 84¢ 75¢ 55¢ 98¢

4. From \$20 in each case, count the change for the sum of the two purchases:

\$4.55	\$3.96	\$7.22	\$8.45	\$3.95	\$9.09	\$7.32	\$1.65
<u>9.20</u>	<u>6.30</u>	<u>4.75</u>	<u>3.32</u>	<u>5.00</u>	<u>6.40</u>	<u>3.95</u>	<u>5.22</u>

5. From \$50 in each case, count the change for the sum of the two purchases:

\$21.75	\$12.50	\$18.75	\$11.45	\$12.76	\$15.24	\$11.99
<u>8.40</u>	<u>10.61</u>	<u>14.65</u>	<u>9.55</u>	<u>7.75</u>	<u>8.32</u>	<u>9.49</u>

6. Assuming that \$100 is offered by the customer in each case, count the change for a purchase which costs:

\$37.25	\$41.53	\$35.68	\$54.75	\$78.12	\$81.19	\$43.62
\$28.35	\$67.31	\$36.45	\$41.70	\$81.54	\$73.22	\$29.44

Horizontal subtraction. A great saving of time on some business forms is effected by the use of **horizontal**, or **line**, subtraction.

For example, on a bill an expression in the form 335-32 is often used. The first number indicates the **gross weight**, that is, the weight of the goods and the box or container; the second number indicates the **tare**, or the weight of the container alone. The **net weight**, that is, the weight of the goods alone, may be found more quickly by horizontal subtraction.

Or on a bank form it might appear that a depositor's balance, on a given date, was \$385, and to the right of this amount the sum of the checks he had drawn might appear as \$133. To find the balance on deposit quickly would require line subtraction.

ORAL EXERCISES

State quickly the net weight in each of the following:

GROSS WEIGHT	TARE	GROSS WEIGHT	TARE
1. 234 lb.	30 lb.	5. 219 lb.	35 lb.
2. 180 lb.	25 lb.	6. 175 lb.	23 lb.
3. 211 lb.	31 lb.	7. 255 lb.	42 lb.
4. 117 lb.	22 lb.	8. 194 lb.	28 lb.

EXERCISES

Find the balance on deposit in each of the following:

BANK BALANCE ON FEB. 1, 19—		CHECKS	BANK BALANCE ON NOV. 1, 19—		CHECKS
1.	\$284.50	\$115.10	6.	\$3785.50	\$2169.25
2.	\$736.85	\$212.90	7.	\$4128.75	\$1735.50
3.	\$289.12	\$198.56	8.	\$5793.25	\$3847.75
4.	\$309.08	\$190.04	9.	\$6234.25	\$3716.75
5.	\$427.68	\$246.79	10.	\$9271.80	\$6247.25

SUBTRACTION APPLIED TO BUSINESS PROBLEMS

The depositors' ledger. In banks that receive deposits against which checks may be drawn a form of record, called the **depositors' ledger**, is used as a means of keeping a statement of each depositor's account. Such a record shows the opening balance (the amount the depositor has to his credit at the beginning of the period), the deposits made, the checks paid, and the closing balance.

If A, B, C, and D have accounts at a bank, the depositors' ledger might appear as follows:

DEPOSITOR	OPENING BALANCE		DEPOSITS		CHECKS		CLOSING BALANCE	
A	82		38		35		85	
B	94		56		32		118	
C	78		39		46		71	
D	49		63		28		84	
	303		196		141		358	

In finding the closing balance for each depositor the deposits are added to the opening balance and then the checks are subtracted. The results are set down in the column headed "Closing Balance," as shown by the black figures.

After all the closing balances have been figured, a page of the ledger may be checked by adding each one of the four columns and seeing if the sum of the opening balances plus the sum of the deposits minus the sum of the checks equals the sum of the closing balances, as shown above by the column totals in black type.

EXERCISES

Copy, complete the following, and check :

1.

DEPOSITOR	OPENING BALANCE		DEPOSITS		CHECKS		CLOSING BALANCE	
A	583		116		237			
B	335		254		286			
C	432		175		215			
D	112		56		87			
E	387		198		295			
F	952		105		784			
G	453		97		335			
H	82		111		76			
I	172		56		116			
J	285		73		180			
K	435		214		386			
L	123		28		54			
TOTALS								

2.

DEPOSITOR	OPENING BALANCE		DEPOSITS		CHECKS		CLOSING BALANCE	
A	396		288		287			
B	512		307		478			
C	473		126		364			
D	207		76		116			
E	445		213		387			
F	556		127		338			
G	764		67		236			
H	93		48		28			
I	140		25		70			
J	219		205		173			
K	147		200		102			
L	88		42		71			
TOTALS								

Ledger accounts. One of the important things in a business concern is keeping the records of amounts owed to the firm and of amounts which the firm owes. A **debit** is a record of money or other value received by the firm and a **credit** is a record of values parted with. A **ledger account** is a collection of related debits and credits which is kept in a record book called a **ledger**. In a simple form of bookkeeping one of the ledger accounts is the **Cash account**, which summarizes the transactions of the firm in which money has been received or spent, as shown below:

Cash

19- May 1	1	368 74	19- May 3	1	217 40
6	2	840 32	8	3	600 -
9	3	161 25	14	4	251 72
11	3	78 56	18	5	183 45
17	5	250 -	26	6	210 74
25	6	175 50	31	Balance	434 79
30	7	23 73			
		1898 10			1898 10
June 1	Balance	434 79			

CASH ACCOUNT

In the above account the first item, \$368.74, represents a sum received, and by custom all such items are entered on the left side. Together these items constitute the debits of the account. The first item, \$217.40, in the right-hand column represents money paid out, and together all the items on the right side constitute the credits of the account. In addition, the account records the dates of the items and the pages of the other record books where these items were first listed.

The **balance** of an account is the difference between the sum of the debits and the sum of the credits. After this difference

is found the balance is entered in its proper column, depending upon whether the debits or the credits is the greater, the page is ruled, and the balance is carried below the rulings, as shown in the model form.

Balances are usually entered in red ink above the rulings.

T account. The illustration at the right is called a **skeleton ledger account**, or a **T account**. In such an account only the amounts are entered; the

other details of a ledger account, the date and the page columns, are omitted. In the work in the following exercises this T-account form should be used.

CASH	
102.20	51.10
38.45	20.00
33.70	51.20
60.00	12.50
35.25	Balance 174.80
40.00	
<u>309.60</u>	<u>309.60</u>
Balance 174.80	

EXERCISES

Prepare T accounts from the following data. Balance and rule each account:

1. Cash received (debits): \$140.20, \$230.50, \$48.75, \$35.95, \$67.80, \$33.40, \$89.60, \$55.50, \$75.95. Cash paid (credits): \$70.10, \$200, \$51.40, \$75.30, \$25.

2. Cash received (debits): \$89.54, \$37.55, \$114.25, \$17.45, \$49.62, \$11.78, \$57.50, \$28.75, \$93.22. Cash paid (credits): \$75, \$25.45, \$100, \$51.50, \$15, \$38.75.

Personal accounts. The accounts which record the transactions that a firm has with individuals or other firms constitute another type of ledger accounts. Such accounts are called **personal accounts**.

For example, suppose that *Student*, as the proprietor of a business, sold merchandise to A. M. Benton, on account, as follows: March 3, \$186; March 8, \$68.45; March 11, \$156.20; March 13, \$121; March 19, \$175; March 27, \$206; March 31, \$110.40.

Suppose also that A. M. Benton paid cash, on account, as follows: March 12, \$186; March 15, \$68.45; March 20, \$156.20; March 22, \$121; March 29, \$175.

These transactions would appear in an account with A. M. Benton in *Student's* ledger as follows:

A. M. Benton

¹⁹⁻				¹⁹⁻			
March 3		2	186 -	March 12		4	186 -
8		2	68 45	15		5	68 45
11		3	156 20	20		5	156 20
13		4	121 -	22		6	121 -
19		5	175 -	29		8	175 -
27		7	206 -				
31		9	110 40				

PERSONAL ACCOUNT RECEIVABLE

In this account the debit side records the sums owed by A. M. Benton to *Student* for the goods which Benton bought, and the credit side records the sums paid by A. M. Benton to *Student* to settle the debts.

Accounts Receivable are personal accounts in which the debit side is the larger; that is, the person or firm owes the business.

Accounts Payable are personal accounts in which the credit side is the larger; that is, the business owes the person or firm.

Suppose that *Student*, in the above example, buys much of his merchandise from Marcy & Co. In *Student's* ledger there might appear the following personal account with Marcy & Co.:

Marcy & Co.

¹⁹⁻				¹⁹⁻			
Jan. 15		4	215 -	Jan. 5		2	215
20		5	116 25	11		3	116 25
25		6	165 80	15		4	165 80
29		8	162 10	19		4	162 10
				23		5	186 35
				28		7	235 -
				31		9	125 -

PERSONAL ACCOUNT PAYABLE

Since the credit side, which records the sums owed by *Student* to Marcy & Co. for goods purchased, is the larger, this account is an Account Payable. The debit side records the sums which *Student* has paid to Marcy & Co. to settle his debts.

EXERCISES

Prepare a T account for each of the following personal accounts. Balance and rule each account:

1. Sales to John M. Winslow (debits): \$126.50, \$58.24, \$36.75, \$41.55, \$17.60, \$38.45. Payments made (credits): \$63.25, \$29.12, \$18.38, \$20.78, \$63.25.

2. Sales to John S. Jackson (debits): \$76.45, \$66.74, \$112.35, \$98.60, \$49.65, \$37.52, \$88.40, \$39.70, \$108.75. Payments made (credits): \$38.23, \$33.37, \$56.18, \$49.30, \$24.83, \$18.76, \$38.22, \$56.17, \$25.

3. Sales to Henry S. Burton (debits): \$56.12, \$35.72, \$48.90, \$89.72, \$43.21, \$59.36, \$11.15, \$90, \$74.35. Payments made (credits): \$56.12, \$25, \$24.45, \$75, \$20, \$59.36, \$100.

4. Payments made to Loomis Bros. (debits): \$65, \$28.50, \$54.32, \$27.50, \$45.90, \$50, \$65.20, \$40.30. Purchases made (credits): Balance \$65, \$108.64, \$55, \$91.80, \$100, \$130.40, \$80.60, \$211.

Gross and net profit. When an article is sold for more than the cost, the difference between the cost and the selling price is the **gross profit** on that article. If the expense of doing business is subtracted from the gross profit, the result is the **net profit** on that article. In outline form,

selling price — cost = gross profit,

and gross profit — expense = net profit.

Thus, if an article which is sold for \$12 costs \$8, the gross profit on the article is \$12 — \$8, or \$4. If the proper share for the article of the expense of doing business is \$2.20, the net profit is \$4 — \$2.20, or \$1.80.

The gross and the net profits on the total operations of a business may be determined in a similar manner.

A TIMED REVIEW

1. Copy and complete the following page from a depositor's ledger:

DEPOSITOR	OPENING BALANCE		DEPOSITS		CHECKS		CLOSING BALANCE	
A	427	54	98	45	312	40		
B	86	34	212	57	192	11		
C	145	20	342	75	285	09		
D	553	65	108	52	413	27		
E	97	48	349	23	210	80		
F	177	66	142	11	78	53		
G	427	50	172	30	335	62		
H	116	75	239	76	158	25		
I	541	10	237	40	455	13		
J	332	05	178	90	206	34		
TOTALS								

The work should be completed in less than 30 min.

2. Complete the following departmental sales summary:

DEPARTMENTAL SALES SUMMARY										
DEPARTMENT	MERCHANDISE SALES		COST OF MDSE. SOLD		GROSS PROFIT		EXPENSES		NET PROFIT	
A	11782	16	7854	84			3102	54		
B	10245	25	6984	40			2566	30		
C	8753	45	6063	90			2190	45		
D	9154	02	6258	53			2392	60		
E	8623	65	6036	75			2155	75		
F	9876	48	6402	55			2634	18		
G	12562	75	8614	35			2956	45		
H	10458	50	6972	65			2641	25		
TOTALS										

The work should be completed in less than 25 min.

Reverse subtraction. In the usual form of subtraction the smaller number is written below the larger, but in the work of an office it often happens that a number has to be subtracted from a number written below it. In such cases it is convenient to subtract in reverse order; that is, to subtract each digit from the one below it.

For example, in stating the gross profit and the expenses of a business period the gross profit is given first and the expenses are given below. The net profit for the business period is found by subtracting the expenses from the gross profit. If the expenses exceed the gross profit, the larger number is then below, and the difference is, of course, a loss and not a profit. It is obvious that it is more convenient and quicker to subtract in reverse order than to write the numbers a second time simply to get the smaller number below the larger.

ORAL EXERCISES

In each of these cases state quickly the difference between the smaller number and the larger:

$$\begin{array}{r} 1. \$420 \\ 750 \\ \hline \end{array}$$

$$\begin{array}{r} 4. \$82 \\ 194 \\ \hline \end{array}$$

$$\begin{array}{r} 7. \$243 \\ 476 \\ \hline \end{array}$$

$$\begin{array}{r} 10. \$610 \\ 576 \\ \hline \end{array}$$

$$\begin{array}{r} 2. \$543 \\ 755 \\ \hline \end{array}$$

$$\begin{array}{r} 5. \$223 \\ 385 \\ \hline \end{array}$$

$$\begin{array}{r} 8. \$47 \\ 33 \\ \hline \end{array}$$

$$\begin{array}{r} 11. \$392 \\ 792 \\ \hline \end{array}$$

$$\begin{array}{r} 3. \$176 \\ 588 \\ \hline \end{array}$$

$$\begin{array}{r} 6. \$382 \\ 694 \\ \hline \end{array}$$

$$\begin{array}{r} 9. \$125 \\ 275 \\ \hline \end{array}$$

$$\begin{array}{r} 12. \$95.25 \\ 77.50 \\ \hline \end{array}$$

EXERCISES

In each of the following, without rewriting, find the difference between the smaller number and the larger, and check each result:

$$\begin{array}{r} 1. 4356 \\ 7432 \\ \hline \end{array}$$

$$\begin{array}{r} 3. 9825 \\ 7814 \\ \hline \end{array}$$

$$\begin{array}{r} 5. \$8216 \\ 9424 \\ \hline \end{array}$$

$$\begin{array}{r} 7. \$24,728 \\ 17,649 \\ \hline \end{array}$$

$$\begin{array}{r} 2. 9435 \\ 5369 \\ \hline \end{array}$$

$$\begin{array}{r} 4. 2583 \\ 5691 \\ \hline \end{array}$$

$$\begin{array}{r} 6. \$5311 \\ 8706 \\ \hline \end{array}$$

$$\begin{array}{r} 8. \$15,275 \\ 20,703 \\ \hline \end{array}$$

9. Gross Profit	\$9,318	11. Gross Profit	\$7,356.20
Expenses	<u>11,496</u>	Expenses	<u>10,373.45</u>
10. Gross Profit	\$16,542	12. Gross Profit	\$13,576.42
Expenses	<u>18,986</u>	Expenses	<u>17,389.55</u>

Complement. The difference between a number and 10, or some higher power of 10 (as 100, 1000, 10,000, and so on), is called the **complement** of that number.

For example, the complement of 3 is 7, of 8 is 2, of 70 is 30, of 48 is 52, of 175 is 825, of 4000 is 6000.

ORAL EXERCISES

State the complement of each of the following :

1.	2.	3.	4.	5.	6.
6	13	9	83	70	555
1	7	2	50	64	895
91	35	3	4	43	889
5	42	8	85	22	942

Complement method of subtraction. If instead of subtracting a number from a larger number, its complement is added to the larger number, the result is too large by some power of 10. If two complements are added, the result is too large by some multiple of 10 (as 20, 30, 200, 300, and so on).

Thus, if instead of subtracting 7 from 9, its complement, 3, is added to 9, the sum is 12, which is 10 too large. If instead of subtracting first 3 and then 2 from 9, the complements 7 and 8 are added to 9, the sum is 24, which is 20 too large.

Subtraction can thus be made a process of adding complements, provided that the proper power or multiple of 10 is discarded from the result.

EXAMPLES. 1. Use the complement method in subtracting 8 from 15.

Solution. The complement of 8 is 2. Then $15 + 2 = 17$, which is 10 too large. Hence the correct difference is 7.

2. Use the complement method in finding the value of the expression

$$21 - 5 + 8 - 3 + 6 - 8 - 2.$$

Solution. Here there are several numbers to be added and several to be subtracted. Wherever a number is to be subtracted set down its complement with the proper power of 10 as shown at the right. Then adding the left-hand column, the result is 57. But four complements have been added and hence the result is 40 too large. The correct value of the expression is therefore 17.

21
5 - 10
8
7 - 10
6
2 - 10
8 - 10
<u>57 - 40</u>

In business the need for finding the value of such an expression as the one in the second example above is rare. The principle of the complement method is used, however, a great deal where arithmetical operations are performed on adding machines.

EXERCISES

Using the complement method, find the value of each of the following expressions:

- | | | |
|---------------|------------------|------------------------------|
| 1. $45 - 6.$ | 5. $62 - 7 - 9.$ | 9. $78 - 6 + 7 - 9 - 2.$ |
| 2. $37 - 9.$ | 6. $74 - 3 - 4.$ | 10. $83 - 5 + 2 - 8 - 7.$ |
| 3. $52 - 39.$ | 7. $78 + 6 - 9.$ | 11. $84 - 9 + 17 - 27 - 21.$ |
| 4. $67 - 42.$ | 8. $64 - 7 - 4.$ | 12. $92 - 11 + 21 - 9 - 17.$ |

MULTIPLICATION

Multiplication. The process of taking (adding together) one of two numbers as many times as there are units in the other is called **multiplication**; that is, multiplication is, in effect, a short method of addition.

For example, $6 \times 4 = 24$, but the same result is obtained by adding $6 + 6 + 6 + 6$.

The number to be multiplied is called the **multiplicand**, and the number by which to multiply is called the **multiplier**. The result obtained by multiplying two numbers is called the **product**.

The term *multiplicand* is rarely used in business practice.

ORAL EXERCISES

1. Multiply each of the following numbers: (*a*) by 2; (*b*) by 5; (*c*) by 4; (*d*) by 6; (*e*) by 3. State only the products:

9	13	15	22	31	8	11	20
12	7	6	14	10	4	31	21
55	33	23	45	65	62	18	17

2. Multiply each of the following numbers: (*a*) by 2; (*b*) by 5; (*c*) by 6; (*d*) by 8. State only the products:

14	22	35	41	50	65	32	61
17	25	27	33	42	31	55	75
16	29	26	18	90	34	64	85

Short methods. Only those short methods of multiplication which have a practical value merit consideration here. In addition to the following methods, the student should always be on the alert for short and simple ways to save time in getting products:

1. *To multiply by 10, 100, 1000, and so on, annex to the number to be multiplied as many ciphers as there are at the right of the multiplier.*

Thus, $654 \times 10 = 6540$, and $4034 \times 1000 = 4,034,000$.

2. *To multiply by 11, multiply by 10 and add the number multiplied to the product.*

Thus, $451 \times 11 = 4510 + 451 = 4961$.

In multiplying a number of two figures by 11, the following rule is helpful: *For the product write the tens' figure of the number multiplied at the left, the units' figure at the right, and the sum of the two between them.*

Thus, in multiplying 36 by 11, write 396, the 3 and 6 being written first and 9, the sum of 3 and 6, being inserted between them.

In multiplying 75 by 11, the sum of 7 and 5 is 12, and hence the product should be written 825, adding the 1 (ten) to the 7 before setting it down.

3. *To multiply by 101, multiply by 100 and add the number multiplied to the product.*

Thus, $635 \times 101 = 63,500 + 635 = 64,135$.

This rule may be extended to cover multiplying by 102, 103, and so on by adding twice the number multiplied to the product, three times the number multiplied, and so on.

4. *To multiply by 99, multiply by 100 and subtract the number multiplied from the product.*

Thus, $67 \times 99 = 6700 - 67 = 6633$.

This rule also may be extended to multiplying by 98, 97, and so on by subtracting twice the number multiplied, three times the number multiplied, and so on.

ORAL EXERCISES

Multiply each of the following numbers by 10; by 100; by 1000:

1.	2.	3.	4.
629	6845	2463	6179
7364	786	809	603
8245	90	3156	2876
975	707	103	89
311	8181	8476	4472

EXERCISES

Multiply each of the following numbers by 11:

1. 34	56	82	107	242	671
2. 93	62	73	982	647	827
3. 44	77	96	263	731	941

Multiply each of the following numbers by 101:

4. 145	62	98	427	1215	1106
5. 103	1640	675	73	482	783
6. 206	303	121	4867	2811	1456

Multiply each of the following numbers by 99:

7. 140	83	842	250	405	643
8. 525	75	581	58	1244	1116
9. 535	19	99	186	4765	927

Checks in multiplication. The most practical check in multiplication is the check of 9's (see page 7). In applying this check, the principle is that the excess of 9's in the product to be checked should equal the excess of 9's in the product of the excesses of 9's in the numbers multiplied.

EXAMPLE. Multiply 458 by 38 and check by casting out the 9's.

<i>Solution.</i> 458	The excess of 9's in 458 is	8
38	The excess of 9's in 38 is	2
<u>3664</u>	The product of these excesses is	<u>16</u>
1374		
<u>17404</u>		

Since the excess of 9's in 17,404 is 7, and that in 16 is also 7, the work is probably correct.

EXERCISES

Perform the following multiplications and check each result:

- | | | |
|------------------------|-------------------------|--------------------------|
| 1. 2792×347 . | 5. 6453×999 . | 9. 8765×619 . |
| 2. 3806×230 . | 6. 4700×3980 . | 10. 3299×178 . |
| 3. 4785×617 . | 7. 8876×4275 . | 11. 6152×426 . |
| 4. 8293×721 . | 8. 9423×387 . | 12. 7934×4008 . |

The chief purpose of Exs. 1-12 above is to give practice in the use of the check of 9's. In the following exercises, in which only the results should be stated, emphasis should be placed upon rapid work and upon the application of short methods wherever possible.

ORAL EXERCISES

1. (a) Multiply each of the following numbers by 7, and to each product add 5:

9	12	5	14	21
7	22	8	11	15
6	20	13	31	16

(b) Multiply each number in (a) by 4, and to each product add 8.

2. (a) Multiply each of the following numbers by 5, and from each product subtract 7:

13	11	7	15	22
9	8	17	12	16
14	18	21	7	6

(b) Multiply each number in (a) by 3, and from each product subtract 11.

3. Multiply each of the following numbers by 11, and from each product subtract 8:

9	32	8	22	25	17	12
11	15	23	34	14	37	21

4. A woman bought one article for \$3, and two articles at \$1.25 each. She gave the clerk a ten-dollar bill. Count the correct change.

5. In each of the following purchases count the correct change if the clerk receives a ten-dollar bill:

2 articles at \$4 each	6 articles at \$1.10 each
3 articles at 75¢ each	9 articles at 50¢ each
11 articles at 81¢ each	12 articles at 40¢ each

6. In each of the following purchases count the correct change if the clerk receives a twenty-dollar bill:

6 articles at \$2.25 each	5 articles at \$3.80 each
4 articles at \$3.75 each	7 articles at \$1.25 each
8 articles at \$2.20 each	4 articles at \$2.90 each

7. In each case state the cost of the following:

14 bu. of apples at \$3 per bushel
8 bbl. of flour at \$7.50 per barrel
16 bu. of wheat at \$1.25 per bushel
50 bu. of oats at 50¢ per bushel

EXERCISES

1. The following columns represent the receipts of cattle, hogs, and sheep at a certain stockyard for six days:

CATTLE	Hogs	SHEEP
8,338	41,109	14,186
5,441	35,562	10,690
1,298	14,393	2,273
20,110	55,798	19,926
20,527	81,771	18,784
<u>9,800</u>	<u>28,354</u>	<u>9,470</u>

a. Without copying the figures, find the total number of cattle received.

b. If the average weight of the cattle was 1009 lb., and the value was 12¢ per pound, what was the total value of the cattle received?

The term "average" as used above means that each one of the cattle may be considered to weigh 1009 lb.

c. Find the total number of hogs received.

d. If the average weight of the hogs was 250 lb., and the value was 10¢ per pound, what was the total value of the hogs received?

e. Find the total number of sheep received.

f. If the average weight of the sheep was 80 lb., and the value was 9¢ per pound, what was the total value of the sheep received?

2. In the following find (*a*) the number of hours worked by the men in each of the four groups, and (*b*) the total number of hours for all the men:

11 men worked 7 hr. per day for 5 da.

12 men worked 4 hr. per day for 6 da.

21 men worked 8 hr. per day for 3 da.

15 men worked 6 hr. per day for 4 da.

3. In the following find (*a*) the amount earned by each of the eight groups, and (*b*) the total sum earned by all the groups:

8 men earned \$25 each	7 men earned \$26 each
11 men earned \$16 each	13 men earned \$23 each
9 men earned \$22 each	18 men earned \$12 each
12 men earned \$18 each	6 men earned \$29 each

4. A number of fields produced the following yields of corn. Find (*a*) the number of bushels produced by each field, (*b*) the total crop produced by all the fields, and (*c*) the total value of the crop at 65¢ per bushel:

8 A., 35 bu. per acre	12 A., 33 bu. per acre
15 A., 38 bu. per acre	24 A., 48 bu. per acre
9 A., 55 bu. per acre	7 A., 49 bu. per acre
18 A., 40 bu. per acre	21 A., 42 bu. per acre

MULTIPLICATION APPLIED TO BUSINESS PROBLEMS

Invoicing. An invoice is a detailed statement of goods bought or sold. The invoice should show a statement of the quantity, the price of each article listed, the total for each type of article, and the total for all the articles combined, as shown in the following form :

SYRACUSE, N. Y., MAY 4, 19—									
MR. WM. A. HORNER									
CANTON, OHIO									
Bought of LAKIN & SON									
323 SOUTH ST.									
	50	BU. BARLEY	90¢	45	—				
	75	" CORN	68¢	51	—				
	100	" OATS	57¢	57	—				
	80	" WHEAT	\$1.40	112	—				
	90	" RYE	85¢	76	50			341	50

INVOICE

Making the extensions. Filling in the totals for each item on an invoice is called **making the extensions**.

EXERCISES

1. Copy and complete this invoice of confectionery :

	6	1-lb. bx. Green Seal	@	74¢					
	12	1-lb. " Blue Seal		70¢					
	6	1-lb. " Milk Chocolate		75¢					
	10	1-lb. " Country Club		45¢					
	12	1-lb. " Kenmore		85¢					
	6	1-lb. " Epicure		85¢					
	15	1-lb. " Green Seal, special		70¢					

The prices in this invoice are given by the pound.

The student may supply a heading like the one in the model form for this invoice, but it is recommended that this be done in one exercise only.

Make the extensions and find the total of each of these invoices:

2.

20 lb. Rice	@	8¢			
12 " Sugar		7¢			
42 " Tea		35¢			
15 " Butter		45¢			
11 " Cheese		40¢			
6 " Dried Beef		70¢			
10 " Honey		52¢			
4 " Candy		65¢			
7 " Crackers		19¢			
6 doz. Eggs		48¢			

As stated before, unless specifically directed to copy, such exercises should be completed without copying.

3.

2 White Sulky Plows	@	\$83.50			
3 Blue Sulky Plows		80.25			
4 Herrick Stalk Cutters		43.50			
5 Jungle Stalk Cutters		41.25			
3 Green Sulky Plows		96.00			
2 Ideal Potato Planters		117.50			
6 Goodwin Steel Rakes		32.25			
5 Nuss Steel Rakes		32.00			
4 Victor Wood Rakes		28.50			
3 Jones 6-Fork Tedders		51.50			
2 Olds Corn Binders		163.50			
2 Young Grain Binders		175.00			

Inventories. An inventory is an itemized statement of the quantity and value of goods on hand. Inventories are usually taken at regular intervals and used in preparing statements setting forth the condition of the business.

A model inventory is shown on page 250.

EXERCISES

Make the extensions and record the totals in each of the following inventories:

QUANTITY	PRICE	EXTENSION	QUANTITY	PRICE	EXTENSION
1. 531 @	59¢		3. 23 @	\$2.50	
133	\$1.00		24	3.00	
18	1.50		24	3.00	
10	2.50		46	2.95	
155	1.00		29	2.50	
59	75¢		57	5.00	
63	1.00		48	5.00	
43	1.00		14	3.95	
12	2.25		4	6.50	
52	1.50		5	10.50	
20	30¢		22	1.75	
11	3.50		12	1.50	
185	25¢		11	2.00	
69	25¢		116	3.50	

QUANTITY	PRICE	EXTENSION	QUANTITY	PRICE	EXTENSION
2. 51 @	50¢		4. 12 @	\$10.50	
11	\$1.98		37	3.75	
57	1.00		10	1.50	
10	2.25		121	2.00	
189	50¢		15	2.50	
23	2.50		114	1.00	
143	25¢		161	1.00	
12	1.10		115	50¢	
46	25¢		336	25¢	
47	10¢		608	45¢	
36	15¢		767	90¢	
78	15¢		5	35¢	
50	19¢		115	25¢	
216	9¢		8	7.98	
142	25¢		536	25¢	
23	75¢		174	65¢	

A TIMED REVIEW

In each of these inventories make the extensions and record the totals:

QUANTITY	PRICE	EXTENSION	QUANTITY	PRICE	EXTENSION
1. 6 @	\$21.50		2. 12 @	\$18.75	
15	11.80		6	15.50	
5	26.40		11	14.00	
24	25.00		14	50.00	
32	37.50		8	27.50	
75	2.40		60	3.50	
11	17.50		9	18.00	
12	31.50		6	27.50	
8	112.50		4	125.00	
9	33.50		18	4.50	
33	14.40		10	47.80	
6	19.75		20	19.50	
18	25.10		12	25.50	
140	7.50		17	5.00	
8	23.75		3	33.50	
5	41.75		120	6.50	
23	23.50		13	13.00	
14	16.75		31	11.00	
9	73.50		5	75.00	
4	187.50		2	245.00	
13	15.50		25	25.00	
12	18.75		12	35.00	
28	10.75		78	1.50	
16	44.50		60	1.60	
22	14.75		32	2.50	
12	62.50		8	62.50	
4	135.50		3	122.50	
7	28.75		9	18.75	
16	75.00		17	1.25	

The extensions and totaling of each inventory should be completed and checked in approximately 30 min.

DIVISION

Division. The process of finding the number of times that one number is contained in another is called **division**. The number which is to be divided is called the **dividend**; the number by which the division is made is called the **divisor**; and the result is called the **quotient**. If the divisor is not contained an exact number of times, the number left over is called the **remainder**.

The terms *dividend* and *quotient* are rarely used in business practice.

Short division. When the divisor is small, 12 or less, the work is often arranged as shown below.

For example, in dividing 2568 by 8, the work may be arranged as follows:

$$\begin{array}{r} 8 \overline{)2568} \\ \underline{321} \end{array}$$

ORAL EXERCISES

1. Divide each of the following numbers by 2; by 3:

6	18	36	60	72	90	108	132
12	30	42	66	84	96	126	144

2. Divide each of the following numbers by 4:

16 yd.	76 ft.	68 in.	72 mi.	24 rd.	104 sq. in.
20 oz.	48 lb.	36 pt.	88 qt.	52 gal.	124 T.

3. Divide each of the following numbers by 6:

24 oz.	12 lb.	60 bu.	90 pk.	30 gal.	114 min.
36 ft.	78 mi.	84 in.	102 rd.	96 sq. mi.	126 yd.

State the result quickly for each of the following:

- If 5 T. of coal cost \$65.90, what is the cost per ton?
- If six loads of grain weigh 8700 lb., what is the weight per load?
- If 8 lb. of meat cost \$1.76, what is the cost per pound?
- If eight men, at the same rate of wages, earn \$104, what does each man earn?
- If 12 doz. eggs cost \$5.40, what is the cost per dozen?

Long division. When the work in division is more extended and the various steps are set down as shown in the example below, the process is called **long division**.

EXAMPLE. Divide 2931 by 172.

Solution. In long division the result is placed above the number to be divided, and the steps in the division are put below.

In this case, when the division is completed there is a remainder of 7. The result may be written as 17, remainder 7, or in the form $17\frac{7}{172}$, which shows that the 7 is still to be divided by 172.

$$\begin{array}{r} 17 \\ 172 \overline{) 2931} \\ \underline{172} \\ 1211 \\ \underline{1204} \\ 7 \end{array}$$

Checks in division. 1. The most simple check of the work of division is as follows:

When there is no remainder, the work is correct if the result multiplied by the divisor equals the number divided. If there is a remainder, it should be added to the product of divisor and result before comparing with the number divided.

Thus, in the above division, $172 \times 17 + 7 = 2924 + 7 = 2931$. Since the number divided was 2931, the work is correct.

2. Another check of the work of division may be applied as follows:

If the excess of 9's in the product of the excesses of 9's in divisor and result increased by the excess of 9's in the remainder (if any) equals the excess of 9's in the number divided, the work is probably correct.

EXAMPLE. Check the above division by casting out the 9's.

Solution

The excess of 9's in 172 is	1	The excess of 9's in 8 is	8
The excess of 9's in 17 is	8	The excess of 9's in 7 is	7
The product of these excesses is	8	The sum of these excesses is	15

It is readily seen that the excess of 9's in 15 is 6, and that the excess of 9's in 2931 is also 6. Hence the work is probably correct.

The student should consult the note on casting out the 9's on page 8.

EXERCISES

Perform each of the following indicated divisions, and check Exs. 1-10 by the first method (page 47) and Exs. 11-24 by casting out the 9's:

- | | | |
|------------------------|-------------------------------|-------------------------------|
| 1. $4183 \div 74$. | 9. $\$11,857 \div 93$. | 17. $\$47,725 \div 49$. |
| 2. $6766 \div 31$. | 10. $\$229,870 \div 254$. | 18. $\$17,366 \div 47$. |
| 3. $6831 \div 112$. | 11. $\$675,250 \div 125$. | 19. $\$28,957 \div 406$. |
| 4. $7633 \div 32$. | 12. $\$136,939 \div 57$. | 20. $\$363,028 \div 147$. |
| 5. $48,683 \div 106$. | 13. $\$207,009 \div 208$. | 21. $\$130,229 \div 288$. |
| 6. $33,025 \div 126$. | 14. $\$1,438,209 \div 136$. | 22. $\$2,457,307 \div 252$. |
| 7. $70,513 \div 170$. | 15. $\$3,476,253 \div 1046$. | 23. $\$6,535,280 \div 6040$. |
| 8. $94,207 \div 142$. | 16. $\$4,523,527 \div 2560$. | 24. $\$8,299,850 \div 650$. |

Dividing by powers and multiples of 10. In dividing a number by a power or multiple of 10, the following points should be observed:

1. *In dividing by 10, 100, 1000, and so on, cut off as many figures, beginning with the units' figure, of the number to be divided as there are ciphers at the right of the divisor. The figures cut off in the number to be divided which are not zeros form the remainder.*

For example, consider the following cases:

$$240 \div 10 = 24\cancel{0} + 1\cancel{0} = 24.$$

$$2471 \div 100 = 24\cancel{71} + 1\cancel{00} = 24\frac{71}{100}.$$

$$42,039 \div 1000 = 42,\cancel{039} + 1\cancel{000} = 42\frac{39}{1000}.$$

2. *In dividing by 20, 200, 30, 300, and so on, cut off the figures, beginning with the units' figure, of the number to be divided, as above, and then divide what is left in the number to be divided by the figure (or figures) in the divisor to the left of the ciphers.*

For example, consider the following cases:

$$480 \div 20 = 48\cancel{0} \div 2\cancel{0} = 24.$$

$$44,000 \div 1100 = 44,\cancel{000} \div 11\cancel{00} = 40.$$

$$4879 \div 300 = 48\cancel{79} \div 3\cancel{00} = 16\frac{16}{300}.$$

ORAL EXERCISES

1. Divide each of these numbers by 10: 460; 590; 420; 5281; 1377; 1639.

2. Divide each of these numbers by 100: 3200; 5300; 7700; 15,873; 16,127; 19,831.

3. Divide each of these numbers by 1000: 65,000; 37,000; 43,000; 176,271; 139,387; 144,167.

4. Divide each of these numbers by 20: 560; 240; 480; 167; 289.

5. Divide each of these numbers by 200: 3800; 4400; 2833; 1247; 1653.

6. Divide each of these numbers by 30: 810; 750; 721; 331; 189.

7. Divide each of these numbers by 300: 8700; 3900; 6329 5157.

DIVISION APPLIED TO BUSINESS PROBLEMS

Average. The process of adding two or more numbers and dividing the sum by the number of items added is called finding the **average** of the numbers.

For example, the sum of 4, 6, 8, and 14 is 32; the average is found by dividing 32 by 4; that is, the average of 4, 6, 8, and 14 is 8.

Averages are much used in business, in such cases as finding the average sales of a number of clerks, the average weight of a number of different packages, and so on. Some illustrations of the use of averages are given in the following exercises.

EXERCISES

1. A merchant's sales for a week were as follows: Monday, \$360; Tuesday, \$240; Wednesday, \$330; Thursday, \$420; Friday, \$300; Saturday, \$450. What were his average daily sales for the 6 da.?

2. In a school each one of the grades had the following enrollment: 1st, 38; 2d, 32; 3d, 30; 4th, 31; 5th, 29; 6th, 28; 7th, 27; 8th, 25. What was the average enrollment of the grades?

3. In a factory ten men received the following amounts in one week: \$32, \$33, \$30, \$31, \$35, \$38, \$34, \$37, \$35, and \$35.

- a. What was their average weekly wage?
- b. How many men received the average wage?
- c. How many received more than the average wage?
- d. How many men received less than the average wage?

4. A dairyman sold eight head of cattle at the following prices: \$350, \$250, \$375, \$225, \$410, \$390, \$400, and \$600.

- a. What was their average selling price?
- b. For how much above the average was the highest-priced animal sold?
- c. For how much below the average was the lowest-priced animal sold?

5. The circulation of a daily newspaper during 6 da. of one week was as follows: 21,250, 21,350, 22,200, 22,400, 22,600, and 21,900. What was the average daily circulation?

6. The following items represent the expenses of a family of four persons for a period of 6 mo.: rent, \$180; food, \$326; clothing, \$194; books and periodicals, \$12; fuel, \$75; sundries, \$32. Find (a) the total expenses for 6 mo.; (b) the average monthly expenses; (c) the average expenses of each member of the family per month.

7. The following items represent the production of grain and hay on a certain farm:

- 35 A. produced 1680 bu. of corn.
- 16 A. produced 672 bu. of oats.
- 12 A. of meadow produced 24 T. of hay.

Find (a) the average yield of each crop per acre, and (b) the value of the yield per acre if corn is worth 65¢ per bushel, oats 50¢ per bushel, and hay \$24 per ton.

8. A man earns \$255 per month. If his expenses average \$132 per month, how many months will it take him to save \$2583 if he has steady employment?

9. A contractor pays wages to a group of men as follows:

- 8 men receive \$256 in a week.
- 12 men receive \$288 in a week.
- 5 men receive \$175 in a week.
- 9 men receive \$207 in a week.
- 6 men receive \$114 in a week.
- 5 men receive \$130 in a week.

Find (a) the average wage per man for each group, and (b) the average wage per man for the total of all the groups.

10. The weight of each player on two rival high-school football teams was as follows:

- Team A: 132 lb., 140 lb., 127 lb., 139 lb., 125 lb.,
133 lb., 137 lb., 132 lb., 136 lb., 129 lb., 133 lb.
- Team B: 124 lb., 135 lb., 130 lb., 136 lb., 129 lb.,
125 lb., 122 lb., 132 lb., 140 lb., 142 lb., 137 lb.

What was the difference in the average weight of the teams?

11. The table below records the wealth produced on the farms of the United States in two separate ten-year periods:

FIRST PERIOD	SECOND PERIOD
\$4 717 070 000	\$9 037 391 000
5 009 595 000	8 819 175 000
5 302 120 000	9 342 790 000
5 594 645 000	9 849 513 000
5 887 170 000	9 894 961 000
6 121 778 000	10 774 491 000
6 273 997 000	13 406 364 000
6 764 210 000	19 331 000 000
7 487 989 000	22 479 000 000
7 890 626 000	24 982 000 000

Without copying the figures, find the average wealth produced per year in each period.

A TIMED REVIEW

1. (a) Complete the following weekly sales report:

WEEKLY SALES REPORT													
DAY	DEPT. A		DEPT. B		DEPT. C		DEPT. D		DEPT. E		DEPT. F		TOTALS
1	50	29	108	03	42	91	87	08	27	47	80	09	
2	65	93	142	74	47	06	59	57	30	35	60	36	
3	30	43	70	19	45	03	79	88	38	64	51	86	
4	35	08	106	80	23	45	164	64	31	73	63	90	
5	52	22	97	04	56	13	69	80	18	69	66	57	
6	224	70	198	77	247	74	170	47	89	11	138	05	
TOTALS													

(b) Find the average daily sales of each department.

(c) Find the average daily sales for the whole store.

The work should be completed in approximately 15 min.

2. (a) Complete the following departmental sales summary:

DEPARTMENTAL SALES SUMMARY									
DEPARTMENT	MERCHANDISE SALES		COST OF MDSE. SOLD		GROSS PROFIT		EXPENSES		NET PROFIT
1	15 783	30	10 259	20			4577	12	
2	18 156	37	12 138	70			4786	42	
3	20 432	58	13 586	25			5824	70	
4	17 952	68	12 208	04			5044	64	
5	13 864	38	9 186	70			3845	84	
6	15 263	46	10 406	53			4093	78	
7	12 263	46	8 420	75			3187	20	
8	13 897	06	8 534	06			4529	18	
TOTALS									

(b) Find the average gross profit of all departments.

(c) Find the average net profit of all departments.

The work should be completed in approximate'y 30 min.

CHAPTER II

FRACTIONS

DIVISORS AND FACTORS

Divisor. If one number is contained in another a certain number of times with no remainder, the first number is called an **integral divisor**, or simply a **divisor**, of the second.

For example, 2 is a divisor of 12, and 5 is a divisor of 20; but 3 is not a divisor of 10.

Divisibility of numbers. The following tests for determining whether or not one number is divisible by another are of practical importance:

1. *A number is divisible by 2 when its right-hand digit is even.*

That is, its right-hand digit must be 0, 2, 4, 6, or 8.

2. *A number is divisible by 3 when the sum of its digits is divisible by 3.*

3. *A number is divisible by 4 when the number expressed by the two right-hand digits is divisible by 4.*

4. *A number is divisible by 5 when its right-hand digit is 0 or 5.*

5. *A number is divisible by 6 when it is even and the sum of its digits is divisible by 3.*

That is, it must be divisible by both 2 and 3.

6. *A number is divisible by 8 when the number expressed by the three right-hand digits is divisible by 8.*

7. *A number is divisible by 9 when the sum of its digits is divisible by 9.*

8. *A number is divisible by 10 when its right-hand digit is 0.*

ORAL EXERCISES

State the numbers by which each of the following is divisible :

- | | | | |
|---------|---------|----------|-----------|
| 1. 26. | 5. 252. | 9. 242. | 13. 1232. |
| 2. 141. | 6. 395. | 10. 195. | 14. 3213. |
| 3. 102. | 7. 792. | 11. 328. | 15. 2742. |
| 4. 536. | 8. 654. | 12. 356. | 16. 3720. |

Factors. The **factors** of a number are the integral divisors of that number.

For example, 2, 3, and 5 are factors of 30 ; 2, 4, and 7 are factors of 28 ; but 5 is not a factor of 21, since 5 is not an integral divisor of it.

Prime number. A **prime number** has no factor except itself and 1. The prime numbers which are factors of a given number are called the **prime factors** of that number.

Thus, 3, 7, and 11 are prime numbers, and the prime factors of 231.

Factoring. The process of finding the prime factors of a number is called **factoring**.

EXAMPLE. Find the prime factors of 2220.

Solution. Since the right-hand digit of 2220 is 0, divide by 5, obtaining 444. Since 444 is an even number, divide by 2, obtaining 222. Since 222 is also an even number, divide again by 2, obtaining 111.

Since the sum of the digits in 111 is 3, divide by 3, obtaining the prime number 37. Hence the prime factors of 2220 are 5, 2, 2, 3, and 37.

Check. The work is checked by finding the product of the factors ; that is, $5 \times 2 \times 2 \times 3 \times 37 = 2220$.

5	)2220
2	)444
2	)222
3	)111
	37

EXERCISES

Find the prime factors in each of the following numbers :

- | | | | | |
|---------|---------|----------|----------|-----------|
| 1. 45. | 5. 170. | 9. 784. | 13. 156. | 17. 840. |
| 2. 92. | 6. 255. | 10. 126. | 14. 770. | 18. 918. |
| 3. 231. | 7. 324. | 11. 252. | 15. 492. | 19. 1248. |
| 4. 342. | 8. 462. | 12. 575. | 16. 819. | 20. 2320. |

Cancellation. Division may be simplified by writing the expression to be divided above a line and the divisor below it, and then rejecting the equal factors, factor for factor, from the expressions above and below the line. This process is called **cancellation**.

EXAMPLE. Divide $28 \times 8 \times 5 \times 9$ by $35 \times 24 \times 6 \times 15$.

$$\text{Solution. } \frac{28 \times 8 \times 5 \times 9}{35 \times 24 \times 6 \times 15} = \frac{\overset{2}{\cancel{28}} \times \overset{4}{\cancel{8}} \times \cancel{5} \times \overset{3}{\cancel{9}}}{\underset{5}{\cancel{35}} \times \underset{3}{\cancel{24}} \times \underset{3}{\cancel{6}} \times \underset{3}{\cancel{15}}} = \frac{2}{15}.$$

In this solution the factors are canceled as follows: 7 from 28 and 35, 8 from 8 and 24, 5 from 5 and 15, and so on, until all equal factors have been canceled. The result is the product of the uncanceled factors above the line divided by the product of those below the line; in this case, $\frac{2}{15}$.

EXERCISES

1. Divide $12 \times 9 \times 15$ by $18 \times 5 \times 3$.
2. Divide $25 \times 35 \times 36$ by $15 \times 14 \times 16$.
3. Divide $18 \times 27 \times 38$ by $14 \times 57 \times 36$.
4. $(126 \times 54 \times 42) \div (36 \times 35) = ?$
5. $(36 \times 640) \div (15 \times 24 \times 16) = ?$
6. $(300 \times 5280) \div (250 \times 24) = ?$
7. If it requires 18 da. at 175 mi. per day to travel a certain distance by automobile, how many miles per day must be traveled in order to cover the distance in 14 da.?
8. If it requires 16 da. of 10 hr. each for 22 men to complete a job, how long will it take 18 men working an 8-hour day to do the same job?
9. How many small packages of 24 sheets each can be made from 60 pkg. of paper, 500 sheets to the package?
10. How long will 15 bu. of oats feed eight horses if each horse is fed three times per day and 2 qt. are fed each time? (1 bu. = 32 qt.)

COMMON FRACTIONS

Fraction. A number which indicates that one or more equal parts of anything are taken is called a **common fraction**, or more usually a **fraction**. It is expressed by two numbers, one above and one below a line, as $\frac{5}{6}$.

The form $5/6$ is also used rather commonly in business.

The number below the line is called the **denominator** and shows the size of the parts considered. The number above the line is called the **numerator** and shows the number of parts considered. The numerator and denominator are the **terms** of the fraction.

Thus, in $\frac{5}{6}$ the denominator is 6, which shows that something divided into six equal parts is considered. The numerator is 5, which shows that five of the six equal parts are under consideration.

A fraction is also an indicated division; that is, $\frac{5}{6}$ is equivalent to the expression $5 \div 6$.

Classes of fractions. A fraction whose numerator is less than its denominator, as $\frac{1}{2}$, $\frac{3}{4}$, or $\frac{7}{8}$, is called a **proper fraction**. One whose numerator is equal to or greater than its denominator, as $\frac{3}{3}$, $\frac{5}{2}$, or $\frac{9}{4}$, is called an **improper fraction**.

Mixed number. A number which is the sum of a whole number and a fraction, as $5\frac{1}{2}$, $3\frac{2}{5}$, or $12\frac{1}{3}$, is called a **mixed number**.

Reduction of fractions. If both numerator and denominator of a fraction have one or more common factors, the fraction may be **reduced to lower terms** by dividing each term by a factor common to both. When the terms cannot be further divided by the same factor, the fraction is said to be in **lowest terms**. Since equal factors have been removed in each division, the value of the fraction is not changed.

EXAMPLE. Reduce $\frac{12}{30}$ to lowest terms.

Solution. 1.
$$\frac{12}{30} = \frac{2 \times \cancel{2} \times \cancel{3}}{\cancel{2} \times \cancel{3} \times 5} = \frac{2}{5}.$$

2. By inspection, $\frac{12}{30} = \frac{2}{5} = \frac{2}{5}.$

The second method, in which the obvious factor to be removed is selected by inspection, is the one most commonly used.

The value of a fraction is not changed when each term is multiplied by the same number. In this case the fraction is **reduced to higher terms**.

Thus, $\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}$; or, by inspection, $\frac{2}{3} = \frac{10}{15}$.

EXERCISES

Reduce to lowest terms:

- | | | | | |
|-------------------------------|-------------------------------|-------------------------------|---------------------------------|---|
| 1. $\frac{6}{8}$. | 4. $\frac{2\frac{1}{4}}{4}$. | 7. $\frac{3\frac{6}{8}}{2}$. | 10. $\frac{4\frac{5}{5}}{5}$. | 13. $\frac{2\frac{5}{16}}{6}$. |
| 2. $\frac{1\frac{8}{2}}{4}$. | 5. $\frac{1\frac{6}{2}}{0}$. | 8. $\frac{2\frac{8}{3}}{6}$. | 11. $\frac{1\frac{2}{12}}{0}$. | 14. $\frac{2\frac{5}{20}}{0\frac{2}{0}0}$. |
| 3. $\frac{2\frac{4}{3}}{0}$. | 6. $\frac{3\frac{0}{5}}{0}$. | 9. $\frac{1\frac{6}{2}}{4}$. | 12. $\frac{7\frac{5}{12}}{5}$. | 15. $\frac{8\frac{2}{5}0\frac{0}{8}0}{0}$. |

Multiply both terms of each fraction as indicated:

- | | | |
|-------------------------|-------------------------|------------------------------------|
| 16. $\frac{5}{6}$ by 2. | 19. $\frac{3}{8}$ by 5. | 22. $\frac{1\frac{1}{6}}{1}$ by 3. |
| 17. $\frac{3}{4}$ by 3. | 20. $\frac{2}{6}$ by 6. | 23. $\frac{5}{36}$ by 2. |
| 18. $\frac{5}{8}$ by 3. | 21. $\frac{5}{6}$ by 5. | 24. $\frac{5}{16}$ by 3. |

25. Reduce $\frac{1}{2}$ to sixths; to twelfths; to twenty-fourths.

26. Reduce to twelfths: $\frac{1}{3}$; $\frac{3}{4}$; $\frac{5}{6}$.

27. Reduce to tenths: $\frac{1}{2}$; $\frac{2}{5}$; $\frac{4}{5}$.

28. Reduce to twenty-fourths: $\frac{3}{8}$; $\frac{5}{6}$; $\frac{3}{4}$; $\frac{7}{12}$.

29. Reduce to thirty-seconds: $\frac{5}{8}$; $\frac{3}{4}$; $\frac{1\frac{5}{16}}{1}$; $\frac{7}{32}$.

30. Express each of the following as an equivalent fraction with the denominator 100: $\frac{2}{5}$; $\frac{1}{4}$; $\frac{1\frac{2}{10}}{20}$; $\frac{7}{50}$; $\frac{3}{4}$; $\frac{1\frac{1}{2}}{20}$; $\frac{6}{25}$; $\frac{9}{10}$.

31. Express each of the following as 16ths of an inch: $\frac{1}{2}$ in.; $\frac{3}{8}$ in.; $\frac{1}{4}$ in.; $\frac{3}{4}$ in.; $\frac{7}{8}$ in.

Reduction of whole and mixed numbers to improper fractions.
Any whole number may be written as an improper fraction with any desired denominator.

Thus, $2 = \frac{2}{1} = \frac{2 \times 3}{1 \times 3} = \frac{6}{3} = \frac{12}{6}$, $5 = \frac{5}{1} = \frac{10}{2}$, and so on.

To write a mixed number as an improper fraction, multiply the whole number by the denominator of the fractional part,

add the numerator of the fraction to the product, and write the result over the denominator of the fractional part.

$$\text{Thus, } 3\frac{1}{2} = \frac{(3 \times 2) + 1}{2} = \frac{7}{2}, \text{ and so on.}$$

EXERCISES

1. Express as halves: 5; 12; 21; 50.
2. Express as thirds: 3; 10; 20.
3. Express as fourths: 2; 5; 10; 16.
4. Express as tenths: 7; 9; 11; 27; 45.

Express each of the following as an improper fraction:

5. $1\frac{2}{6}$; $3\frac{3}{10}$; $1\frac{1}{2}$; 4.
6. $5\frac{5}{8}$; $9\frac{15}{16}$; $25\frac{7}{8}$; $7\frac{1}{4}$.
7. $2\frac{2}{8}$; $5\frac{1}{12}$; $4\frac{3}{8}$; $6\frac{1}{2}$; 2.
8. $5\frac{7}{12}$; $10\frac{3}{8}$; $9\frac{1}{4}$; 7; $4\frac{3}{16}$.
9. $3\frac{5}{8}$; $11\frac{3}{4}$; $15\frac{7}{16}$; $26\frac{9}{16}$; $42\frac{7}{16}$.
10. $15\frac{5}{8}$; $25\frac{3}{16}$; $19\frac{17}{20}$; $26\frac{15}{32}$; $18\frac{23}{60}$.

Reduction of improper fractions to whole or mixed numbers.

Since a fraction is an indicated division, an improper fraction may be written as an equivalent whole or mixed number by dividing the numerator by the denominator, expressing the remainder (if any) as a fraction in lowest terms.

$$\text{Thus, } \frac{105}{15} = 7, \text{ and } \frac{38}{4} = 9\frac{2}{4} = 9\frac{1}{2}.$$

EXERCISES

Express each of the following as a whole or mixed number:

1. $\frac{24}{2}$; $\frac{32}{3}$; $\frac{122}{4}$; $\frac{96}{5}$; $\frac{78}{6}$.
2. $\frac{35}{8}$; $\frac{96}{12}$; $\frac{112}{16}$; $\frac{150}{32}$; $\frac{175}{60}$.

Express each of the following as an integral or mixed expression:

3. $\frac{11}{8}$ yd.; $\frac{75}{12}$ ft.; $\frac{568}{320}$ mi.; $\frac{9460}{5280}$ mi.
4. $\frac{720}{60}$ min.; $\frac{5760}{60}$ min.; $\frac{4308}{60}$ hr.; $\frac{563}{24}$ da.
5. $\frac{46}{9}$ sq. yd.; $\frac{763}{160}$ A.; $\frac{376}{27}$ cu. yd.; $\frac{19284}{1728}$ cu. ft.
6. $\frac{569}{16}$ lb.; $\frac{9280}{2000}$ T.; $\frac{12350}{2240}$ T.; $\frac{1800}{640}$ sq. mi.

ADDITION OF FRACTIONS

Adding similar fractions. Fractions which have equal denominators are called **similar fractions**. If fractions are similar, they can be added by adding the numerators and writing the result over the common denominator. The resulting fraction should be reduced to its lowest terms, to a whole number, or to a mixed number with its fractional part in lowest terms.

Thus, $\frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$, and $\frac{7}{12} + \frac{1}{2} = \frac{7}{12} + \frac{6}{12} = \frac{13}{12} = 1\frac{1}{12} = 1\frac{1}{12}$.

ORAL EXERCISES

Add the following:

1. $\frac{3}{4}$, $\frac{1}{4}$, and $\frac{1}{4}$.

6. $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{6}{5}$, and $\frac{2}{5}$.

2. $\frac{1}{6}$, $\frac{5}{6}$, and $\frac{1}{6}$.

7. $\frac{11}{16}$, $\frac{9}{16}$, $\frac{5}{16}$, and $\frac{3}{16}$.

3. $\frac{1}{8}$, $\frac{3}{8}$, $\frac{7}{8}$, and $\frac{5}{8}$.

8. $\frac{7}{24}$, $\frac{5}{24}$, $\frac{11}{24}$, and $\frac{17}{24}$.

4. $\frac{7}{10}$, $\frac{9}{10}$, $\frac{3}{10}$, and $\frac{1}{10}$.

9. $\frac{5}{24}$, $\frac{13}{24}$, $\frac{11}{24}$, and $\frac{1}{24}$.

5. $\frac{5}{12}$, $\frac{3}{12}$, $\frac{1}{12}$, and $\frac{7}{12}$.

10. $\frac{1}{32}$, $\frac{5}{32}$, $\frac{27}{32}$, and $\frac{19}{32}$.

Common multiple. A multiple of a number is the product of that number and some whole number.

Thus, 45 is a multiple of 5, since $5 \times 9 = 45$. It is also a multiple of 9, since $9 \times 5 = 45$; or of 15, since $15 \times 3 = 45$.

A **common multiple** of two or more numbers is a multiple of each of them.

Thus, 45 is a common multiple of 15, 9, and 5, since $45 = 15 \times 3$, or $45 = 9 \times 5$.

ORAL EXERCISES

1. State several multiples of 5; of 8; of 10; of 16; of 24.

2. State several common multiples of 4 and 5; of 6 and 4; of 6 and 9; of 5 and 15.

3. Is 40 a common multiple of 4 and 5? Is there a less number which contains both of them? What is it?

4. Is 120 a common multiple of 10 and 12? Is there a less number which contains both 10 and 12? What is it?

5. What number less than 48 contains both 6 and 8?

Least common multiple. The least number which is a common multiple of two or more numbers is called their **least common multiple (l.c.m.)**.

Thus, while 48 is a common multiple of 6, 8, and 3, their least common multiple is 24.

The l.c.m. of two or more numbers is found by separating each number into its prime factors and then finding the product of the different factors, using each the greatest number of times that it occurs in any one of the numbers.

EXAMPLE. Find the l.c.m. of 12, 30, and 20.

Solution. $12 = 2 \times 2 \times 3.$

$30 = 2 \times 3 \times 5.$

$20 = 2 \times 2 \times 5.$

Hence the l.c.m. $= 2 \times 2 \times 3 \times 5 = 60.$

EXERCISES

In each of the following find the l.c.m.:

1. 4 and 6.

6. 60, 75, and 90.

2. 10 and 15.

7. 45, 60, and 80.

3. 12, 15, and 20.

8. 160, 200, and 240.

4. 12, 16, and 18.

9. 180, 240, and 320.

5. 20, 30, and 50.

10. 125, 225, and 425.

Least common denominator. If the common denominator of two or more fractions is the least common multiple of all the denominators, the fractions have the **least common denominator (l.c.d.)**. Fractions which do not have a common denominator may be changed to fractions which have the l.c.d. by finding the l.c.m. of the denominators, and then changing each fraction to an equivalent fraction with this denominator.

EXAMPLE. Change $\frac{3}{10}$ and $\frac{4}{25}$ to fractions which have the l.c.d.

Solution. The l.c.m. of 10 and 25 is 50.

Then $\frac{3}{10} = \frac{3}{10} \times \frac{5}{5} = \frac{15}{50}$ and $\frac{4}{25} = \frac{4}{25} \times \frac{2}{2} = \frac{8}{50}.$

Hence $\frac{3}{10}$ and $\frac{4}{25}$ expressed with the l.c.d. are $\frac{15}{50}$ and $\frac{8}{50}$ respectively.

EXERCISES

Express as fractions which have the l.c.d.:

1. $\frac{1}{4}$, $\frac{3}{8}$, and $\frac{5}{6}$.
2. $\frac{2}{5}$, $\frac{4}{15}$, and $\frac{3}{10}$.
3. $\frac{2}{9}$, $\frac{5}{6}$, and $\frac{1}{12}$.
4. $\frac{5}{3}$, $\frac{3}{4}$, and $\frac{7}{16}$.
5. $\frac{3}{5}$, $\frac{5}{12}$, and $\frac{1}{30}$.
6. $\frac{3}{4}$, $\frac{2}{8}$, $\frac{5}{16}$, and $\frac{7}{32}$.
7. $\frac{3}{4}$, $\frac{2}{6}$, $\frac{5}{12}$, and $\frac{7}{24}$.
8. $\frac{4}{3}$, $\frac{2}{9}$, $\frac{2}{15}$, and $\frac{8}{45}$.
9. $\frac{5}{8}$, $\frac{11}{16}$, $\frac{13}{32}$, and $\frac{25}{64}$.
10. $\frac{5}{16}$, $\frac{13}{32}$, $\frac{15}{8}$, and $\frac{3}{4}$.
11. $\frac{3}{16}$, $\frac{15}{64}$, $\frac{15}{32}$, and $\frac{5}{8}$.
12. $\frac{5}{6}$, $\frac{7}{12}$, $\frac{29}{30}$, and $\frac{7}{30}$.

Adding fractions. If two or more fractions do not have a common denominator, the fractions must be expressed as similar fractions before they can be added.

EXAMPLE. Add $\frac{1}{5}$, $\frac{3}{10}$, and $\frac{7}{8}$.

Solution. The l.c.d. is 40, since the l.c.m. of 5, 10, and 8 is 40.

Then $\frac{1}{5} = \frac{8}{40}$, $\frac{3}{10} = \frac{12}{40}$, $\frac{7}{8} = \frac{35}{40}$.

Hence $\frac{8}{40} + \frac{12}{40} + \frac{35}{40} = \frac{55}{40} = 1\frac{11}{8} = 1\frac{3}{2}$.

EXERCISES

Add the following:

1. $\frac{2}{3}$ and $\frac{5}{6}$.
2. $\frac{3}{4}$ and $\frac{7}{8}$.
3. $\frac{2}{5}$ and $\frac{2}{3}$.
4. $\frac{5}{6}$ and $\frac{7}{8}$.
5. $\frac{1}{2}$, $\frac{4}{5}$, and $\frac{2}{3}$.
6. $\frac{1}{3}$, $\frac{5}{6}$, and $\frac{3}{8}$.
7. $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$.
8. $\frac{2}{3}$, $\frac{1}{5}$, and $\frac{5}{6}$.
9. $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{9}{10}$.
10. $\frac{2}{3}$, $\frac{5}{8}$, and $\frac{7}{12}$.
11. $\frac{1}{2}$, $\frac{5}{6}$, and $\frac{3}{10}$.
12. $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{3}{10}$.
13. $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{1}{6}$.
14. $\frac{3}{10}$, $\frac{7}{8}$, and $\frac{5}{6}$.
15. $\frac{7}{12}$, $\frac{3}{4}$, and $\frac{5}{8}$.

Addition of common business fractions. Fractions which are used most commonly in business have small denominators. With a little practice such fractions can be added almost as rapidly as whole numbers.

EXAMPLE. Add $\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{3}$, and $\frac{5}{6}$.

Solution. The l.c.d. is obviously 12.

Now, mentally reducing the fractions to 12ths, add, thinking 6, 15, 23, $\frac{33}{12}$, $2\frac{9}{12}$, $2\frac{3}{4}$.

Check. Add the fractions again, beginning at the right.

ORAL EXERCISES

Add the following :

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{9}$	$\frac{2}{3}$	$\frac{7}{12}$	$\frac{5}{16}$	
$\frac{3}{4}$	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{1}{4}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{2}{9}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{5}{8}$	
$\frac{3}{4}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{5}{8}$	$\frac{3}{10}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{5}{6}$	$\frac{5}{12}$	$\frac{15}{16}$	
$\frac{1}{2}$	$\frac{5}{6}$	$\frac{2}{3}$	$\frac{7}{8}$	$\frac{9}{10}$	$\frac{7}{15}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	
11.	12.	13.	14.	15.	16.	17.	18.	19.	20.	21.
$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{4}{15}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{5}{8}$
$\frac{1}{2}$	$\frac{2}{3}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{4}$
$\frac{1}{2}$	$\frac{5}{6}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{2}{9}$	$\frac{2}{3}$	$\frac{4}{5}$	$\frac{1}{3}$	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{1}{2}$
$\frac{3}{4}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{3}{4}$	$\frac{4}{9}$	$\frac{3}{4}$	$\frac{7}{10}$	$\frac{2}{5}$	$\frac{5}{6}$	$\frac{3}{8}$	$\frac{1}{8}$
$\frac{1}{4}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{3}{10}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{16}$
$\frac{3}{4}$	$\frac{5}{6}$	$\frac{1}{5}$	$\frac{5}{8}$	$\frac{1}{9}$	$\frac{7}{12}$	$\frac{1}{5}$	$\frac{7}{15}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{9}{16}$
$\frac{1}{2}$	$\frac{1}{3}$	$\frac{9}{10}$	$\frac{7}{8}$	$\frac{5}{9}$	$\frac{5}{12}$	$\frac{9}{10}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{3}{4}$	$\frac{3}{8}$
$\frac{3}{4}$	$\frac{1}{6}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{3}{4}$	$\frac{9}{16}$

Adding mixed numbers. In adding mixed numbers the fractions are added first and then the whole numbers. The two sums are then added to obtain the result.

EXAMPLE. Add $1\frac{2}{3}$, $5\frac{1}{4}$, and $4\frac{2}{5}$.

Solution. By inspection the l. c. d. of the fractional parts is seen to be 15. Each mixed number is then rewritten with its fractional part expressed as 15ths. The sum of the fractions is found through thinking $6, 18, \frac{2}{3}, 1\frac{1}{3}$. Adding this to 10, the sum of the whole numbers, the result is found to be $11\frac{1}{3}$.

$$\begin{array}{r}
 1\frac{2}{3} = 1\frac{10}{15} \\
 5\frac{1}{4} = 5\frac{4}{15} \\
 4\frac{2}{5} = 4\frac{6}{15} \\
 \hline
 11\frac{1}{3}
 \end{array}$$

ORAL EXERCISES

State the sum of each of the following :

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
$11\frac{1}{2}$	$21\frac{1}{3}$	$42\frac{2}{3}$	$31\frac{1}{3}$	$51\frac{1}{2}$	$21\frac{1}{4}$	$73\frac{3}{4}$	$121\frac{1}{5}$	$191\frac{1}{8}$	$251\frac{3}{16}$
$23\frac{3}{4}$	$35\frac{5}{6}$	$51\frac{1}{2}$	$21\frac{1}{9}$	$35\frac{5}{6}$	$31\frac{1}{3}$	$93\frac{3}{8}$	$131\frac{1}{2}$	$153\frac{3}{4}$	$151\frac{1}{4}$

EXERCISES

Add the following :

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
$2\frac{1}{2}$	$3\frac{1}{2}$	$6\frac{1}{4}$	$4\frac{5}{8}$	$10\frac{1}{3}$	$5\frac{2}{3}$	$16\frac{3}{10}$	$20\frac{3}{8}$	$14\frac{1}{3}$	$9\frac{3}{2}$
$5\frac{3}{4}$	$2\frac{1}{3}$	$4\frac{2}{3}$	$7\frac{3}{4}$	$9\frac{1}{9}$	$11\frac{3}{4}$	$1\frac{9}{10}$	$7\frac{5}{16}$	$12\frac{2}{5}$	$3\frac{1}{10}$
$3\frac{1}{4}$	$8\frac{2}{3}$	$5\frac{3}{4}$	$8\frac{1}{2}$	$6\frac{5}{9}$	$6\frac{5}{12}$	$9\frac{1}{2}$	$3\frac{3}{16}$	$7\frac{2}{3}$	$4\frac{3}{8}$
$1\frac{1}{2}$	$9\frac{1}{2}$	$3\frac{5}{6}$	$6\frac{1}{4}$	$4\frac{2}{3}$	$10\frac{5}{6}$	$7\frac{2}{5}$	$2\frac{1}{4}$	$3\frac{4}{15}$	$2\frac{5}{8}$
11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
$7\frac{3}{4}$	$1\frac{5}{6}$	$7\frac{1}{6}$	$3\frac{3}{8}$	$3\frac{4}{9}$	$2\frac{1}{6}$	$3\frac{4}{5}$	$8\frac{1}{2}$	$2\frac{7}{15}$	$8\frac{1}{4}$
$9\frac{1}{4}$	$7\frac{1}{6}$	$5\frac{1}{3}$	$5\frac{7}{8}$	$1\frac{1}{9}$	$12\frac{1}{2}$	$2\frac{1}{10}$	$6\frac{3}{4}$	$5\frac{3}{5}$	$1\frac{1}{8}$
$6\frac{1}{4}$	$3\frac{1}{3}$	$2\frac{1}{2}$	$7\frac{1}{2}$	$2\frac{2}{3}$	$1\frac{7}{12}$	$7\frac{3}{10}$	$9\frac{5}{8}$	$9\frac{4}{5}$	$7\frac{5}{2}$
$4\frac{1}{2}$	$5\frac{1}{2}$	$8\frac{1}{4}$	$1\frac{3}{4}$	$8\frac{7}{9}$	$7\frac{1}{4}$	$3\frac{2}{5}$	$7\frac{1}{16}$	$6\frac{2}{3}$	$3\frac{5}{16}$

In each of the following add the fractional parts mentally :

21.	22.	23.	24.	25.	26.
$25\frac{1}{2}$	$34\frac{1}{2}$	$26\frac{1}{2}$	$28\frac{1}{4}$	$40\frac{1}{5}$	$5\frac{1}{4}$
$48\frac{2}{3}$	$62\frac{1}{4}$	$17\frac{1}{3}$	$63\frac{1}{2}$	$97\frac{3}{5}$	$81\frac{1}{2}$
$52\frac{1}{2}$	$59\frac{3}{4}$	$83\frac{2}{3}$	$40\frac{3}{8}$	$31\frac{1}{2}$	$60\frac{1}{8}$
$67\frac{1}{3}$	$27\frac{1}{2}$	$5\frac{5}{6}$	$73\frac{1}{2}$	$5\frac{1}{5}$	$73\frac{3}{16}$
$81\frac{2}{3}$	$70\frac{1}{4}$	$81\frac{1}{6}$	$8\frac{5}{8}$	$93\frac{2}{5}$	$71\frac{7}{16}$
$29\frac{1}{2}$	$61\frac{1}{2}$	$90\frac{2}{3}$	$24\frac{1}{4}$	$32\frac{4}{5}$	$32\frac{5}{8}$

Fractions in the dry-goods business. In the dry-goods business, in writing fractions which represent fractions of yards of goods, the denominators are usually omitted, it being understood that such a number (unless otherwise stated) represents fourths of a yard.

For example, 40^1 , 27^2 , and 45^3 stand for $40\frac{1}{4}$ yd., $27\frac{2}{4}$ yd., and $45\frac{3}{4}$ yd. respectively. In measuring very valuable goods, 8ths and 16ths are sometimes used, but in such cases it is always stated that the fractions are 8ths or 16ths.

When goods are billed, the number of yards in the pieces are written in a horizontal line on the bill. They are added much the same as in horizontal or line addition with whole numbers.

EXAMPLE. A bill for 4 pc. of silk was filled in as follows :

4 pc., 20^1 , 22^3 , 15^2 , 18^1 .

Find the total number of yards billed.

Solution. $20^1 + 22^3 + 15^2 + 18^1 = 75^7 = 76^3$.

Hence the total number of yards billed was $76\frac{3}{4}$.

EXERCISES

Find the number of yards in each of the following :

1. 3 pc. dress goods, 15^2 , 23^3 , and 18^1 .

2. 2 pc. aeroplane linen, 45^2 and 41^3 .

3. 4 pc. duck, 36^2 , 38^3 , 40^1 , and 42^3 .

4. 3 pc. plaid, 41^2 , 39^3 , and 40^1 .

5. 6 pc. dress silk, 29^3 , 31^1 , 32^1 , 30^2 , 31^3 , and 32^1 .

6. 5 pc. stripe, 39^3 , 40^1 , 42^3 , 41^2 , and 41^1 .

7. A merchant bought 25 pc. of silk at \$1 per yard. The number of yards in the pieces were as follows: 48^1 , 45^2 , 47^3 , 46 , 48^2 , 49^1 , 53^2 , 54^3 , 57^1 , 60^3 , 56^3 , 55^1 , 54^2 , 57^3 , 52^2 , 52 , 56^3 , 57 , 50^2 , 52^3 , 57^1 , 51 , 54^3 , 50^3 , 55^3 . If he sold the entire lot at \$1.60 per yard, how much was his gross profit ?

SUBTRACTION OF FRACTIONS

Subtracting fractions. If two fractions have a common denominator, one may be subtracted from the other by subtracting the numerator of the first from the numerator of the second, and writing the difference over the common denominator.

Thus, $\frac{5}{8} - \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$, and $\frac{7}{8} - \frac{3}{8} = \frac{4}{8} = \frac{1}{2}$.

When the fractions do not have a common denominator, they must first be reduced to similar fractions before the subtraction is performed.

Thus, $\frac{1}{8} - \frac{1}{2} = \frac{1}{8} - \frac{4}{8} = \frac{7}{8}$, and $\frac{3}{4} - \frac{1}{8} = \frac{6}{8} - \frac{1}{8} = \frac{5}{8}$.

Mixed numbers are subtracted in a similar way after the fractional parts are reduced to similar fractions.

Thus, $12\frac{5}{8} - 7\frac{1}{4} = 12\frac{5}{8} - 7\frac{2}{8} = 5\frac{3}{8}$.

When the fractional part of the number to be subtracted is greater than that from which it is to be subtracted, the latter is increased by a unit taken from the whole number.

$$\text{Thus, } 9\frac{1}{4} - 6\frac{1}{2} = 9\frac{1}{4} - 6\frac{2}{4} = 8\frac{5}{4} - 6\frac{2}{4} = 2\frac{3}{4}.$$

ORAL EXERCISES

Perform the indicated subtractions:

- | | | |
|--|-------------------------------------|---------------------------------------|
| 1. $\frac{5}{9} - \frac{2}{9}$. | 13. $\frac{1}{2} - \frac{1}{4}$. | 25. $3\frac{9}{16} - 1\frac{5}{32}$. |
| 2. $\frac{7}{8} - \frac{5}{8}$. | 14. $\frac{1}{3} - \frac{1}{6}$. | 26. $2\frac{5}{6} - 1\frac{5}{8}$. |
| 3. $\frac{5}{8} - \frac{1}{8}$. | 15. $\frac{5}{6} - \frac{1}{3}$. | 27. $7\frac{4}{5} - 5\frac{3}{4}$. |
| 4. $\frac{9}{10} - \frac{3}{10}$. | 16. $\frac{3}{4} - \frac{1}{2}$. | 28. $8\frac{2}{3} - 6\frac{5}{8}$. |
| 5. $\frac{7}{12} - \frac{5}{12}$. | 17. $\frac{7}{8} - \frac{3}{4}$. | 29. $13\frac{1}{2} - 2\frac{1}{3}$. |
| 6. $\frac{11}{12} - \frac{7}{12}$. | 18. $\frac{9}{10} - \frac{2}{5}$. | 30. $14\frac{2}{3} - 3\frac{1}{6}$. |
| 7. $5\frac{3}{5} - 2\frac{1}{5}$. | 19. $\frac{2}{3} - \frac{1}{2}$. | 31. $19\frac{5}{6} - 15\frac{3}{4}$. |
| 8. $9\frac{7}{10} - 4\frac{3}{10}$. | 20. $\frac{11}{12} - \frac{5}{6}$. | 32. $20\frac{3}{4} - 15\frac{2}{3}$. |
| 9. $12\frac{5}{8} - 7\frac{3}{8}$. | 21. $\frac{3}{4} - \frac{5}{8}$. | 33. $22\frac{1}{6} - 19\frac{1}{2}$. |
| 10. $19\frac{1}{6} - 14\frac{5}{6}$. | 22. $\frac{2}{3} - \frac{1}{9}$. | 34. $35\frac{2}{5} - 15\frac{1}{2}$. |
| 11. $20\frac{5}{16} - 7\frac{9}{16}$. | 23. $\frac{5}{12} - \frac{1}{8}$. | 35. $20\frac{3}{4} - 14\frac{2}{3}$. |
| 12. $12\frac{5}{6} - 4\frac{1}{6}$. | 24. $\frac{7}{8} - \frac{7}{16}$. | 36. $19\frac{4}{5} - 17\frac{3}{4}$. |

MULTIPLICATION OF FRACTIONS

Multiplying a whole number and a fraction. To multiply a whole number and a fraction, multiply the whole number by the numerator of the fraction and divide the product by the denominator, as in the following example.

EXAMPLE. Multiply 565 by $\frac{3}{5}$.

$$\text{Solutions.} \quad 1. \quad 565 \times \frac{3}{5} = \frac{565 \times 3}{5} = \frac{1695}{5} = 339.$$

$$2. \quad 565 \times \frac{3}{5} = \frac{\overset{113}{\cancel{565}} \times 3}{\cancel{5}} = 339.$$

It is evident that cancellation, as used in the second solution, simplifies the work and should always be used where possible.

ORAL EXERCISES

1. A certain crop of wheat was sold for \$1200. Find the value of $\frac{1}{2}$ of it; $\frac{1}{3}$ of it; $\frac{2}{3}$ of it; $\frac{1}{4}$ of it.

2. A business man's estate was valued at \$24,000. Find the value of $\frac{1}{2}$ of it; $\frac{1}{3}$ of it; $\frac{5}{6}$ of it; $\frac{1}{4}$ of it.

3. In a certain company the investment which amounts to \$72,000 is held as follows: A, $\frac{1}{6}$; B, $\frac{2}{5}$; C, $\frac{1}{10}$; and D, $\frac{1}{3}$. Find the amount invested by each man.

4. The investment in a certain company worth \$18,000 is held by six men, five of whom hold $\frac{1}{6}$, $\frac{2}{5}$, $\frac{1}{10}$, $\frac{1}{9}$, and $\frac{1}{6}$ respectively. What is the value of the sixth man's investment?

EXERCISES

In each of the following find the product:

- | | | | |
|------------------------------|-------------------------------|---------------------------------|---------------------------------|
| 1. $18 \times \frac{4}{9}$. | 5. $75 \times \frac{3}{5}$. | 9. $125 \times \frac{4}{5}$. | 13. $\frac{4}{7} \times 156$. |
| 2. $24 \times \frac{3}{8}$. | 6. $81 \times \frac{5}{9}$. | 10. $121 \times \frac{3}{11}$. | 14. $\frac{5}{6} \times 96$. |
| 3. $56 \times \frac{7}{8}$. | 7. $55 \times \frac{3}{11}$. | 11. $140 \times \frac{5}{7}$. | 15. $\frac{7}{12} \times 346$. |
| 4. $52 \times \frac{3}{4}$. | 8. $72 \times \frac{3}{8}$. | 12. $184 \times \frac{3}{8}$. | 16. $\frac{3}{16} \times 346$. |

Find the value of each of the following:

- | | | |
|-----------------------------|------------------------------|------------------------------|
| 17. $\frac{3}{4}$ of \$450. | 20. $\frac{3}{16}$ of \$540. | 23. $\frac{7}{12}$ of \$624. |
| 18. $\frac{3}{8}$ of \$648. | 21. $\frac{5}{12}$ of \$246. | 24. $\frac{5}{9}$ of \$734. |
| 19. $\frac{5}{9}$ of \$765. | 22. $\frac{7}{8}$ of \$925. | 25. $\frac{7}{16}$ of \$528. |

26. Multiply 280 by $4\frac{3}{4}$.

In multiplying by a mixed number, the partial products which result from multiplying by the fractional part and the integral part are found separately and then combined; that is,

$$280 \times 4\frac{3}{4} = 280 \times \frac{3}{4} + 280 \times 4.$$

The work may be set down as shown at the right.

280
$\frac{4\frac{3}{4}}$
210
$\frac{1120}{?}$

- | | | |
|----------------------------------|--------------------------------------|--------------------------------------|
| 27. $245 \times 5\frac{5}{6}$. | 31. $475 \times 15\frac{1}{2}$. | 35. $500 \times 82\frac{3}{8}\phi$. |
| 28. $352 \times 8\frac{3}{5}$. | 32. $200 \times 12\frac{1}{4}\phi$. | 36. $475 \times \$1.24\frac{1}{5}$. |
| 29. $200 \times 12\frac{1}{8}$. | 33. $250 \times 45\frac{1}{8}\phi$. | 37. $220 \times 47\frac{1}{2}\phi$. |
| 30. $350 \times 9\frac{3}{8}$. | 34. $325 \times 67\frac{3}{4}\phi$. | 38. $\$55.60 \times 66\frac{2}{3}$. |

Find the cost of each of the following :

- | | |
|--|--|
| 39. $5\frac{3}{4}$ yd. of cloth at 75¢. | 44. 500 bu. of wheat at \$1.21 $\frac{3}{4}$. |
| 40. $2\frac{3}{8}$ yd. of cloth at 89¢. | 45. 250 bu. of oats at 62 $\frac{1}{2}$ ¢. |
| 41. $12\frac{1}{3}$ lb. of veal at 28¢. | 46. 1250 bu. of rye at 68 $\frac{3}{4}$ ¢. |
| 42. $25\frac{3}{4}$ lb. of sugar at 9¢. | 47. 250 qt. of milk at 9 $\frac{1}{4}$ ¢. |
| 43. 55 lb. of beef at 8 $\frac{1}{3}$ ¢. | 48. 125 gal. of milk at 27 $\frac{3}{4}$ ¢. |

Multiplying two fractions. Two or more fractions are multiplied by multiplying the numerators together for the numerator of the product and the denominators together for the denominator of the product. The cancellation of all pairs of common factors from numerator and denominator shortens the work and gives the result in its lowest terms.

The same rule applies in case one of the numbers is a mixed number, but in this case the mixed number should first be reduced to an improper fraction before multiplying.

EXAMPLES. 1. Multiply $\frac{3}{8}$ by $1\frac{5}{8}$.

Solution.
$$\frac{3}{8} \times 1\frac{5}{8} = \frac{3}{8} \times \frac{13}{8} = \frac{39}{64} = \frac{1}{6}.$$

2. Find the product of $3\frac{1}{2} \times \frac{3}{4} \times 10$.

Solution.
$$3\frac{1}{2} \times \frac{3}{4} \times 10 = \frac{7}{2} \times \frac{3}{4} \times 10 = \frac{21}{8} \times 10 = \frac{210}{8} = 26\frac{3}{4}.$$

EXERCISES

In each of the following find the product :

- | | | |
|--|---|---|
| 1. $\frac{4}{5} \times \frac{5}{6} \times \frac{7}{8}.$ | 5. $\frac{7}{8} \times \frac{1}{4} \times 16 \times 4.$ | 9. $12\frac{1}{2} \times 8 \times 2\frac{3}{5}.$ |
| 2. $\frac{5}{8} \times \frac{3}{10} \times \frac{4}{7}.$ | 6. $2\frac{1}{2} \times 1\frac{1}{5} \times \frac{2}{3}.$ | 10. $3\frac{1}{7} \times 14 \times 14.$ |
| 3. $\frac{4}{5} \times 10 \times \frac{2}{3}.$ | 7. $3\frac{1}{3} \times \frac{3}{5} \times 10.$ | 11. $3\frac{1}{7} \times 21 \times 8.$ |
| 4. $20 \times \frac{4}{5} \times \frac{3}{8} \times 2.$ | 8. $1\frac{5}{6} \times 2\frac{6}{7} \times 21.$ | 12. $4\frac{1}{2} \times 5\frac{1}{3} \times 7\frac{1}{2}.$ |

13. A man sells an article costing \$12 at $1\frac{3}{4}$ times the cost. Find the selling price.

14. Cloth costing \$1.75 per yard is sold at $1\frac{1}{3}$ times the cost. Find the selling price.

15. Cloth costing $\$1.62\frac{1}{2}$ per yard is sold at $\$2.37\frac{1}{2}$ per yard. Find the gross profit on the sale of $22\frac{3}{4}$ yd.

16. Find the cost of $1\frac{3}{4}$ T. of copper at $19\frac{1}{2}\phi$ per pound.

The ordinary ton of 2000 lb. is always understood, unless the contrary is stated.

17. A woman bought 5 yd. of cloth at $37\frac{1}{2}\phi$ per yard, $7\frac{1}{2}$ yd. at 49ϕ , and $4\frac{1}{2}$ yd. at 99ϕ . Find the total amount of the purchase.

18. Find the cost at $8\frac{1}{4}\phi$ per yard of a shipment of ticking containing $223\frac{3}{4}$ yd.

DIVISION OF FRACTIONS

Dividing a fraction by a whole number. Since the numerator of a fraction indicates the number of fractional parts, a fraction may be divided by a whole number by dividing the numerator by that number and writing the result over the denominator.

Thus, $\frac{4}{8} \div 2 = \frac{2}{8}$, and $\frac{9}{16} \div 3 = \frac{3}{16}$.

Since the denominator of a fraction designates the size of the fractional parts, if the number of parts is increased the value of the fraction is correspondingly decreased; that is, multiplying the denominator of a fraction by a number is equivalent to dividing the numerator by that number.

Thus, $\frac{15}{16} \times \frac{1}{3} = \frac{15}{16 \times 3} = \frac{15}{48} = \frac{5}{16}$, or $\frac{15}{16} \div 3 = \frac{15 \div 3}{16} = \frac{5}{16}$.

ORAL EXERCISES

Perform the indicated divisions:

- | | | | | |
|-------------------------|---------------------------|---------------------------|--------------------------|----------------------------|
| 1. $\frac{4}{8} \div 2$ | 4. $\frac{15}{16} \div 5$ | 7. $\frac{22}{5} \div 11$ | 10. $\frac{2}{3} \div 5$ | 13. $\frac{5}{8} \div 10$ |
| 2. $\frac{5}{6} \div 5$ | 5. $\frac{12}{5} \div 4$ | 8. $\frac{27}{4} \div 9$ | 11. $\frac{3}{7} \div 6$ | 14. $\frac{14}{25} \div 7$ |
| 3. $\frac{4}{9} \div 2$ | 6. $\frac{21}{50} \div 7$ | 9. $\frac{24}{5} \div 12$ | 12. $\frac{4}{5} \div 8$ | 15. $\frac{2}{5} \div 4$ |

Dividing by a fraction. As was seen above, dividing a fraction by 3, or $\frac{3}{1}$, was the same as multiplying it by $\frac{1}{3}$. Hence, in dividing by a fraction, invert the divisor, and proceed as in multiplication, canceling when possible.

EXAMPLE. Divide $\frac{9}{12}$ by $\frac{5}{4}$.

Solution.
$$\frac{9}{12} \div \frac{5}{4} = \frac{\overset{3}{\cancel{9}}}{\underset{\underset{3}{\cancel{12}}}{12}} \times \frac{4}{5} = \frac{3}{5}.$$

EXERCISES

Perform the indicated divisions:

- | | | | |
|--------------------------------------|--------------------------------------|--|------------------------------|
| 1. $\frac{5}{6} \div \frac{3}{4}$. | 5. $15 \div \frac{2}{3}$. | 9. $\frac{3}{4} \div \frac{5}{8}$. | 13. $39 \div \frac{3}{8}$. |
| 2. $\frac{9}{10} \div \frac{2}{5}$. | 6. $24 \div \frac{9}{5}$. | 10. $\frac{3}{40} \div \frac{6}{25}$. | 14. $24 \div \frac{5}{6}$. |
| 3. $\frac{7}{9} \div \frac{7}{3}$. | 7. $\frac{5}{6} \div \frac{7}{12}$. | 11. $\frac{7}{48} \div \frac{7}{12}$. | 15. $96 \div \frac{3}{4}$. |
| 4. $\frac{5}{6} \div \frac{5}{4}$. | 8. $\frac{3}{8} \div \frac{5}{16}$. | 12. $\frac{5}{64} \div \frac{9}{32}$. | 16. $72 \div \frac{5}{12}$. |

Dividing with mixed numbers. In dividing a mixed number by a whole number, it is often possible to divide the integral part and the fractional parts separately without difficulty.

Thus, $15\frac{2}{5} \div 5 = 15 \div 5 + \frac{2}{5} \div 5 = 3\frac{2}{25},$
and $13\frac{1}{4} \div 4 = 13 \div 4 + \frac{1}{4} \div 4 = 3\frac{1}{4} + \frac{1}{16} = 3\frac{5}{16}.$

In case the integral part is not readily divisible, the usual method is to reduce the mixed number to a fraction and divide as with fractions.

Thus, $7\frac{1}{5} \div 9 = \frac{36}{5} \div 9 = \frac{4}{5},$ and $4\frac{2}{3} \div 12 = \frac{14}{3} \times \frac{1}{12} = \frac{7}{18}.$

In dividing by a mixed number, the divisor should be reduced to a fraction. The division is then carried out the same as in division by a fraction.

EXAMPLES. 1. Divide 12 by $2\frac{2}{3}$.

Solution.
$$12 \div 2\frac{2}{3} = 12 \div \frac{8}{3} = \frac{\overset{3}{\cancel{12}}}{8} \times \frac{3}{2} = \frac{9}{2} = 4\frac{1}{2}.$$

2. Divide $4\frac{1}{5}$ by $2\frac{4}{5}$.

Solution.
$$4\frac{1}{5} \div 2\frac{4}{5} = \frac{21}{5} \div \frac{14}{5} = \frac{\overset{3}{\cancel{21}}}{5} \times \frac{5}{\underset{\underset{2}{\cancel{14}}}{14}} = \frac{3}{2} = 1\frac{1}{2}.$$

EXERCISES

Perform the indicated divisions:

- | | | | |
|-----------------------------|------------------------------|---|---|
| 1. $10\frac{2}{5} \div 5$. | 6. $7\frac{1}{3} \div 4$. | 11. $5\frac{2}{5} \div \frac{4}{15}$. | 16. $27\frac{3}{8} \div 9\frac{1}{8}$. |
| 2. $12\frac{1}{2} \div 4$. | 7. $15 \div 1\frac{2}{3}$. | 12. $4\frac{1}{2} \div \frac{2}{9}$. | 17. $45\frac{3}{16} \div 9\frac{1}{16}$. |
| 3. $18\frac{1}{3} \div 5$. | 8. $14 \div 1\frac{3}{4}$. | 13. $7\frac{1}{3} \div 1\frac{2}{9}$. | 18. $17\frac{1}{2} \div 2\frac{1}{2}$. |
| 4. $10\frac{2}{3} \div 8$. | 9. $20 \div 3\frac{1}{3}$. | 14. $8\frac{1}{3} \div 1\frac{5}{12}$. | 19. $14\frac{5}{9} \div 13\frac{2}{3}$. |
| 5. $6\frac{1}{5} \div 9$. | 10. $36 \div 1\frac{4}{5}$. | 15. $6\frac{4}{5} \div 2\frac{1}{2}$. | 20. $16\frac{3}{8} \div 4\frac{3}{4}$. |

Fractional parts. It is often necessary to find what fractional part one number or quantity is of another. A problem of this type is simply a case of dividing one number by another.

EXAMPLE. What fractional part of 50 is $7\frac{1}{2}$?

$$\text{Solution.} \quad 7\frac{1}{2} \div 50 = 1\frac{5}{2} \div 50 = \frac{15}{2} \div 50 = \frac{15}{2} \times \frac{1}{50} = \frac{3}{20}.$$

EXERCISES

In each of the following find what fractional part the second number is of the first:

- | | | |
|---------------|----------------------------|-----------------------------|
| 1. 125; 20. | 7. 100; $62\frac{1}{2}$. | 13. 100; $166\frac{2}{3}$. |
| 2. 180; 65. | 8. 100; $133\frac{1}{3}$. | 14. 100; $6\frac{1}{4}$. |
| 3. 360; 48. | 9. 100; $12\frac{1}{2}$. | 15. 100; $114\frac{2}{7}$. |
| 4. 144; 78. | 10. 100; $37\frac{1}{2}$. | 16. 200; $108\frac{1}{3}$. |
| 5. 2000; 664. | 11. 100; $16\frac{2}{3}$. | 17. 250; $137\frac{1}{2}$. |
| 6. 2240; 432. | 12. 100; $66\frac{2}{3}$. | 18. 300; $166\frac{2}{3}$. |

19. 18 is $\frac{3}{4}$ of what number? $\frac{2}{3}$ of what number? $\frac{5}{12}$ of what number?

If 18 is $\frac{3}{4}$ of some number, then 6 is $\frac{1}{4}$ of that number, and the number is —.

20. 150 is $\frac{3}{4}$ of what number? $\frac{2}{3}$ of what number?

21. \$64 is $\frac{4}{5}$ of what amount? $\frac{8}{9}$ of what amount?

22. After spending $\frac{3}{8}$ of his money a man had \$42 left. How much had he at first?

23. A man sold a horse, losing $\frac{2}{9}$ of what he paid for it. If he received \$112 for the horse, what did it cost him? What was his loss?

24. A man gained $\frac{1}{8}$ of the selling price on goods which he sold for \$120. What did the goods cost him? How much did he gain?

REVIEW EXERCISES

1. A invested \$750 in a business and B invested \$1250 in it. In a certain month they cleared \$80. How much should each receive if they share the profits in proportion to their investments?

2. A man lost \$125 on an investment of \$1000. What fractional part of the investment did he lose?

3. A boy spent $\frac{1}{3}$ of his money in one store, $\frac{1}{4}$ in another, and had \$2 left. How much did he have at first?

4. Two men rent a pasture for cattle at \$24 per month. One man puts in 5 cows and the other puts in 3 cows. How much should each man pay per month?

5. If a chocolate bar weighs $1\frac{3}{8}$ oz., how many bars can be made from 22 lb. of chocolate (1 lb. = 16 oz.)?

6. If 1 lb. 1 oz. of dough makes 1 lb. of bread, how many loaves of bread weighing $12\frac{1}{2}$ oz. can be made from 4250 lb. of dough?

7. In Ex. 6 how many pounds of dough are required to make 6840 loaves weighing 30 oz. each?

8. A carpenter estimates that it will take 63 hr. to do a certain piece of work. If he works $6\frac{3}{4}$ hr. per day, how many days will be required to finish the job?

9. If an automobile goes $19\frac{1}{2}$ mi. on a gallon of gasoline, how much gasoline will it take for a trip of 685 mi.?

10. Boiled linseed oil weighs $7\frac{1}{2}$ lb. per gallon. At $11\frac{1}{4}\phi$ per gallon find the cost of a barrel of oil weighing 766 lb.

11. A man bought a secondhand automobile for $\frac{3}{5}$ of its cost, when new. He then sold it and gained $\frac{1}{5}$ of what he paid. If he gained \$87 on the deal, what was the price of the machine when new?

12. A farm worth \$24,840 is owned jointly by two partners. The interest of one partner is $\frac{4}{9}$ of the value of the farm. What is the value of each partner's interest?

13. If the rate of foreign exchange is such that a French franc is worth $19\frac{1}{2}\text{¢}$, what is the value in francs of \$225?

14. At one time the rate of foreign exchange was such that \$1 was worth $97\frac{1}{2}$ German marks. What was the value in dollars of 250 marks at that time?

15. The inventory of a stock of dry goods showed the following pieces of cloth on hand: 45^1 , 38^2 , 27^3 , 35^1 , 52^1 , 54^2 , 58^3 , 46^1 , 44^1 , 41^3 , 38^1 . Find the total cost of the cloth at \$1.15 per yard.

16. When A failed in business it was found that his resources were \$4560 and his liabilities \$7600. What fractional part of his liabilities was he able to pay? How much was received by a creditor to whom A owed \$1225?

17. If $9\frac{1}{4}$ lb. of alfalfa seed were sown to the acre, what did the seed for $15\frac{1}{2}$ A. cost at 19¢ per pound?

18. A man sold a city lot with a frontage of 75 ft. for \$6375. At the same rate per front foot, what is the value of the adjoining lot which is 65 ft. wide?

19. If a widow with a yearly income of \$900 allows $\frac{2}{5}$ of her income for food, $\frac{1}{6}$ for clothing, and saves $\frac{1}{2}$ of the remainder, what are her monthly savings?

20. A farmer who kept separate accounts for his crops paid \$25 for binding twine which he used for 20 A. of wheat and 35 A. of oats. How much did he charge to the account of each crop?

21. A farmer raised 336 T. of sugar beets on 24 A. of land. On another piece containing 28 A. he raised 462 T. Find the average number of tons per acre for the entire crop.

22. A shipment of hams weighed 1825 lb. Find the total cost of the hams if the invoice price was $28\frac{1}{2}\phi$ per pound and the freight was $\frac{1}{2}\frac{8}{5}\phi$ per pound.

23. Two boys purchase a lawn mower, one putting in \$15 and the other \$10. Working together, they earn \$40 in 6 da. by cutting the neighbors' lawns. If they divide $\frac{3}{5}$ of their earnings equally and the remainder according to the amount which each invested in the lawn mower, how much does each boy receive?

DECIMAL FRACTIONS

Common and decimal fractions. The fraction three tenths may be written in two ways, $\frac{3}{10}$ or 0.3. The first form is a common fraction, while the second is called a **decimal fraction**. In a common fraction the denominator is always written, while in the decimal fraction the size of the denominator is determined by the number of figures to the right of the decimal point.

For example, in 0.5 the denominator is 10; in 0.82 the denominator is 100; and in 0.063 the denominator is 1000.

The 0 before the decimal point in 0.82 is used simply to avoid confusing .82 with 82 in case the decimal point is carelessly placed.

While the term *decimal fraction* is often used, the term **decimal** is the more common. The term **mixed decimal** is sometimes used in a manner similar to the term *mixed number*, but, in general, such a number is called simply a **decimal**. In either case, the figures to the right of the decimal point indicate a fraction whose denominator is not written but which is always some power of 10.

Thus, 0.15 means $\frac{15}{100}$, 1.02 means $1\frac{2}{100}$, and 8.075 means $8\frac{75}{1000}$.

Reading decimals. In reading decimals the word "and" is used to indicate the decimal point, but is not used elsewhere in the number.

Thus, the decimal 506.032 is read "five hundred six and thirty-two thousandths," or "five-o-six-point-o-three-two." Unless otherwise directed, the first method of reading is to be used.

The place value of the figures at the right and left of the decimal point is shown in the following table:

<i>Period :</i>	MILLIONS			THOUSANDS			UNITS			THOUSANDTHS			MILLIONTHS			
<i>Place Value :</i>	Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths
<i>Figure :</i>	7	8	9,	5	6	4,	3	0	1	.	7	2	0	3	5	2

ORAL EXERCISES

Read the following:

- | | | | |
|--------------------|---------------|----------------|--------------|
| 1. 2.7. | 4. 12.217. | 7. 0.0483. | 10. 0.02031. |
| 2. 3.65. | 5. 530.024. | 8. 0.6067. | 11. 2.00463. |
| 3. 7.04. | 6. 1.0005. | 9. 4.47321. | 12. 0.00590. |
| 13. 250.15 sq. mi. | 15. \$2.2465. | 17. 11.058 oz. | |
| 14. 4.82 mills. | 16. 57.365 T. | 18. 30.085 ft. | |

19. Under normal conditions the English pound sterling is worth \$4.8665 in United States money.

20. The circumference of a circle is approximately 3.1416 times the length of its diameter.

EXERCISES

Using decimals, write the following in figures:

- Seven tenths; forty-five hundredths; two hundred fifty thousandths.
- Seventy-five thousandths; seven hundred fifty-five ten thousandths; fifty-five ten thousandths.
- Twenty-five and five tenths; seventy-two and five hundredths; four hundred twenty-five and twenty-five hundredths.

4. Six hundred thousand, five hundred twelve and five hundredths; one million, one thousand, six hundred fifty-five and fifteen hundredths; five hundred eighty-five millionths.

5. Two and one hundred fifty-seven thousandths; twenty-five and three hundred twenty ten-thousandths; four hundred twenty-five thousand, five hundred fifty millionths.

Adding decimals. In adding decimals, the numbers are written so that the decimal points are in a vertical column. The addition is then performed as with whole numbers, and the decimal point in the sum is placed under the decimal points in the numbers added.

Thus, in adding 2.75, 32.6, and 0.025 arrange the numbers as shown at the right. Adding as with whole numbers and placing the decimal point in the sum under the decimal points above, the result is found to be 35.375.

2.75
32.6
0.025
35.375

EXERCISES

Find the sum in each of the following:

- 5.5, 90, 27.25, 37.5, and 2.05.
- 6.5, 75, 4.26, 5.05, and 74.06.
- 42, 2.35, 7.6, 0.84, 0.08, and 2.1.
- 0.003, 0.125, 0.057, 0.016, and 0.103.
- 2.51, 9.06, 11.01, 15.006, and 17.235.
- 176.2, 143.01, 296.05, 4.025, and 17.830.

7. \$175.70	8. \$27.38	9. \$109.82	10. 47,615.8
256.25	16.54	562.43	23,479.1
401.17	71.39	759.23	54,690.1
619.87	85.43	641.08	30,462.8
516.73	46.17	509.71	50,792.6
24.95	109.27	76.25	17,604.3
92.70	98.62	82.50	27,505.0
76.15	207.53	141.92	39,628.7
<u>327.91</u>	<u>405.28</u>	<u>210.76</u>	<u>73,215.0</u>

11. 10.608	12. 9.0673	13. 8.4322	14. 170.060
45.259	1.9215	7.6648	297.
9.919	2.7504	0.0907	423.155
4.762	0.8210	3.0154	0.025
11.217	0.5116	2.1072	219.345
9.346	2.1098	0.7560	507.
6.285	1.7327	4.8320	276.325
4.235	0.5280	2.1908	0.395
<u>12.526</u>	<u>4.8295</u>	<u>0.5128</u>	<u>124.535</u>

Subtracting decimals. In subtracting numbers which involve decimals, the number to be subtracted is written with its decimal point under that of the number from which it is to be taken. The subtraction is then performed as with whole numbers and the decimal point in the result is placed below the other decimal points.

Thus, in subtracting 52.973 from 98.65, the numbers are set down as shown at the right. In subtracting, think of 98.65 as 98.650, since annexing ciphers to the figures at the right of the decimal point does not change the value of the decimal; that is, $0.1 = 0.10 = 0.100$, which is at once evident if the fractional equivalents $\frac{1}{10} = \frac{10}{100} = \frac{100}{1000}$ are considered.

98.65
<u>52.973</u>
45.677

EXERCISES

Subtract the following :

- | | |
|-------------------------|-------------------------|
| 1. 5.27 from 9.63. | 3. 12.3475 from 20.169. |
| 2. 7.1634 from 15.2967. | 4. 28.1739 from 35.29. |

Perform the following operations :

- From 46.3752 take 37.8401.
- From 27.9301 take 20.109.
- From 8.3629 take 5.24307.
- From 12.463 take 9.1435.
- One of two given amounts is \$480.21; their sum is \$729.85. Find the other amount.
- Find the remainder in a subtraction if 21.638 is taken from 86.0703.

11. Find the number subtracted if the number from which it was taken and the difference are 0.12564 and 0.09016 respectively.

12. From the sum of two thousand fifty and thirty-nine thousandths and five hundred forty-one and twenty-seven hundredths, take nine hundred thirty-eight and forty-five hundredths.

13. A merchant purchased a delivery truck on Jan. 1 for \$1250. During the year he paid out \$85.30 for tires, \$42.25 for repairs, \$181.45 for gasoline and oil, and \$60 for garage rent. He sold the truck at the end of the year for \$975. Find the total expense of operating the car for the year.

Multiplying decimals. Decimals are multiplied in the same way as whole numbers; then, beginning at the right of the product, as many places are pointed off as there are decimal places in the two numbers multiplied.

Thus, $72.5 \times 0.2 = 14.50$, or 14.5; $45.25 \times 0.3 = 13.575$; and $\$1250 \times 0.06 = \75.00 , or \$75.

In multiplying a decimal by a power of 10, the rule just given can be carried out by simply moving the decimal point to the right as many places as there are ciphers at the right of the multiplier.

Thus, $42.375 \times 10 = 423.75$; $42.375 \times 100 = 4237.5$; $42.375 \times 10,000 = 423,750$; and so on.

ORAL EXERCISES

Perform the indicated multiplications:

- | | |
|---|---|
| 1. 2.5×10 ; 2.5×100 . | 6. 1.95×10 ; 1.95×100 . |
| 2. 3.6×100 ; 0.36×100 . | 7. 10.4×100 ; 10.4×1000 . |
| 3. 2.7×100 ; 2.7×1000 . | 8. 0.2×10 ; 0.02×10 . |
| 4. 0.51×10 ; 0.51×100 . | 9. 0.15×10 ; 0.015×10 . |
| 5. 16.3×10 ; 16.3×1000 . | 10. 1.7×10 ; 0.017×10 . |
| 11. 7.9×10 ; 0.079×10 ; 0.079×1000 . | |
| 12. 6.1×100 ; 6.1×1000 ; 0.61×1000 . | |
| 13. 50×0.1 ; 75×0.10 ; 20×0.2 . | |
| 14. 7.9×0.1 ; 7.9×0.10 ; 7.9×0.100 . | |
| 15. 45×0.2 ; 3.6×0.2 ; 24×0.3 . | |
| 16. 100×1.5 ; 200×2.5 ; 150×0.4 . | |

17. 1.5×0.4 ; 7.2×0.5 ; 7.2×0.05 .

18. 7.5×0.6 ; 12.5×0.8 ; 4.8×0.01 .

19. 2.35×0.1 ; 2.35×0.01 ; 2.35×0.02 .

20. 2.5×1.2 ; 2.5×0.12 ; 0.25×0.12 .

Estimating results. With practice the student can soon estimate the approximate result of a multiplication with sufficient accuracy to serve as a check on absurd errors in his work.

For example, if the student is asked to find the cost of 28 yd. of cloth at \$2.35 per yard, he should see at a glance that the cost is about \$65. At \$2.50 per yard the cost would be \$70, and hence at \$2.35 the cost is slightly less than \$70. Actually, the cost is \$65.80, but the student should not waste time trying to make his estimates too close, since the approximate result will serve as an efficient check.

EXERCISES

1. The following table shows the production of certain grains in the United States and their value per bushel during a recent ten-year period:

OATS		RYE		BARLEY	
Production in bushels	Value per bushel	Production in bushels	Value per bushel	Production in bushels	Value per bushel
807 156 000	\$0.47	31 851 000	\$0.74	166 765 000	\$0.56
1 007 353 000	.41	32 239 000	.74	170 284 000	.55
1 186 341 000	.34	34 897 000	.72	173 832 000	.58
922 298 000	.45	33 119 000	.83	160 240 000	.87
1 418 337 000	.32	35 664 000	.66	223 834 000	.51
1 121 768 000	.39	41 381 000	.63	178 189 000	.53
1 141 060 000	.44	42 779 000	.87	194 953 000	.54
1 549 030 000	.36	54 050 000	.83	228 851 000	.52
1 251 837 000	.52	48 862 000	1.22	182 309 000	.88
1 592 740 000	.66	62 933 000	1.66	211 759 000	1.13

(a) Estimate the value of the oats crop for each year in millions of dollars.

(b) Find the exact value of the oats crop for each year and compare it with the estimated result.

(c) Find the total value of the oats crop for the ten years.

2. Solve (a), (b), and (c) of Ex. 1 for the rye crop.
3. Solve (a), (b), and (c) of Ex. 1 for the barley crop.
4. In the table of Ex. 1 find the average value per bushel of each grain during the ten-year period.

Dividing a decimal by a whole number. A decimal can be divided by a whole number much the same as in the division of whole numbers by first placing the decimal point in the result in line with the decimal point in the number divided (below in short division and above in long division). The first figure of the result is placed in line with the last figure of that part of the number divided which first contains the divisor, ciphers being placed to the left between this figure and the decimal point if necessary.

EXAMPLES

1. Divide 1.80 by 15.

$$\text{Solution. } 15 \overline{)1.80} \\ 0.12$$

3. Divide 215.36 by 32.

$$\text{Solution. } 32 \overline{)215.36} \\ \begin{array}{r} 6.73 \\ 192 \\ \hline 233 \\ 224 \\ \hline 96 \\ 96 \\ \hline \end{array}$$

2. Divide 0.0752 by 8.

$$\text{Solution. } 8 \overline{)0.0752} \\ 0.0094$$

ORAL EXERCISES

Divide the following:

- | | | |
|--------------|-----------------|------------------|
| 1. 85 by 5. | 6. 7.2 by 8. | 11. 4.32 by 8. |
| 2. 8.5 by 5. | 7. 5.6 by 7. | 12. 0.32 by 4. |
| 3. 45 by 5. | 8. 4.2 by 6. | 13. 0.084 by 7. |
| 4. 4.5 by 5. | 9. 10.40 by 8. | 14. 0.096 by 12. |
| 5. 72 by 9. | 10. 9.60 by 12. | 15. 0.048 by 6. |

EXERCISES

Perform the indicated divisions:

- | | | |
|---------------------|----------------------|-----------------------|
| 1. $11.25 \div 9$. | 4. $3.724 \div 7$. | 7. $300.3 \div 21$. |
| 2. $21.69 \div 3$. | 5. $481.5 \div 5$. | 8. $141.68 \div 23$. |
| 3. $3.78 \div 14$. | 6. $2.256 \div 16$. | 9. $33.25 \div 25$. |

10. $8.2017 \div 39$. 13. $52.245 \div 215$. 16. $985.8 \div 186$.
 11. $224.116 \div 43$. 14. $493.44 \div 257$. 17. $8.9033 \div 553$.
 12. $46.656 \div 216$. 15. $1.4178 \div 139$. 18. $303.81 \div 123$.

Inexact division. When there is a remainder in the division of a decimal by a whole number, the remainder is sometimes written as a common fraction in lowest terms when the fraction is a simple one such as a half, third, fourth, eighth, and so on.

For example, the result of $1.61 \div 4$ may be written $0.40\frac{1}{4}$.

In general, however, it is better to continue the division by annexing ciphers at the right of the number to be divided until the division has been carried to a desired number of places.

Thus, $1.61 \div 4 = 4 \overline{)1.6100}$, the result being exact in the fourth decimal place.

0.4025

When there is no possibility of an exact division, the result is carried to the desired number of places and then its last figure is corrected, according to what the next figure in the result would be if the division were carried one place more. If the next figure is less than 5, no change is made; if the next figure is 5 or greater, the last figure in the result is increased by 1.

EXAMPLE. Divide 383.76 by 141, giving the result correct to three decimal places.

Solution. Arranging the divisor and the number to be divided as before, divide until the result has three decimal places. The result is then 2.721 with a remainder 99. It is now readily seen that if the division were carried one place more, the next figure would be the result of dividing 990 by 141, or 7. It is thus seen that the last figure of the result should be a 2 instead of a 1.

Hence the result correct to three decimal places, or to the nearest thousandth, is given as 2.722.

	2.721
141 $\overline{)383.760}$	
	282
	<u>1017</u>
	987
	<u>306</u>
	282
	<u>240</u>
	141
	<u>99</u>

If the remainder in the above example had been less than 70, say 65, the result would have been given, correct to the nearest thousandth, as 2.721.

EXERCISES

Perform the following divisions, giving each result correct to the nearest hundredth :

$$1. 124.673 \div 119. \quad 3. 198.675 \div 429. \quad 5. 3247.32 \div 127.$$

$$2. 1267.85 \div 276. \quad 4. 6756.12 \div 731. \quad 6. 938.763 \div 452.$$

Perform the following divisions, giving each result correct to the nearest thousandth :

$$7. 826.511 \div 131. \quad 9. 8974.303 \div 321. \quad 11. 64.821 \div 909.$$

$$8. 7487.624 \div 125. \quad 10. 55.496 \div 876. \quad 12. 20.62 \div 802.$$

Dividing by decimals. If both terms of a fraction are multiplied by the same number, the value of the fraction is not changed. Similarly, if both the divisor and the number to be divided are multiplied by the same number, the result of the division is not affected.

Accordingly, to divide by a number which involves a decimal, first multiply both the divisor and the number to be divided by that power of 10 which makes the divisor a whole number and then proceed as before. Since multiplying a decimal by a power of 10 is accomplished by moving the decimal point to the right, the above rule may be expressed as follows: Before dividing move the decimal point in both the divisor and the number to be divided as many places to the right as there are decimal places in the divisor.

EXAMPLE. Divide 5.375 by 1.25.

Solution. $5.375 \div 1.25$ is equivalent to $537.5 \div 125$; or following the second rule, move the decimal points as shown at the right. Then dividing by the whole number 125, the result is found to be 4.3.

		4.3
1 25.	)	537.5
		5 00
		37 5
		37 5

Estimating results. Absurd errors in results, often due to careless placing of the decimal point, can be avoided by estimating the result in advance. With practice such estimates can

be made rapidly and accurately, and serve as valuable checks on the student's work.

For example, in the above division it is readily estimated that the result will be a little more than 4, since 125 is contained 4 times in 500.

EXERCISES

In each of the following estimate the result, perform the division, and then compare the result with the estimate :

- | | | |
|------------------------|-------------------------|------------------------------|
| 1. $3.23 \div 1.7$. | 6. $38.95 \div 1.90$. | 11. $2.289 \div 0.109$. |
| 2. $2.99 \div 2.3$. | 7. $4.320 \div 2.7$. | 12. $19.38 \div 0.019$. |
| 3. $14.76 \div 0.36$. | 8. $4.944 \div 2.4$. | 13. $0.10280 \div 0.105$. |
| 4. $174.4 \div 0.16$. | 9. $43.89 \div 0.021$. | 14. $0.7752 \div 0.076$. |
| 5. $317.4 \div 1.38$. | 10. $6603 \div 0.031$. | 15. $0.041303 \div 0.0401$. |

- | 16. | 17. | 18. |
|----------------------|---------------------|-----------------------|
| $72 \div 12$. | $9.6 \div 12$. | $225 \div 1.25$. |
| $7.2 \div 1.2$. | $96 \div 1.2$. | $22.5 \div 1.25$. |
| $7.2 \div 0.12$. | $96 \div 0.12$. | $22.5 \div 0.125$. |
| $7.2 \div 0.012$. | $9.6 \div 0.012$. | $0.225 \div 0.125$. |
| $0.72 \div 0.012$. | $0.96 \div 0.012$. | $0.225 \div 0.0125$. |
| $0.072 \div 0.012$. | $0.096 \div 0.12$. | $2.25 \div 0.0125$. |

Perform the following divisions, giving each result correct to the nearest thousandth :

- | | |
|-----------------------------|----------------------------|
| 19. $12.723 \div 1.75$. | 22. $876.542 \div 2.879$. |
| 20. $27.94 \div 2.19$. | 23. $64.256 \div 0.237$. |
| 21. $399.2847 \div 32.64$. | 24. $2.9876 \div 0.0075$. |
25. If the product of two numbers is 2.205 and one of them is 10.5, find the other number.
26. If 250 yd. of goods cost \$218.75, what is the cost per yard ?
27. When eggs are sold at 42¢ per dozen, find the selling price per egg.
28. At what price was sugar purchased if 22 lb. cost \$2.10 ?
29. Find the price of wheat when 2150 bu. cost \$2203.75.

Common fractions and decimals. A common fraction may be expressed as a decimal by dividing the numerator by the denominator. Where the division is not exact, the result may be given correct to any required number of decimal places.

Thus, $\frac{3}{4} = 3 \div 4 = 0.75$; $\frac{24}{1600} = 24 \div 1600 = 0.015$; and $\frac{5}{12} = 5 \div 12 = 0.4167$, correct to four decimal places.

EXERCISES

Express each of the following as a decimal, correct to the nearest thousandth:

- | | | | |
|--------------------|----------------------|--------------------------------|------------------------|
| 1. $\frac{1}{3}$. | 6. $\frac{1}{8}$. | 11. $\frac{5}{16}$. | 16. $8\frac{9}{16}$. |
| 2. $\frac{1}{4}$. | 7. $\frac{2}{3}$. | 12. $\frac{3}{16}$. | 17. $12\frac{3}{2}$. |
| 3. $\frac{7}{8}$. | 8. $\frac{3}{5}$. | 13. $\frac{7}{8}$. | 18. $14\frac{2}{7}$. |
| 4. $\frac{3}{8}$. | 9. $2\frac{5}{16}$. | 14. $\frac{1}{6}\frac{1}{4}$. | 19. $3\frac{1}{7}$. |
| 5. $\frac{5}{6}$. | 10. $\frac{7}{12}$. | 15. $2\frac{9}{32}$. | 20. $32\frac{9}{11}$. |

Express each of the following as a decimal, correct to the nearest hundredth:

- | | | |
|---------------------------|----------------------------|-----------------------------|
| 21. $11\frac{45}{64}$. | 24. $38\frac{96}{231}$. | 27. $89\frac{2335}{2240}$. |
| 22. $14\frac{87}{128}$. | 25. $141\frac{109}{144}$. | 28. $76\frac{1117}{5280}$. |
| 23. $26\frac{516}{640}$. | 26. $97\frac{528}{1728}$. | 29. $115\frac{19}{1760}$. |

30. If an investment of \$520 returns \$23.40 in a year, the return is what decimal fraction of the investment?

31. If property costing \$2900 is sold for \$3037, express the gross profit as a decimal fraction of the cost; of the selling price.

Buying and selling by the hundred and thousand. In business, prices are frequently quoted by the hundred or by the thousand. In such cases it is usually best to reduce a given quantity to hundreds or thousands by dividing by 100 or by 1000, and then to multiply this result by the quoted price per hundred or per thousand.

The division of numbers by powers of 10 is accomplished by moving the decimal point in the number to be divided (or by

inserting the decimal point in a whole number) as many places to the left as there are ciphers in the divisor, annexing ciphers at the left of the number divided if necessary.

Thus, $107 \div 100 = 1.07$; $135.2 \div 100 = 1.352$; and $1.267 \div 1000 = 0.001267$.

ORAL EXERCISES

Perform the indicated divisions:

- | | | |
|----------------------|-----------------------|-------------------------|
| 1. $207 \div 100$. | 4. $1420 \div 100$. | 7. $14,200 \div 1000$. |
| 2. $256 \div 1000$. | 5. $1759 \div 1000$. | 8. $1.25 \div 1000$. |
| 3. $87.5 \div 100$. | 6. $12.85 \div 100$. | 9. $97.375 \div 1000$. |

10. Find the cost of 1200 lb. of graham flour at \$4.50 per hundredweight.

11. Find the freight charges on 1220 lb. of merchandise at \$5 per hundredweight.

EXERCISES

Find the freight charges on each of the following:

- 1250 lb. bacon at 50¢ per hundredweight.
- 550 lb. salt at 30¢ per hundredweight.
- 150 lb. cheese at 45¢ per hundredweight.
- What is the cost of 5000 ft. of lumber at \$45.75 per thousand feet?
- If a certain grade of shingles sells for \$12.50 per thousand, what is the cost of 4250 shingles?
- At \$120 per thousand, find the cost of building tile for a house requiring 7500 tiles.
- If pressed brick sells at \$28 per thousand, find the cost of 25,000 bricks.
- At \$15 per thousand, what is the cost of 5500 bricks?
- Find the cost of 14,500 tiles at \$130 per thousand.
- What is the cost of 25,000 bricks at \$21.50 per thousand?
- Find the cost of 15,500 shingles at \$9.50 per thousand.
- At 85¢ per hundredweight find the freight charges on a shipment of fruit weighing 8450 lb.

13. If cement sells at \$4.45 per barrel, what is the cost of 16,240 barrels?

14. Find the total cost of the following:

850 lb. of nails at \$8.50 per cwt.

1800 ft. of lumber at \$35 per M

6400 ft. of flooring at \$40 per M

28,500 shingles at \$8.50 per M

The abbreviations "cwt." and "M" stand for 'hundredweight' and 'thousand' respectively.

15. Find the total cost of the following:

860 machine bolts at \$4.25 per C

325 machine bolts at \$3.75 per C

475 carriage bolts at \$2.50 per C

125 carriage bolts at \$3.25 per C

The abbreviation "C" stands for "hundred."

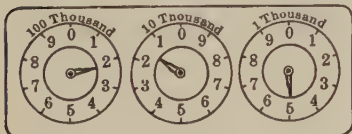
16. What is the cost of 28,000 cu. ft. of gas at \$1.25 per M (1000 cu. ft.)?

17. A gas meter read 11,250 cu. ft. on Jan. 2; on Feb. 2 it read 14,280 cu. ft. Find the cost of the gas for the month at \$1.10 per M.

Subtract the January reading from the February reading to find the amount of gas used.

18. In the diagrams below the gas-meter readings for two months are shown. For June 1 the reading is 21,500 cu. ft.

JUNE 1



JULY 1



Read the meter for July 1, and compute the gas bill for the month at \$1.35 per M.

19. If an electric-meter reading was 1340 kilowatt hours (K.W.H.) on June 1, and 1285 K.W.H. on May 1, find the amount of the bill at $13\frac{1}{2}\text{¢}$ per K.W.H.

20. Find the total cost of the electricity used as indicated by the following electric-meter readings :

a. Present reading, 176 K.W.H.; previous reading, 162 K.W.H.; rate per K.W.H., $8\frac{1}{2}\phi$.

b. Present reading, 2117 K.W.H.; previous reading, 2082 K.W.H.; rate per K.W.H., $7\frac{1}{2}\phi$.

c. Present reading, 6794 K.W.H.; previous reading, 6712 K.W.H.; rate per K.W.H., $9\frac{1}{3}\phi$.

REVIEW EXERCISES

1. Multiply 37.05 by 19.94, and divide the product by 74.1.

2. If 64 is divided by 0.08, find the result.

3. If the result of a division is 618.56 and the divisor is 15.8, what number was divided ?

4. If $\frac{1}{3}$ of the sales price of a lot of shirts is gross profit, what did each shirt cost if the selling price is \$36 per dozen ?

5. Find the retail price of each hat in a shipment billed at \$39.60 per dozen, if 0.40 of the cost is to be added to the cost for gross profit.

6. At 35¢ per hour, how many hours does a boy have to work to earn \$61.25 ?

7. If it takes 4 T. of hay per day to feed 1000 sheep, how much hay is required to feed 8750 sheep for 30 da. ? At \$14.50 per ton, what is the cost of the hay ?

8. A man paid \$1250 for a tractor. If he agrees to plow for \$3.75 per acre, how many acres must he plow to earn the cost of the tractor ?

9. How much would a farmer save by buying 1500 used potato sacks at 15¢ each instead of new ones at 21¢ each ? Expressing the result first as a common fraction and then as a decimal, find the part of the higher price that is saved.

10. At \$1.28 per hundredweight, what should be paid to a farmer for a car of potatoes weighing 32,580 lb. ?

11. What is the profit on a bill of furniture sold for \$156 if that figure represents 1.45 of the cost?

12. A can of cream tested 0.46 butter fat. What amount should be paid for 65 lb. of this cream if butter fat is worth 38¢ per pound?

13. Verify the following bill:

GAS AND ELECTRIC BILL

IF PAID AT OFFICE PLEASE BRING THIS BILL WITH YOU. IF PAID BY CHECK DETACH STUB AND MAIL SAME WITH CHECK. CHECK REMITTANCE WILL BE ACKNOWLEDGED ONLY WHEN REQUESTED.

USED BETWEEN READING DATES NOTED BELOW			
S. D. Shaylor		Nov. 10	
2610 Durant St.		To	
		Dec. 10	
		19--	
To GENERAL GAS AND ELECTRIC COMPANY, DR.			
METER READINGS			
GAS			
70,000		@ \$1.07 per 1000 cu. ft.	
59,800	10,200	Cu. Ft.	\$ 10.91
ELECTRIC			
6782			
6758	24	K. W. H. @ 8¢	\$ 1.92
TOTAL			\$ 12.83
NOTICE: THIS BILL IS LEFT AS A NOTICE OF AMOUNT NOW DUE AND PAYABLE AT THE COMPANY'S OFFICE			
RECEIVED PAYMENT		DATE PAID	
M. R. S.		DEC. 20, 19--	
FOR THE COMPANY		GAS BALANCE \$	
		ELEC. BALANCE \$	
		TOTAL \$	

14. A can of cream weighed 54 lb. net and tested 0.52 butter fat. If butter fat is worth 44¢ per pound, what is the value of the cream?

15. At \$7.50 per hundredweight, what is the cost of 15 lb. of sugar? of 35 lb. of sugar?

16. At \$9.50 per hundredweight, how many pounds of sugar can be bought for \$1? for \$0.75?

17. A merchant sold a stock of goods at 0.15 below cost, receiving \$1449.25 for the lot. What was his loss?

18. At \$18 per ton, what is the value of a load of coal weighing 3600 lb.?

The price per 1000 lb. is one half the price per ton. Hence the value may be found by using $3.6 \times \$9$.

19. The gross weight of a load of hay was 5830 lb.; the weight of the wagon was 1650 lb. At \$12.50 per ton what was the value of the load?

20. At $\$1.61\frac{2}{3}$ per hundredweight, find the value of the wheat in the following scale ticket:

STORAGE ELEVATOR CO.			
SOUTH BEND, IDAHO			
No. <u>285</u>	Sept. 14, 19--		
Load of <u>Wheat</u>	Gross weight	<u>4905</u>	lb.
From <u>H. G. Haynes</u>	Wagon weight	<u>1455</u>	lb.
To <u>L. S. Smith</u>	Net weight	<u>3450</u>	lb.
Received by <u>Frank C. Raymond</u>			

21. Find the total cost of 2850 lb. of Utah coal at \$18 per ton; 2400 lb. of Wyoming coal at \$19 per ton; and 1850 lb. of New Mexico coal at \$20 per ton.

ALIUOT PARTS

Aliquot part. An aliquot part of a number is a number which is contained in the given number an integral number of times.

For example, 50¢, 25¢, and $12\frac{1}{2}$ ¢ are all aliquot parts of \$1, since they are contained 2, 4, and 8 times respectively in \$1.

Much work in making computations can be saved by the use of aliquot parts, as they afford opportunities for many different short cuts.

For example, if it is required to know how many collars at 25¢ each can be bought for \$4, it is necessary to divide \$4 by \$0.25. If, however, it

is recognized at once that $25¢$ is $\frac{1}{4}$ of $\$1$, it is evident that $\$4$ will buy four times as many collars as there are units in the dollars expended; that is, dividing mentally by the fraction $\frac{1}{4}$, it is seen that $4 \times 4 = 16$.

Fractional part. Certain other numbers which are not contained in a given number an integral number of times are also used in short methods of making computations. These parts are called **fractional parts**, as distinguished from aliquot parts.

For example, since $25¢$ is $\frac{1}{4}$ of $\$1$, $75¢$ is $\frac{3}{4}$ of $\$1$. Similarly, $37\frac{1}{2}¢$, $62\frac{1}{2}¢$, $87\frac{1}{2}¢$, and so on are all fractional parts of $\$1$, being respectively $\frac{3}{8}$, $\frac{5}{8}$, and $\frac{7}{8}$ of $\$1$.

If it is required to find how many neckties at $75¢$ each can be bought for $\$6$, the result can be easily found by thinking $6 \times \frac{4}{3} = 8$.

Parts of $\$1$. The most commonly used aliquot and fractional parts of $\$1$ are given in the following table. These parts should be thoroughly learned so that they are instantly recognized.

$25¢$ is $\frac{1}{4}$ of $\$1$	$66\frac{2}{3}¢$ is $\frac{2}{3}$ of $\$1$	$37\frac{1}{2}¢$ is $\frac{3}{8}$ of $\$1$
$50¢$ is $\frac{1}{2}$ of $\$1$	$16\frac{2}{3}¢$ is $\frac{1}{6}$ of $\$1$	$62\frac{1}{2}¢$ is $\frac{5}{8}$ of $\$1$
$75¢$ is $\frac{3}{4}$ of $\$1$	$83\frac{1}{3}¢$ is $\frac{5}{6}$ of $\$1$	$87\frac{1}{2}¢$ is $\frac{7}{8}$ of $\$1$
$33\frac{1}{3}¢$ is $\frac{1}{3}$ of $\$1$	$12\frac{1}{2}¢$ is $\frac{1}{8}$ of $\$1$	$8\frac{1}{3}¢$ is $\frac{1}{12}$ of $\$1$

EXAMPLE. At $25¢$ per yard, how many yards of cloth can be bought for $\$8$?

Solution. Since $25¢$ is $\frac{1}{4}$ of $\$1$, the result is found by thinking $8 \div \frac{1}{4} = 8 \times 4 = 32$.

Hence 32 yd. of cloth can be bought for $\$8$.

ORAL EXERCISES

1. Find the number of yards that can be bought for each of the following amounts at $50¢$ per yard:

(a)	(b)	(c)	(d)	(e)	(f)	(g)
$\$12$	$\$24$	$\$30$	$\$15$	$\$20$	$\$18$	$\$21$
$\$10$	$\$11$	$\$14$	$\$17$	$\$32$	$\$60$	$\$44$
$\$22$	$\$13$	$\$19$	$\$25$	$\$40$	$\$26$	$\$50$

2. In Ex. 1 find the number of yards that can be bought at $25¢$ per yard; at $33\frac{1}{3}¢$ per yard; at $8\frac{1}{3}¢$ per yard; at $16\frac{2}{3}¢$ per yard; at $12\frac{1}{2}¢$ per yard.

3. Find the cost of each of the following purchases at $12\frac{1}{2}\phi$ per yard:

(a)	(b)	(c)	(d)	(e)
48 yd.	12 yd.	24 yd.	90 yd.	84 yd.
36 yd.	72 yd.	60 yd.	100 yd.	144 yd.
96 yd.	66 yd.	64 yd.	120 yd.	114 yd.

Since 1 yd. costs $\$1\frac{1}{2}$, 48 yd. cost $48 \times \$1\frac{1}{2}$, or $\$6$.

4. In Ex. 3 find the cost of each purchase at $37\frac{1}{2}\phi$ per yard; at $62\frac{1}{2}\phi$ per yard; at $87\frac{1}{2}\phi$ per yard; at 75ϕ per yard; at $83\frac{1}{3}\phi$ per yard.

5. State the number of pounds that can be bought for:

(a)	(b)
\$12 @ $37\frac{1}{2}\phi$ per pound	\$14 @ $66\frac{2}{3}\phi$ per pound
\$8 @ $8\frac{1}{3}\phi$ per pound	\$16 @ $12\frac{1}{2}\phi$ per pound
\$23 @ 50ϕ per pound	\$21 @ $87\frac{1}{2}\phi$ per pound
\$18 @ 75ϕ per pound	\$42 @ 25ϕ per pound
\$12 @ 25ϕ per pound	\$25 @ $83\frac{1}{3}\phi$ per pound
\$14 @ $33\frac{1}{3}\phi$ per pound	\$40 @ $62\frac{1}{2}\phi$ per pound

6. State the cost of each of the following:

(a)	(b)
24 yd. @ 25ϕ	54 yd. @ $16\frac{2}{3}\phi$
37 yd. @ 50ϕ	30 yd. @ $83\frac{1}{3}\phi$
21 yd. @ $66\frac{2}{3}\phi$	20 yd. @ 25ϕ
32 yd. @ $12\frac{1}{2}\phi$	40 yd. @ $37\frac{1}{2}\phi$
39 yd. @ $33\frac{1}{3}\phi$	22 yd. @ 25ϕ

7. State the number of yards that can be bought for:

(a)	(b)
\$22 @ $16\frac{2}{3}\phi$ per yard	\$18 @ $37\frac{1}{2}\phi$ per yard
\$35 @ 50ϕ per yard	\$28 @ $87\frac{1}{2}\phi$ per yard
\$26 @ $66\frac{2}{3}\phi$ per yard	\$16 @ $8\frac{1}{3}\phi$ per yard
\$25 @ $83\frac{1}{3}\phi$ per yard	\$28 @ $33\frac{1}{3}\phi$ per yard
\$15 @ 75ϕ per yard	\$30 @ $62\frac{1}{2}\phi$ per yard
\$17 @ 25ϕ per yard	\$45 @ $12\frac{1}{2}\phi$ per yard

Aliquot parts of other numbers. In the previous work the aliquot and fractional parts were all parts of \$1. It is obvious that the method of using these parts can be extended to other numbers.

For example, just as 25¢ is $\frac{1}{4}$ of \$1, so 0.25 is $\frac{1}{4}$ of 1, $2\frac{1}{2}$ is $\frac{1}{4}$ of 10, 25 is $\frac{1}{4}$ of 100, and so on.

By applying the same principles it is possible to develop many short methods of multiplication and division where the multiplier or divisor is some part of a power of 10.

Short methods of multiplication. The following illustrate several short methods, which the student should learn and apply in his work. Many others could be given, but with the principle firmly fixed the student will be able to see many cases where he can shorten the work of multiplication and division.

1. *To multiply by 25, multiply by 100 and take $\frac{1}{4}$ of the product.*

Since $25 = \frac{100}{4}$, $37 \times 25 = 3700 \div 4 = 925$.

2. *To multiply by 50, multiply by 100 and take $\frac{1}{2}$ of the product.*

Since $50 = \frac{100}{2}$, $49 \times 50 = 4900 \div 2 = 2450$.

3. *To multiply by 125, multiply by 1000 and take $\frac{1}{8}$ of the product.*

Since $125 = \frac{1000}{8}$, $376 \times 125 = 376,000 \div 8 = 47,000$.

4. *To multiply by 250, multiply by 1000 and take $\frac{1}{4}$ of the product.*

Since $250 = \frac{1000}{4}$, $116 \times 250 = 116,000 \div 4 = 29,000$.

5. *To multiply by 500, multiply by 1000 and take $\frac{1}{2}$ of the product.*

Since $500 = \frac{1000}{2}$, $319 \times 500 = 319,000 \div 2 = 159,500$.

Similar methods can be developed for multiplying by $1\frac{1}{4}$, $2\frac{1}{2}$, $12\frac{1}{2}$, $33\frac{1}{3}$, $16\frac{2}{3}$, $6\frac{1}{4}$, $8\frac{1}{3}$, and so on, by following the same plan; that is, *in multiplying by an aliquot part of a power of 10, multiply by the power of 10 and take that fractional part of the product that the multiplier is of the power of 10.*

Consider the following examples :

Since $1\frac{1}{4} = \frac{10}{8}$, $28 \times 1\frac{1}{4} = 280 \div 8 = 35$.

Since $16\frac{2}{3} = \frac{100}{6}$, $48 \times 16\frac{2}{3} = 4800 \div 6 = 800$.

Since $6\frac{1}{4} = \frac{100}{16}$, $984 \times 6\frac{1}{4} = 98,400 \div 16 = 6150$.

The student will find many similar cases and should seek to extend the use of aliquot parts wherever it will save work.

EXERCISES

1. Multiply in turn by 25 and by 50: 274; 57; 144; 327; 306; 255.

2. Multiply in turn by 125 and by 250: 76; 368; 502; 184; 224; 840.

3. Multiply in turn by 500: 84; 126; 96; 376; 328; 156.

Multiply each of the following numbers by $1\frac{1}{4}$; by $2\frac{1}{2}$; by $3\frac{1}{3}$; by $33\frac{1}{3}$; by $16\frac{2}{3}$; by $166\frac{2}{3}$:

4. 240	120	360	720	960	126
5. 480	540	840	900	320	726
6. 60	300	640	660	980	864

Perform the following multiplications:

7. $54 \times 16\frac{2}{3}$	$64 \times 6\frac{1}{4}$	$36 \times 166\frac{2}{3}$
8. $44 \times 2\frac{1}{2}$	$36 \times 16\frac{2}{3}$	$160 \times 112\frac{1}{2}$
9. $52 \times 1\frac{1}{4}$	$27 \times 333\frac{1}{3}$	120×1.25
10. $96 \times 3\frac{1}{3}$	$75 \times 666\frac{2}{3}$	$744 \times 8\frac{1}{3}$

Multiplying with fractional parts. When multiplying with such fractional parts of 100 as $37\frac{1}{2}$, $62\frac{1}{2}$, $87\frac{1}{2}$, $66\frac{2}{3}$, $83\frac{1}{3}$, and so on, short methods similar to those of aliquot parts can be used.

1. To multiply by $37\frac{1}{2}$, multiply by 300 and take $\frac{1}{8}$ of the product.

$$\text{Since } 37\frac{1}{2} = \frac{3}{8} \text{ of } 100 = \frac{300}{8}, 48 \times 37\frac{1}{2} = \frac{48 \times 300}{8} = 1800.$$

2. To multiply by $66\frac{2}{3}$, multiply by 200 and take $\frac{1}{3}$ of the product.

$$\text{Since } 66\frac{2}{3} = \frac{2}{3} \text{ of } 100 = \frac{200}{3}, 96 \times 66\frac{2}{3} = \frac{96 \times 200}{3} = 6400.$$

In solving the following exercises the student should develop methods similar to those given above.

EXERCISES

Copy and find the total cost in each of the following :

1. 24 yd. @ 25¢	2. 32 yd. @ $12\frac{1}{2}$ ¢	3. 98 yd. @ 4¢
36 yd. $33\frac{1}{3}$ ¢	48 yd. 75¢	19 yd. 15¢
42 yd. 50¢	30 yd. $83\frac{1}{3}$ ¢	114 yd. $37\frac{1}{2}$ ¢
21 yd. $66\frac{2}{3}$ ¢	12 yd. $8\frac{1}{3}$ ¢	109 yd. 13¢

In all cases where a result is a fraction of a cent, discard the fraction if it is less than $\frac{1}{2}$ ¢; if $\frac{1}{2}$ ¢ or greater, give the result as the next full cent.

Without copying, find the total cost in each of the following :

4. 42 yd. @ 11¢	6. 36 yd. @ $8\frac{1}{3}$ ¢	8. 120 yd. @ $33\frac{1}{3}$ ¢
33 yd. 22¢	72 yd. $37\frac{1}{2}$ ¢	144 yd. $66\frac{2}{3}$ ¢
52 yd. $33\frac{1}{3}$ ¢	96 yd. $12\frac{1}{2}$ ¢	166 yd. 25¢
25 yd. 25¢	48 yd. $62\frac{1}{2}$ ¢	230 yd. 11¢
5. 38 yd. @ 20¢	7. 160 yd. @ $6\frac{1}{4}$ ¢	9. 45 yd. @ \$1.66 $\frac{2}{3}$
47 yd. 30¢	78 yd. $2\frac{1}{2}$ ¢	75 yd. \$1.33 $\frac{1}{3}$
56 yd. 40¢	93 yd. $3\frac{1}{3}$ ¢	88 yd. \$1.12 $\frac{1}{2}$
74 yd. 50¢	84 yd. $8\frac{1}{3}$ ¢	24 yd. \$1.87 $\frac{1}{2}$

Without copying, find the total cost in each of the following :

10. 24 A. @ \$50	57 A. @ \$66 $\frac{2}{3}$	68 A. @ \$100
48 A. \$125	117 A. \$50	72 A. \$62.50
120 A. \$75	32 A. \$87.50	84 A. \$150
54 A. \$83 $\frac{1}{3}$	60 A. \$125	36 A. \$133 $\frac{1}{3}$
11. 36 T. @ \$12.50	25 T. @ \$16.40	25 T. @ \$25
50 T. \$17.50	20 T. \$20	72 T. \$11
25 T. \$15	6 $\frac{1}{4}$ T. \$16	30 T. \$12.50
23 T. \$11	42 T. \$25	15 T. \$15

Short methods in division. The short methods used in multiplying with aliquot and fractional parts can be reversed and applied to division, as follows:

1. *To divide by 25, multiply by 4 and divide by 100.*

Since $25 = \frac{100}{4}$, $96 \div 25 = \frac{96 \times 4}{100} = 3.84$; the division by 100 being accomplished by simply pointing off two places in the product.

2. To divide by $66\frac{2}{3}$, multiply by 3 and divide by 200.

$$\text{Since } 66\frac{2}{3} = \frac{200}{3}, 38 \div 66\frac{2}{3} = \frac{38 \times 3}{200} = 0.57.$$

The student should now see that any short method of division is simply the method for multiplying by the same number inverted, so that it is unnecessary to learn any separate methods for division.

EXERCISES

1. Divide 180 by $1\frac{1}{4}$.

5. Divide 750 by $66\frac{2}{3}$.

2. Divide 650 by $2\frac{1}{2}$.

6. Divide 1200 by $37\frac{1}{2}$.

3. Divide 480 by $16\frac{2}{3}$.

7. Divide 1250 by 250.

4. Divide 540 by $33\frac{1}{3}$.

8. Divide 1260 by $83\frac{1}{3}$.

9. At \$1.25 per yard, how many yards of cloth can be bought for \$10 ? for \$20 ? for \$40 ? for \$50 ? for \$100 ?

10. At \$2.50 per yard, how many yards of cloth can be bought for \$100 ? for \$200 ? for \$400 ? for \$500 ? for \$600 ? for \$1000 ?

11. At $\$1.66\frac{2}{3}$ per yard, how many yards of cloth can be bought for \$10 ? for \$20 ? for \$30 ? for \$60 ? for \$100 ?

12. At \$37.50 per acre, how many acres of land can be bought for \$250 ? for \$450 ? for \$700 ? for \$850 ? for \$1000 ?

13. At \$62.50 per acre, how many acres of land can be bought for \$625 ? for \$750 ? for \$1250 ? for \$2000 ? for \$3000 ? for \$4000 ? for \$5000 ?

14. Find the number of tons that can be purchased for each of the following amounts at \$12.50 per ton, at \$20 per ton; at \$25 per ton:

\$150

\$200

\$300

\$75

\$125

\$250

\$120

\$240

\$360

15. Find the number of acres that can be bought for each of the following amounts at \$75 per acre ; at \$125 per acre ; at \$200 per acre :

\$1000	\$750	\$500
\$1200	\$1500	\$600
\$900	\$800	\$2000

Aliquot and fractional parts in business. Aliquot and fractional parts are used extensively in extending inventories and invoices.

EXERCISES

Without copying, make the extensions and find the totals in each of the following inventories :

1.			2.		
QUANTITY	PRICE	EXTENSIONS	QUANTITY	PRICE	EXTENSIONS
27 yd.	@ $66\frac{2}{3}\phi$		43 yd.	@ $33\frac{1}{3}\phi$	
33 "	75ϕ		42 "	$83\frac{1}{3}\phi$	
57 "	25ϕ		40 "	98ϕ	
126 "	$8\frac{1}{3}\phi$		51 "	80ϕ	
56 "	$87\frac{1}{2}\phi$		67 "	30ϕ	
64 "	$37\frac{1}{2}\phi$		$59\frac{1}{2}$ "	20ϕ	
19 "	40ϕ		114 "	9ϕ	
20 "	97ϕ		42 "	55ϕ	
98 "	25ϕ		55 "	8ϕ	
210 "	$33\frac{1}{3}\phi$		48 "	\$1.25	
30 "	35ϕ		164 "	25ϕ	
58 "	8ϕ		68 "	75ϕ	
39 "	\$1. $33\frac{1}{3}$		$33\frac{1}{4}$ "	40ϕ	
165 "	5ϕ		86 "	10ϕ	
101 "	13ϕ		36 "	\$1.25	
73 "	70ϕ		34 "	40ϕ	
25 "	25ϕ		30 "	72ϕ	
80 "	23ϕ		45 "	45ϕ	
150 "	\$1.50		75 "	75ϕ	
74 "	$33\frac{1}{3}\phi$		18 "	$66\frac{2}{3}\phi$	
186 "	25ϕ		23 "	75ϕ	

3.			4.		
QUANTITY	PRICE	EXTENSIONS	QUANTITY	PRICE	EXTENSIONS
36 yd.	@ 8¢		79 yd.	@ 50¢	
84 "	12 $\frac{1}{2}$ ¢		64 "	37 $\frac{1}{2}$ ¢	
160 "	\$1.12 $\frac{1}{2}$		123 "	40¢	
75 "	\$1.33 $\frac{1}{3}$		93 "	66 $\frac{2}{3}$ ¢	
29 "	3 $\frac{1}{2}$ ¢		56 "	\$1.25	
112 "	2 $\frac{1}{2}$ ¢		124 "	12 $\frac{1}{2}$ ¢	
88 "	8¢		79 "	7¢	
119 "	50¢		95 "	25¢	
25 "	69¢		50 "	99¢	
36 $\frac{1}{2}$ "	8¢		35 $\frac{1}{2}$ "	12¢	
120 "	87 $\frac{1}{2}$ ¢		320 "	6 $\frac{1}{4}$ ¢	
54 "	2 $\frac{1}{2}$ ¢		81 "	3 $\frac{1}{3}$ ¢	
30 "	9 $\frac{1}{2}$ ¢		108 "	16 $\frac{2}{3}$ ¢	
44 "	\$1.10		37 "	\$1.50	
35 "	\$2.50		32 "	\$1.75	
125 "	25¢		155 "	30¢	
80 "	17¢		60 "	19¢	
47 "	11¢		52 "	22¢	
75 "	\$1.33 $\frac{1}{3}$		58 "	\$1.50	

Without copying, make the extensions and find the totals in each of these invoices :

5.

5	bags Beans	160 lb. each	@	6 $\frac{1}{4}$ ¢			
5	" Potatoes	120 lb. each		3 $\frac{1}{2}$ ¢			
5	bunches Bananas	37, 45, 38, 50, 56 lb.		8 $\frac{1}{3}$ ¢			
5	bags Onions	100 lb. each		1 $\frac{1}{4}$ ¢			
5	cans Crackers	6 $\frac{1}{2}$, 6 $\frac{1}{4}$, 6 $\frac{3}{4}$, 7, 6 $\frac{1}{4}$ lb.		18¢			
25	boxes Prunes	25 lb. each		12 $\frac{1}{2}$ ¢			
5	" Chocolate	24 $\frac{1}{2}$ lb. each		44¢			
5	" Walnut Meats	50 lb. each		50¢			
5	chests Tea	132 lb. each		33 $\frac{1}{3}$ ¢			
24	bags Flour	$\frac{1}{8}$ bbl. each		\$10.50			
5	bbl. Sugar	354, 360, 364, 365, 363 lb.		6 $\frac{1}{4}$ ¢			

The prices in this invoice are given by the pound, except in the case of the flour, where the unit is the barrel, which weighs 196 lb. net.

6.

6	Motor Meters	@ \$15.00			
12	Nickeled Bumpers	12.50			
12	Bevel Plate Mirrors	3.50			
4	Inclosed Drive Mirrors	5.75			
10	Set Clocks, Rim Wind	14.75			
4	Route Books	2.75			
6	Adjustable Carburetor Foot Rests	1.60			
6	Adjustable Luggage Carriers	7.75			
8	Auto Cushions	2.25			
12	Windshield Cleaners	2.45			
4	Electric Hand Warmers	10.00			
5	Hand Fire Extinguishers	9.75			
12	Auto 3-1 Lamps	10.50			
18	Wool Dusters	1.50			
10	Tire Pressure Gauges	1.75			
4	Sets Tire Chains	6.25			
8	" Tire Covers	3.50			
6	Spare-Tire Chains and Locks	2.75			
8	Leather Door Pads	1.65			

7.

LOT NO.	QUANTITY	DESCRIPTION	PER DOZEN			
204	4 $\frac{8}{12}$ doz.	Railroad Overalls	@ \$16.50			
205	2 $\frac{2}{12}$ "	Coats	16.50			
4	4 $\frac{1}{12}$ "	Railroad Overalls	22.00			
4x	6 $\frac{4}{12}$ "	" "	23.00			
5	1 $\frac{6}{12}$ "	Coats	22.50			
5x	$\frac{2}{12}$ "	"	23.75			
14	1 $\frac{8}{12}$ "	Railroad Overalls	15.75			
14x	$\frac{6}{12}$ "	" "	16.70			
15	1 $\frac{8}{12}$ "	Coats	15.35			
504	1 $\frac{9}{12}$ "	Railroad Overalls	19.50			
504x	$\frac{10}{12}$ "	" "	20.50			
505	$\frac{10}{12}$ "	Coats	19.50			
505x	$\frac{4}{12}$ "	"	20.50			
164	3 $\frac{7}{12}$ "	Railroad Overalls	20.00			
165	$\frac{4}{12}$ "	Coats	22.00			
34	1 $\frac{2}{12}$ "	Railroad Overalls	13.50			
34x	1 $\frac{11}{12}$ "	" "	14.50			
35	$\frac{2}{12}$ "	Coats	13.50			
35x	1 $\frac{4}{12}$ "	"	14.50			

8.

6 pc.	45, 52 ¹ , 41, 50 ² , 56 ³ , 49	American Silk @ \$1.25			
10 "	42 ¹ , 42, 47 ² , 44, 46 ² , 41,				
	45 ³ , 48 ¹ , 43, 40 ³	Striped Denim	1.12 $\frac{1}{2}$		
10 "	49 ² , 40, 43 ³ , 45, 42 ² ,				
	48, 49 ¹ , 45, 47 ² , 50	Fancy Gingham	.16 $\frac{3}{4}$		
5 "	49 ³ , 44, 46 ² , 40, 46 ¹	Wash Silk	.50		
10 "	52, 47 ³ , 42 ¹ , 45, 48 ² ,				
	49 ¹ , 47, 42 ³ , 43, 44 ²	English Serge	1.65		
6 "	43, 41 ² , 44, 47 ² , 40, 48	Fancy Plaids	.62 $\frac{1}{2}$		
5 "	44 ³ , 45, 46 ² , 43, 47 ¹	Fancy Stripe	.20		

9.

10 pc.	52 ¹ , 50, 52, 56, 42 ³ ,				
	48, 53 ² , 54, 50 ¹ , 51 ²	Gunner's Duck @ \$0.25			
5 "	47, 41, 48, 46, 49	Scotch Cheviot	.62 $\frac{1}{2}$		
10 "	52, 45, 40, 41, 42,				
	43, 52, 51, 52, 47	Taffeta Silk	1.25		
10 "	52, 51, 60, 60, 61,				
	60, 62, 61, 58, 53	Special Cotton	.46		
10 "	42, 48, 52, 58, 62,				
	38, 49, 51, 54, 46	Percale Shirting	.13 $\frac{1}{2}$		
6 "	47, 52, 62, 43, 45, 46	Amazon Cloth	1.37 $\frac{1}{2}$		

10.

$\frac{1}{2}$	doz. Dinner Plates	\$9.25			
$\frac{1}{2}$	doz. Breakfast Plates	8.45			
1 $\frac{1}{2}$	doz. Bread and Butter Plates	6.65			
1 $\frac{1}{3}$	doz. Sauce Dishes	6.45			
2 $\frac{1}{4}$	doz. Tea Cups and Saucers	14.35			
3 $\frac{1}{2}$	doz. Platters	3.48 ea.			

For additional problems see page 399.

CHAPTER III

PRACTICAL MEASUREMENTS

DENOMINATE NUMBERS

Measurements in business. Many types of business computations are made with numbers which represent measures of some sort, such as length, area, weight, and the like. To make such computations with skill and accuracy requires familiarity with the tables of common measures.

ORAL EXERCISES

1. Read and review the tables of measures given on pages 399-402 of the Appendix, noting particularly the different abbreviations used for the measures.

Name two or more units used in measuring each of the following:

- | | |
|------------------------------|----------------------------|
| 2. Length. | 8. United States money. |
| 3. Counting by 12. | 9. English money. |
| 4. Counting sheets of paper. | 10. Avoirdupois weight. |
| 5. Volume (cubic measure). | 11. Angular measure. |
| 6. Time. | 12. Area (square measure). |
| 7. Liquid measure. | 13. Dry measure. |

Denominate numbers. All numbers are either **concrete** or **abstract**; concrete when applied to a particular object or measure, and abstract when not so applied. A **denominate number** is a concrete number which specifies a number of units of some measure.

For example, 4 and 8 are abstract numbers, 4 barns and 8 houses are concrete numbers, and 4 ft. and 8 lb. are denominate numbers.

Reducing denominate numbers. Sometimes denominate numbers specify two or more units (or denominations) of measure, as 5 ft. 9 in., 3 lb. 8 oz., and the like.

It is occasionally necessary to express such numbers in terms of smaller or larger units. The first case is called **reducing to lower denominations** and the second **reducing to higher denominations**.

EXAMPLES. 1. Reduce 5 ft. 9 in. to inches.

Solution. 5 ft. = 12 in. $\times 5 = 60$ in. 5 ft. 9 in. = 60 in. + 9 in. = 69 in.

2. Reduce 102 oz. to pounds and ounces.

Solution. 102 oz. $\div 16$ oz. = 6 with a remainder of 6 oz.

Hence 102 oz. = 6 lb. 6 oz.

The reduction of denominate numbers has lost most of its practical importance, since, with the exception of feet and inches, such numbers are now rarely used. Today 16 lb. 4 oz. is more often referred to as $16\frac{1}{4}$ lb., and similarly for other measures. Computations with such numbers are made with the fractional forms, or, with increasing frequency, with the decimal forms.

The reducing of such numbers as 337 in. to 9 yd. 1 ft. 1 in. is practically obsolete. If the length is wanted in feet and inches, it is given as 28 ft. 1 in.; if in yards and inches, it is given as 9 yd. 13 in. The purpose of the following exercises is to test the student's familiarity with the tables of measures and to illustrate the principle of reducing awkward expressions to more convenient and usual forms.

EXERCISES

Reduce the following expressions:

1. 15 lb. 10 oz. to ounces; 12.4 lb. to ounces.
2. 2 T. 655 lb. to pounds; 2.8 cwt. to pounds.
3. 140 rd. 2 ft. to feet; 1 mi. 440 yd. to feet.
4. 2 cu. ft. 750 cu. in. to cubic inches.
5. 8 sq. rd. 20 sq. yd. 7 sq. ft. to square feet.
6. $2\frac{1}{2}$ A. to square rods; 12.8 A. to square rods.
7. 4 hr. 58 min. 12 sec. to seconds.
8. $65\frac{3}{4}$ gal. to pints; 27.3 gal. to quarts.

9. $2\frac{3}{8}$ yd. to inches; 10.37 ft. to inches.
10. 24 bu. 7 qt. to pints; $34\frac{5}{8}$ bu. to quarts.
11. 465 min. to hours and minutes; to hours and a fraction.
12. 66 oz. to pounds and ounces; to pounds and a fraction.
13. 350 sq. in. to square feet and square inches.
14. 400 pt. to gallons; $27\frac{1}{2}$ pt. to quarts and a fraction.
15. 4800 seconds of arc to degrees and a fraction.

Reduce the following expressions as indicated, giving the results with decimals correct to the nearest hundredth:

16. 400 ft. to rods.
17. 23,450 lb. to tons.
18. 5430 sq. in. to square feet.
19. 50,000 sq. ft. to acres.
20. 5243 qt. to bushels.
21. 529 min. to hours.
22. 279 articles to gross.
23. 2768 cu. ft. to cubic yards.
24. 25 ft. 10 in. to feet; 2 ft. $4\frac{7}{8}$ in. to feet.
25. 2 gal. $3\frac{1}{2}$ qt. to gallons; 5 pk. $2\frac{1}{4}$ qt. to pecks.
26. 35 s. 8 d. and £4 18 s. 6 d. to pounds sterling.

Adding and subtracting denominate numbers. If denominate numbers involving two or more units are set down with the similar denominations in columns, addition and subtraction can be performed with these numbers the same as with whole numbers. The results should be reduced to convenient form, generally that of higher denominations.

As already stated, except for feet and inches, the fundamental operations with such numbers are of little practical value, as it is easier and quicker to work with the numbers in fractional or decimal form.

EXAMPLES. 1. Add 6 ft. 3 in., 11 ft. 9 in., and 13 ft. 5 in.

Solution. Set the numbers down with the different denominations in the proper columns. Adding as with the whole numbers, the result is found to be 30 ft. 17 in., or, in more convenient form, 31 ft. 5 in.

6 ft.	3 in.
11	9
13	5
30 ft.	17 in.
or 31 ft.	5 in.

2. From 29 ft. 3 in. subtract 13 ft. 11 in.

Solution. Set the numbers down in their proper columns. Since it is impossible to take 11 in. from 3 in., the 3 in. is increased by a unit taken from the first column. Then since 1 ft. = 12 in., the upper number is thought of as 28 ft. 15 in. The subtraction is then performed as usual, the result being 15 ft. 4 in.

29 ft.	3 in.
13	11
15 ft.	4 in.

EXERCISES

Add the following :

- 1 yr. 6 mo. 5 da. and 2 yr. 7 mo. 15 da.
- 5 lb. 10 oz., 3 lb. 12 oz., and 7 lb. 14 oz.
- 2 mi. 50 rd., 1 mi. 100 rd., and 1 mi. 80 rd.
- 5 yd. 2 ft., 4 yd. 1 ft., 3 yd., 4 yd. 1 ft., and 1 yd. 1 ft.
- 25 T. 500 lb., 18 T. 350 lb., 1 T. 150 lb., 10 T. 1500 lb., 21 T. 1225 lb., 15 T. 146 lb., and 18 T. 947 lb.
- 4 gal. 2 qt. 1 pt., 3 gal. 1 qt., 2 gal. 1 pt., 1 qt. 1 pt., 8 gal. 2 qt., 3 qt. 1 pt., 6 gal. 2 qt. 1 pt., and 2 gal. 1 pt.
- 1 ft. 10 in., 3 ft. 8 in., 12 ft. 4 in., 15 ft. 3 in., 11 ft., 4 ft. 15 in., 9 ft. 6 in., 10 ft. 2 in., 7 ft. 4 in., 12 ft. 7 in., 20 ft. 9 in., and 16 ft. 11 in.
- 30 sq. ft. 100 sq. in., 10 sq. ft. 48 sq. in., 18 sq. ft. 125 sq. in., 96 sq. ft. 55 sq. in., 82 sq. ft. 120 sq. in., and 16 sq. ft. 10 sq. in.
- 12 cu. ft. 500 cu. in., 21 cu. ft. 725 cu. in., 4 cu. ft. 1260 cu. in., 15 cu. ft. 144 cu. in., 2 cu. yd. 25 cu. ft. 1500 cu. in., and 1 cu. yd. 1450 cu. in.

Subtract the following :

- | | | |
|------------------------------|---|----------------------|
| 10. 10 yd. 2 ft. 12 in. | 11. 12 yd. 1 ft. 7 in. | 12. 12 T. 1420 lb. |
| <u>6 yd. 1 ft. 8 in.</u> | <u>8 yd. 2 ft. 6 in.</u> | <u>8 T. 1675 lb.</u> |
| 13. 16 hr. 8 min. 39 sec. | 14. 15 sq. yd. 5 sq. ft. 100 sq. in. | |
| <u>7 hr. 49 min. 52 sec.</u> | <u>12 sq. yd. 8 sq. ft. 125 sq. in.</u> | |

15. From a bin containing 50 T. of coal the following amounts were removed: 10 T. 250 lb., 15 T. 1250 lb., and 4 T. 600 lb. How much coal remained in the bin?

16. From 25 bu. of potatoes two lots were taken as follows: 8 bu. 3 pk. and 12 bu. 2 pk. How much remained?

17. From a bolt of cloth containing 50 yd., the following amounts had been sold: 3 yd. 2 ft., $5\frac{1}{2}$ yd., 8 yd. 1 ft. 3 in., and 6 yd. 9 in. How much cloth remained unsold?

18. From a piece of beef weighing $39\frac{1}{2}$ lb. a butcher sold the following cuts: 4 lb. 12 oz., $7\frac{1}{2}$ lb., $6\frac{3}{8}$ lb., and 5 lb. 11 oz. How much was left?

Time between two dates. Compound subtraction is sometimes used in determining the time between two dates.

EXAMPLE. Find the time between March 17 and Nov. 10 of the same year.

Solution. March 17 is the 17th day of the 3d month and Nov. 10 is the 10th day of the 11th month. Hence the dates may be written as shown at the right. In such cases the month is considered as 30 da. In subtracting, the upper date is thought of as 10 mo. 40 da., and the result is found to be 7 mo. 23 da.

mo.	da.
11	10
3	17
7	23

EXERCISES

Find the time between the following dates in the same year:

- | | |
|-------------------------|------------------------|
| 1. April 1 and July 15. | 4. Feb. 11 and Nov. 2. |
| 2. May 3 and Aug. 12. | 5. Aug. 25 and Dec. 5. |
| 3. May 23 and Dec. 15. | 6. Jan. 17 and Nov. 3. |

Find the time between the following dates:

7. Oct. 12, 1923, and July 5, 1924.
8. Feb. 12, 1923, and April 15, 1925.
9. July 28, 1923, and Nov. 11, 1925.
10. May 14, 1925, and May 4, 1927.

In Exs. 7-10 use the method of the above example with a third column for the year.

Multiplying and dividing with denominate numbers. Multiplication and division is performed with denominate numbers by multiplying or dividing the parts of the numbers separately and then expressing the results in a convenient form.

EXAMPLES. 1. Find the distance around a square which is 5 ft. 8 in. on a side.

Solution. Since each of the sides is 5 ft. 8 in., the distance around the square is 5 ft. 8 in. $\times$ 4. Multiplying the parts of the number separately, as shown at the right, the result is 20 ft. 32 in. Expressed in better form, the result is 22 ft. 8 in.

5 ft.	8 in.
	4
20 ft.	32 in.
or 22 ft.	8 in.

2. The total weight of 5 loads of coal was 7 T. 500 lb. What was the average weight per load?

Solution. The average is found by dividing the weight by 5. Dividing the 7 T. as shown at the right, it is found that there is a remainder of 2 T., or 4000 lb. Adding the 4000 lb. to the 500 lb. and dividing by 5, the average weight is found to be 1 T. 900 lb.

5	) 7 T. 500 lb.
	1 T. 900 lb.

In many cases it is easier to reduce the number to the lower denomination before dividing. The result is then reduced to higher denominations if it will give a more convenient form. Thus, in the above example, 7 T. 500 lb. might have been expressed as 14,500 lb., and then the result, 2900 lb., reduced to 1 T. 900 lb. In the following exercises the student should perform the divisions by the most convenient method.

EXERCISES

Multiply the following:

1. 3 ft. 9 in. by 10.

5. 12 rd. 4 yd. 1 ft. by 3.

2. 5 lb. 12 oz. by 8.

6. 8 T. 5 cwt. by 4.

3. 3 qt. 2 pt. by 5.

7. 5 A. 80 sq. rd. by 5.

4. 4 gal. 3 qt. by 4.

8. 4 bu. 2 pk. 3 qt. by 6.

9. 40 sq. rd. 25 sq. yd. 20 sq. ft. by 3.

10. £3 12 s. 6 d. by 5.

Divide the following :

- | | |
|-------------------------|-------------------------|
| 11. 12 T. 500 lb. by 4. | 14. 38 qt. 1 pt. by 12. |
| 12. 73 ft. 8 in. by 6. | 15. 9 yd. 32 in. by 8. |
| 13. 62 lb. 8 oz. by 5. | 16. 32 bu. 7 pk. by 6. |
17. Divide 25 cu. ft. 1200 cu. in. into three equal parts.
 18. Divide 42 yd. 2 ft. 9 in. into five equal parts.
 19. Divide 42 cu. ft. 1500 cu. in. into six equal parts.
 20. Divide 14 A. 122 sq. rd. into five equal parts.

Statutory weights. The practice of selling farm and garden produce by the bushel is becoming less common, and instead there is a growing tendency to sell such commodities by weight. The weights per bushel of these commodities are fixed by law in the different states, and are known as **statutory weights**. A few of these weights, together with the exceptions in certain of the states, are shown in the following table :

COMMODITIES	STATUTORY WEIGHT IN POUNDS	EXCEPTIONS
Barley	48	Alabama, Georgia, Kentucky, Pennsylvania, 47 ; Arizona, 45 ; California, 50.
Beans	60	
Clover seed	60	
Corn, shelled	56	Arizona, 54 ; California, 52.
Oats	32	Maine, New Jersey, Virginia, 30 ; Maryland, 26.
Potatoes, Irish	60	Maryland, Pennsylvania, Virginia, 56.
Rye	56	California, 54.
Wheat	60	

EXERCISES

- Find the value in Texas of 2500 lb. of oats at 40¢ per bushel.
- Find the value of the oats in Ex. 1 if they are held at the same price in New Jersey ; in Maryland.

3. Find the value in New York of 15,250 lb. of potatoes at \$1.12 per bushel; of 25,400 lb. of potatoes at \$1.04 per bushel.

4. Find the values in Ex. 3 if the potatoes are held in Virginia. What is the difference in values between the two states?

5. Find the total value of the following:

1260 lb. of wheat at \$1.20 per bushel

3540 lb. of wheat at \$1.14 per bushel

2435 lb. of wheat at \$1.12 per bushel

6. Find the total value of the following:

245 lb. of clover seed at \$5.00 per bushel

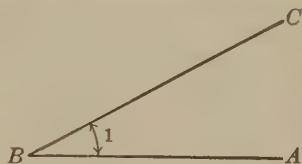
960 lb. of clover seed at \$4.75 per bushel

1550 lb. of clover seed at \$4.25 per bushel

RECTILINEAR PLANE FIGURES

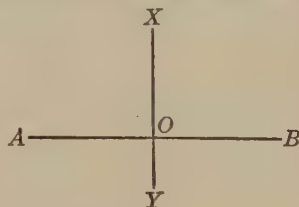
Angle. An angle is a figure formed by two straight lines which meet in a point.

The figure at the right shows an angle which is designated by the letters A , B , and C , or the number 1 placed within the angle. The angle is read: "the angle ABC ," or "the angle 1." The point B where the lines meet is called the **vertex** of the angle, and the lines BA and BC the **sides** of the angle.



Right angle. If one line cuts another so as to make four equal angles, the angles formed are called **right angles**. The lines are then said to be **perpendicular** to each other.

In the figure at the right, the angles XOA , AOY , YOB , and BOX are right angles, since the four angles are equal.



Degree. The angle which is equivalent to one ninetieth of a right angle is taken as the unit of measure of angles. This unit is called a **degree** (1°).

There are 90° in a right angle and 360° in the sum of all the angles about a point, such as the point O in the figure above.

Plane surface. The top surface of a school desk, the surface of the blackboard, or the surface of still water are examples of a **plane surface** or a **plane**. In each case there is measurable length and breadth, but no thickness.

Plane figure. A **plane figure** is a figure all of whose lines lie in the same plane surface.

A figure drawn on the blackboard is a good illustration of a plane figure.

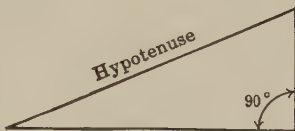
Triangle. A **triangle** is a closed plane figure which is bounded by three straight lines. A triangle, therefore, has three sides and three angles.

The sum of the three angles of a triangle is shown in geometry to be 180° .



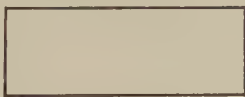
TRIANGLE

Right triangle. A **right triangle** is a triangle which has a right angle, that is, an angle of 90° . The side opposite the right angle is called the **hypotenuse**.



RIGHT TRIANGLE

Rectangle. A **rectangle** is a four-sided closed plane figure whose sides are straight lines and whose angles are right angles.



RECTANGLE

Square. A **square** is a rectangle all of whose sides are equal.



SQUARE

Perimeter. The **perimeter** of a closed plane figure is the sum of the lengths of the lines which form it; that is, it is the distance around the figure.

EXERCISES

1. The sides of a triangle are 3 ft. 2 in., 4 ft. 8 in., and 4 ft. 9 in. Find the perimeter of the triangle.
2. Each of the three sides of a triangle is 2 ft. 7 in. Find the perimeter of the triangle.
3. Find the perimeter of a square 2 ft. 4 in. on a side.

4. The adjoining sides of a rectangular field are 40 rd. 2 ft. and 20 rd. 11 ft. respectively. Find the perimeter of the field.

The opposite sides of a rectangle are equal.

5. Measure the length and width of your classroom and find the perimeter of the room.

6. How much picture molding must be used to go once around a room 16 ft. 6 in. by 20 ft. 9 in.?

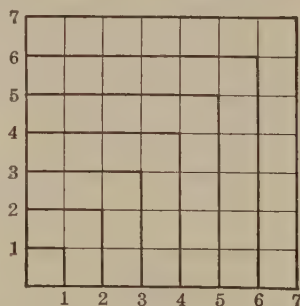
7. How much molding is required in making frames for ten pictures each 18 in. by 22 in.?

8. How much barbed wire is needed to fence in two rectangular fields with a four-strand fence if one of the fields is 250 ft. by 350 ft. and the other is 640 ft. by 1200 ft.?

9. How many pounds of wire are needed in Ex. 8 if the wire runs 16 ft. to the pound? What is the total cost of the wire at \$5.50 per hundredweight?

Area of a square. A square which is 1 in. on a side is said to have an **area** of 1 sq. in. (1 square inch); a square which is 1 ft. on a side an area of 1 sq. ft.; and so on. The square inch, square foot, square yard, and so on are the **units of area**, according as the figures considered are measured in inches, feet, or yards.

If one of the small squares in the figure at the right represents 1 sq. in., it is readily seen by counting that a square 2 in. on a side contains 4 small squares, or 4 sq. in.; a square 3 in. on a side has an area of 9 sq. in.; and so on. It is thus seen that



The area of a square in square units is equal to the number of units in its side times the number of units in its side.

This statement may be shortened as follows:

Area of a square = (number of units in side) $\times$ (number of units in side) square units.

If it is understood that A shall stand for "area in square units" and s for "number of units in its side," the above expression may be still further shortened by writing

$$A = s^2,$$

in which s^2 means $s \times s$. Since, if the units in s are inches, the area is in square inches, if s is in feet, the area is in square feet, and so on, there can be no misunderstanding of what A represents.

Such a method of expressing a rule is called a **formula**, and is easier to remember than a rule. In applying a formula to a problem it is necessary to substitute values for the letters, compute the result, and assign to it its proper denomination.

EXAMPLE. Find the area of a square which is 9 in. on a side.

Solution. In this case, s in the formula represents 9 in.

Then $A = s^2 = 9 \times 9 = 81$.

Since s is a dimension in inches, A is in square inches, and hence the area of the square is 81 sq. in.

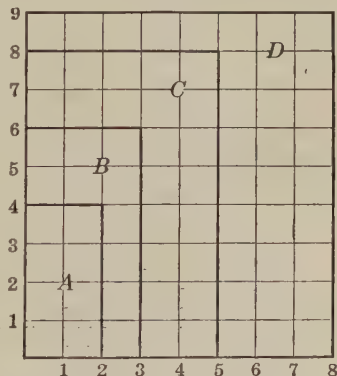
Area of a rectangle. In the figure below, if each small square represents 1 sq. in., it is readily seen by counting that rectangle A , which is 4 in. long and 2 in. wide, contains 8 small squares, or 8 sq. in. Similarly, rectangle B contains 6×3 , or 18 sq. in.; rectangle C , 8×5 , or 40 sq. in., and rectangle D , 9×8 , or 72 sq. in. Thus it is seen that

Area of a rectangle = (number of units in length) $\times$ (number of units in width) square units.

If A is allowed to stand for "area in square units," l for "number of units in length," and w for "number of units in width," the above rule may be written as a formula as follows:

$$A = lw,$$

in which it is understood that the writing of l and w without any sign means that their product is to be taken.



EXAMPLE. Find the area of a rectangle which is 11 in. long and 7 in. wide.

Solution. Here l in the formula is 11 in. and w is 7 in.

Then $A = lw = 11 \times 7 = 77$.

Hence the area of the rectangle is 77 sq. in.

EXERCISES

Find the areas of squares which have the following sides :

1. 11 in. 3. 3 ft. 3 in. 5. 44 rd.

2. 2 ft. 6 in. 4. $3\frac{1}{2}$ yd. 6. $1\frac{1}{2}$ mi.

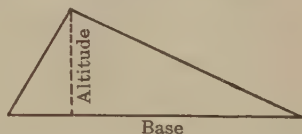
Find the areas of rectangles of the following dimensions :

7. 14 in. $\times$ 12 in. 9. $18' \times 14\frac{1}{4}'$. 11. $15\frac{1}{4}$ rd. $\times$ $18\frac{3}{4}$ rd.

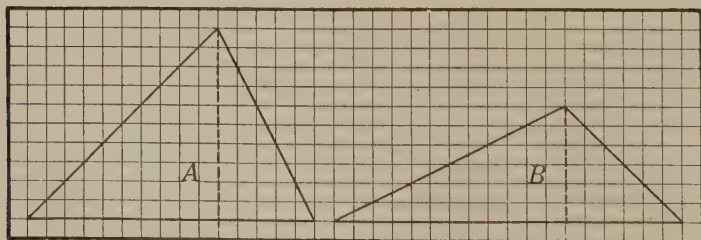
8. $5' 6'' \times 2' 3''$. 10. $40\frac{1}{2}$ rd. $\times$ $22\frac{1}{2}$ rd. 12. $120' \times 98\frac{3}{4}'$.

Dimensions are often given in the form used in Ex. 8, in which $5' 6''$ means 5 ft. 6 in.

Area of a triangle. The side upon which a triangle appears to stand is called the **base**, and the perpendicular distance from the vertex to the base of the triangle is called the **height** or **altitude**.



If each small square in the figure below represents 1 sq. in., it is seen by counting that triangle *A* contains 65 whole squares and 20 fractional squares which are



apparently equal to 10 whole squares ; that is, the area appears to be 75 sq. in. Similarly, the area of triangle *B* is seen, by counting, to be 54 sq. in. But $75 = \frac{1}{2} \times 15 \times 10$ and $54 = \frac{1}{2} \times 18 \times 6$; that

is, the same result is found by taking half the product of the base and altitude of each triangle.

It is proved in geometry that

Area of a triangle $= \frac{1}{2} \times (\text{number of units in base}) \times (\text{number of units in altitude})$ square units.

Letting A represent "area in square units," b the "number of units in the base," and h the "number of units in the height," the area of a triangle may be expressed by the following formula:

$$A = \frac{1}{2} bh.$$

Accuracy of measurement. All measurements are, in general, only approximate. That is, if the base of a triangle is given as 11.6 in., it generally means that the length is nearer 11.6 in. than it is 11.5 in. or 11.7 in. *In any computations with ordinary measurements no result can be found to any higher degree of accuracy than the least accurate of any measurement upon which the computation is based.*

EXAMPLE. Find the area of a triangle whose base is 14.1 in. and whose height is 5.2 in.

Solution. In this case, b in the formula is 14.1 in. and h is 5.2 in.

Hence $A = \frac{1}{2} bh = \frac{1}{2} \times 14.1 \times 5.2 = 36.66.$

But the dimensions of the triangle are approximate only to tenths of an inch, and hence the area should be given as 36.7 sq. in.

It is evident that a great deal of time and work can be saved in making computations with practical measurements by keeping the results down to the degree of accuracy required by the conditions of the problems.

Thus, in finding the cost of painting a given surface, it would be obviously absurd to find the area correct to six decimal places, and then to figure the cost from such a result.

Where dimensions are given as whole numbers the student should estimate what is a reasonable degree of accuracy for the conditions involved. Ordinarily in the work here given no greater accuracy than to tenths of an inch will be required.

Thus, a triangle of base 11 in. and height 7 in. may be considered as having been measured to the nearest 0.1 in.

EXERCISES

Find the areas of triangles with bases and altitudes as follows, giving each result to a proper degree of accuracy :

- | | |
|------------------------------|--|
| 1. 4 in.; 7 in. | 6. 12.7 ft.; 16.9 ft. |
| 2. 9 in.; 15 in. | 7. 20 rd.; $16\frac{1}{2}$ rd. |
| 3. 2 ft.; $4\frac{1}{2}$ ft. | 8. 12.43 ft.; 9.89 ft. |
| 4. 24.3 in.; 18.2 in. | 9. $12\frac{1}{4}$ yd.; $6\frac{1}{2}$ yd. |
| 5. 15.1 in.; 14.7 in. | 10. 22.7 rd.; 16.5 rd. |

11. Draw a triangle and measure its base and altitude. From these measurements compute the area of the triangle.

12. A triangular city lot has a frontage (base) of 54 ft. and a depth (altitude) of 160 ft. What is the lot worth at 75¢ per square foot?

13. A square piece of land 208 ft. on a side contains approximately 1 A. If the piece is divided into two equal triangular lots, what will be the base and the depth of each lot? the area of each lot? the value of each lot at $62\frac{1}{2}$ ¢ per square foot?

Practical application of area. The figuring of areas enters into the work of computing the cost of plastering, papering, painting, roofing, and the like. The general principle is the same in any case; the area of the surface to be covered is first figured and from that the cost of the job at a given price per unit.

The customs in regard to approximating results and in making allowances for uncovered areas, such as windows and doors, vary so much that no text-book can give all the details. The students should be encouraged to find out the local customs with respect to these trades and to make up problems which will apply them. The following exercises give an idea of the general principles applied to such work.

EXERCISES

1. Find the number of feet of baseboard required for a room $20' \times 18'$; for a room $16' 9'' \times 14' 6''$; for a room $22\frac{1}{2}' \times 12\frac{1}{4}'$.

2. If each of the rooms in Ex. 1 had three doors, each $3'$ wide, find the number of feet needed after deducting a full allowance for the doors.

3. Find the areas of the ceilings of the rooms in Ex. 1.

4. Find the cost of calcimining each of the ceilings in Ex. 3 at 11¢ per square yard.

5. Find the area of the side walls and the ceiling of a room $20' \times 15' \times 10'$; of a room $15' \times 12' \times 10'$.

In writing dimensions the length is always given first, then the width, and then the height.

6. If no allowance is made for openings, find the cost of plastering the rooms in Ex. 5 at 60¢ per square yard.

Compute the number of square yards to the nearest tenth.

7. Allowing 140 sq. ft. for openings, find the cost of plastering a room $18' \times 10' \times 9'$ at 55¢ per square yard.

Sometimes an allowance is made for only half the area of the openings, sometimes for the full area, and sometimes for a specified area as above.

8. A room $18' \times 12' \times 10'$ has 4 windows, each $3' \times 6'$, and two doors, each $3' \times 8'$. Making an allowance of half the area of the openings, find the cost of plastering the walls and ceiling at 45¢ per square yard.

9. Find the dimensions of your classroom, and compute the cost of plastering the room at 75¢ per square yard, making a full allowance for all areas not now plastered.

10. Wall paper is usually sold in rolls 18 in. wide and 16 yd. long. One method of estimating the amount of wall paper required for a room is to allow three 18-inch rolls for every 200 sq. ft. Find the amount of paper needed for a room which is $18' \times 10' \times 8' 6''$.

A roll of paper 16 yd. long is called a "double roll." Single rolls containing 8 yd. are sometimes used. Any fraction of a roll should be counted as a whole roll, as fractions of rolls cannot be purchased or returned.

11. Making an allowance of 120 sq. ft. for openings, find the cost of the paper for a room $22' \times 16' \times 9'$ at 75¢ per roll.

The double roll of 16 yd. is understood unless the contrary is stated.

12. Find the perimeter of a room which is $18' \times 14' \times 9'$. How many strips of wall paper are needed if an allowance of $15'$ is made for openings in figuring the perimeter?

Any fraction of a strip should be counted as a whole strip.

13. Find the number of rolls of paper needed to paper the room in Ex. 12 and the cost of the paper at 85¢ per roll.

In this method of estimating, the pieces left over from the rolls and the extra amount purchased by buying whole rolls takes care of the spaces above and below the windows and above the doors.

14. Find the number of rolls of paper needed to paper the room of Ex. 7 by the method given in Ex. 10.

15. Find the number of rolls of paper needed to paper the room in Ex. 8 by the method of Exs. 12 and 13.

In this case only the width of the doors and windows should be considered.

16. Wall-paper borders are sold by the yard. Find the cost of the border for a room $17\frac{1}{2}' \times 12'$ at 45¢ per yard; at $37\frac{1}{2}\text{¢}$ per yard; at $66\frac{2}{3}\text{¢}$ per yard.

17. Find the dimensions of your classroom (or use those found in Ex. 9, above). Making allowance for the doors and windows in figuring the perimeter, but making no allowance for the black-board, find the number of rolls of paper needed to paper the room. Find the cost of the paper at 65¢ per roll.

18. Roofing is generally figured by the **square**, which is an area of 100 sq. ft. How many squares of composition roofing are needed for the flat roof of a garage $54' \times 37\frac{1}{2}'$?

19. What is the cost of tinning an A-roof, each side of which is $30' \times 15'$, at $\$3$ per square?

20. The number of slate shingles required per square varies according to the size of the slate. Estimating to the nearest 100 slates, how many slates are required for an A-roof, each side of which is $26' \times 48'$, if 533 slates per square are used?

21. Wooden shingles are sold in bunches of 250. How many bunches of shingles should be ordered for an A-roof, each side of which is $18' \times 38'$, if 3.2 bunches are allowed per square?

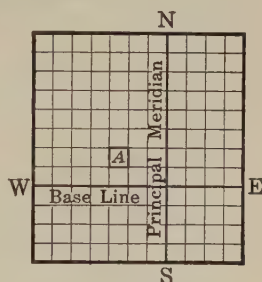
Fractional parts of bunches are not sold.

22. How many bunches of shingles are needed for the roof in Ex. 19 if shingles which lay 2.8 bunches per square are used? Find the cost of the shingles at \$14.50 per M.

23. What is the cost of laying a slate roof in Ex. 19 if the cost per square is \$8.25?

Land measure. In certain sections of the United States the land is laid out in townships, as in the figure just below.

Starting with a **principal meridian** (a north-and-south line) and a **base line** (an east-and-west line) lines are run parallel to the meridian and the base line at intervals of 6 mi. These lines divide the land into **townships**, each 6 mi. square, which are numbered north and south from the base line. The north-and-south rows of townships are called **ranges**, which are numbered east and west from the principal meridian.



LAND DIVIDED INTO
TOWNSHIPS

Scale, 1 unit = 6 mi.

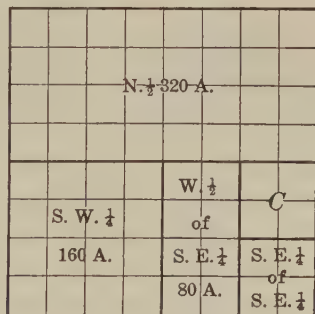
Township A in the above figure is located by the following description: Tp. 2 N., R. 3. W.; that is, the second township north of the base line in the third range west of the principal meridian.

A township is divided into 36 sections, each 1 mi. square, which are numbered as shown in the first figure at the right. The sections, which contain 640 A. each, are divided into

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

A TOWNSHIP

Scale, 1 unit = 1 mi.



A SECTION

Scale, 1 unit = $\frac{1}{4}$ mi.

to **halves** and **quarters**, and the quarter sections may be still further divided as shown in the diagram of a section above.

If the township diagram above is considered as township A in the first figure, and the section diagram above as section 20 of township A, then

tract C in the section is described as follows: N. E. $\frac{1}{4}$ of S. E. $\frac{1}{4}$, Sec. 20, Tp. 2 N., R. 3 W.; that is, the northeast quarter of the southeast quarter of section 20 in the second township north of the base line in the third range west of the principal meridian.

EXERCISES

1. Draw a map similar to the first figure on page 115 and locate the following townships: Tp. 2 N., R. 2 E.; Tp. 5 N., R. 4 W.; Tp. 2 N., R. 3 E.

2. Draw a large township map and locate the following tracts: S. $\frac{1}{2}$, Sec. 10; E. $\frac{1}{2}$ of S. W. $\frac{1}{4}$, Sec. 8; N. W. $\frac{1}{4}$ of S. E. $\frac{1}{4}$, Sec. 28.

Find the value of each of the following tracts:

3. E. $\frac{1}{2}$, Sec. 2, at \$85 per acre.
4. S. $\frac{1}{2}$, Sec. 10, at \$70 per acre.
5. N. E. $\frac{1}{4}$, Sec. 18, at \$175 per acre.
6. N. $\frac{1}{2}$ of N. W. $\frac{1}{4}$, Sec. 30, at \$80 per acre.
7. S. W. $\frac{1}{4}$ of S. W. $\frac{1}{4}$, Sec. 31, at \$155 per acre.
8. N. W. $\frac{1}{4}$ of S. E. $\frac{1}{4}$, Sec. 6, at \$112.50 per acre.

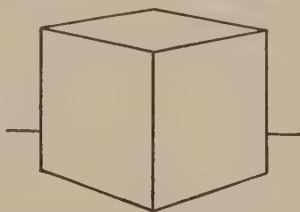
RECTANGULAR SOLIDS

Rectangular solid. A solid is a figure which has length, breadth, and thickness. A **rectangular solid** is a solid in which the surfaces that form its faces are rectangles (or squares). Each edge of a rectangular solid is perpendicular to the other edges which it meets.

A covered crayon box, a brick, an evenly trimmed block of ice, and the like are examples of rectangular solids.

Cube. A cube is a rectangular solid which has six equal square faces.

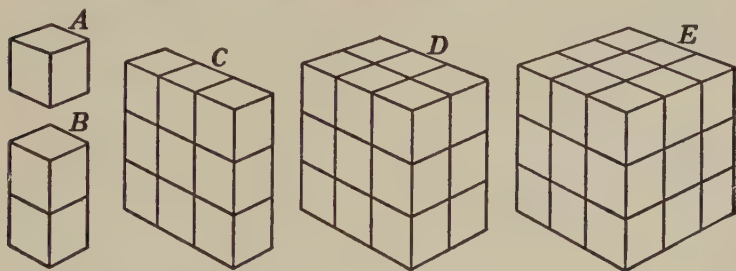
Volume of a rectangular solid. A cube 1 in. on an edge is said to have a **volume** of 1 cu. in. (1 cubic inch); a cube which is 1 ft. on an edge, a volume of 1 cu. ft., and so on. The cubic inch, cubic



RECTANGULAR SOLID

foot, cubic yard, and so on are the **units of volume**, according as the figure is measured in inches, feet, yards, and so on.

If *A* in the figure below is $1'' \times 1'' \times 1''$, it contains 1 cu. in. It is readily seen by counting the blocks that *B* contains $1 \times 1 \times 2$,



or 2 cu. in.; that *C* contains $3 \times 1 \times 3$, or 9 cu. in.; that *D* contains $3 \times 2 \times 3$, or 18 cu. in.; and that *E* contains $3 \times 3 \times 3$, or 27 cu. in.

It is thus evident that in a rectangular solid the volume is the product of the three dimensions; that is,

Volume = (number of units in length) $\times$ (number of units in width) $\times$ (number of units in height) cubic units.

Letting *V* represent "volume in cubic units," *l* the "number of units in the length," *w* the "number of units in the width," and *h* the "number of units in the height," the above statement may be expressed by the following formula:

$$V = lwh.$$

The three dimensions must, of course, be expressed in the same units of measure.

EXAMPLE. Find the volume of a box which is 5' long, 2' wide, and $4\frac{1}{2}'$ deep.

Solution. In this case, *l* in the formula is 5', *w* is 2', and *h* is $4\frac{1}{2}'$

Then $V = lwh = 5 \times 2 \times 4\frac{1}{2} = 45$.

Since the dimensions are given in feet, the volume is in cubic feet, and hence the volume of the box is 45 cu. ft.

EXERCISES

Find the volumes of rectangular solids with the following dimensions:

1. $8' \times 6' \times 2'$.

6. $3' 3'' \times 2' 6'' \times 5'$.

2. $18' \times 15' \times 14'$.

7. $4' \times 2' 6'' \times 5' 9''$.

3. $17'' \times 11'' \times 10''$.

8. $5\frac{1}{2}' \times 4' \times 6\frac{1}{2}'$.

4. $4'' \times 2'' \times 3''$.

9. $3' 6'' \times 2' 6'' \times 4' 6''$.

5. $5\frac{1}{2}' \times 48'' \times 12'$.

10. $3' 9'' \times 5' \times 8\frac{1}{2}'$.

Practical application of volume. The figuring of volumes enters into the work of computing the size of excavations, the capacity of all types of bins and containers, the amount of material required for concrete or brick construction, and many other similar cases.

In practical work with volumes, the accuracy of the given dimensions should be considered and results should not be given to a greater number of decimal places than the conditions warrant.

The student should consult the section on accuracy of measurement on page 111.

EXERCISES

1. An open quarry is 120 ft. long, 80 ft. wide, and 30 ft. deep. Find the amount of stone excavated, first in cubic feet and then in cubic yards.

2. A load of dirt, sand, or gravel is equivalent to a cubic yard. How many loads are removed in excavating a cellar $20' \times 16' \times 10'$?

3. How many loads of earth are removed in digging a ditch 40 rd. long, 10 ft. wide, and 6 ft. deep?

4. A cord of wood is 8' long, 4' wide, 4' high. How many cubic feet does a cord contain? How many cords are contained in a pile $32' \times 4' \times 8'$? in a pile $50' \times 4' \times 6'$?

5. A classroom is $26' \times 22' \times 12'$. How many cubic feet of air does the room hold?

6. Each pupil in a classroom should have at least 30 cu. ft. of fresh air per minute. If there are 24 pupils in the classroom of Ex. 5, how many times per hour should the air be changed so as to provide the proper amount of fresh air?

7. If the inlet for fresh air in a classroom is $2' \times 3'$, at what rate (feet per minute) should air pass into the classroom so that a class of 48 pupils shall receive the proper amount of fresh air?

8. Find the dimensions of your classroom and the number of pupils in the class. How many times per hour should the air in the room be changed in order that each pupil shall have 30 cu. ft. of fresh air per minute?

9. A cubic foot of water weighs $62\frac{1}{2}$ lb. and ice is 0.92 as heavy as water. If the ice chamber of a refrigerator is $28'' \times 18'' \times 20''$, what is the weight of a piece of ice that will exactly fill the chamber?

10. An artificial ice plant makes its ice in cakes 18'' long and 16'' wide. Using the data given in Ex. 9, find how thick the cakes should be frozen so that each cake shall weigh approximately 100 lb.

11. For ordinary estimates it is considered that 22 bricks laid in mortar occupy a cubic foot. How many bricks are required for a wall 72' long, 18'' thick, and 6' high? for a wall 16' long, 22'' thick, and $8\frac{1}{2}'$ high?

The above estimate is based upon the size of the common brick which is 8'' long, 4'' wide, and 2'' thick. Bricks vary in size, but the common brick is used as the basis for measuring brickwork.

12. How many bricks are required for a wall 12'' wide and 6' high set inside the edges of a lot $100' \times 120'$, if a length of 20' is allowed for openings? Find the cost of the brick at \$20 per M.

13. In figuring the volume for the purpose of computing the labor cost of brickwork it is customary to "count the corners twice"; that is, the wall in Ex. 12 would be figured as a 12-inch wall equivalent in length to the perimeter of the lot, on account of the extra work in laying the corners. Similarly, only one

half of the volume of the openings is deducted. Using this method, find the labor cost of laying the wall in Ex. 12 at \$6 per 400 bricks.

14. How many bricks are required for the walls of a building $36' \times 30' \times 16'$ if the walls are to be 18" thick, and allowance is made for 12 windows, each $3' \times 6'$, and 4 doors, each $3' \times 8'$?

15. Using the rules given in Ex. 13, find the labor cost of laying the walls in Ex. 14 at \$5.50 per 400 bricks.

16. For practical estimates it is generally considered that a bushel has a volume of $1\frac{1}{4}$ cu. ft. What is the capacity in bushels of a bin $8' \times 12' \times 6'$? of a bin $15' \times 8' \times 9' 6''$?

17. The **stricken bushel**, which is used in measuring grain, actually contains 2150.42 cu. in. If the bins in Ex. 16 are filled with grain, what are their capacities to the nearest 0.1 bu.?

18. The **heaped bushel**, which is used in measuring large fruits, contains 2747.71 cu. in. If the bins in Ex. 16 are filled with potatoes, what are their capacities to the nearest 0.1 bu.?

19. To the nearest hundredth, how many cubic feet are there in a stricken bushel? in a heaped bushel?

20. Express, correct to the nearest hundredth, a cubic foot as a decimal of a stricken bushel; of a heaped bushel.

21. Using $1\frac{1}{4}$ cu. ft. as the equivalent of a bushel, how high should a bin with a base $12' \times 6'$ be built so that it will hold 500 bu.?

22. If hard coal runs 35 cu. ft. to the ton, find the number of tons of coal in a bin $12' \times 8' \times 6'$.

23. How many tons of soft coal weighing 49 lb. per cubic foot can be put into a bin $10' \times 16' \times 8'$?

24. A haymow 24 ft. long, 16 ft. wide, and 18 ft. high is filled with well-settled clover hay. If 550 cu. ft. of this hay weigh a ton, find the weight of the hay in the mow.

25. A cubic foot of water is approximately equivalent to $7\frac{1}{2}$ gal. How many gallons of water can be stored in a rectangular reservoir 120 ft. long, 72 ft. wide, and 18 ft. deep?

26. A small river is 175' wide and has an average depth of 8'. If the river flows at $2\frac{1}{2}$ mi. per hour, how many gallons of water flow past a given point in 1 hr.?

27. How many cubic feet of storage space will be required for 1020 cases of goods packed in boxes $2' 4'' \times 18'' \times 14''$?

28. A manufacturer packs a certain type of goods in packing cases $3' 6''$ long, and $2' 4''$ wide, and 2' deep. Placing the cases the most economical way, how many can be put in a freight car which is $38' 6''$ long, $8' 3''$ wide, and $7' 9''$ high, inside measurements?

Measuring lumber. The unit in measuring lumber is the board foot, or in large quantities a thousand board feet (written M bd. ft.). A **board foot** is the amount of lumber contained in a board 1' long, 1' wide, and 1" thick. Thus in a given piece of lumber,

Number of board feet = number of feet in length $\times$ number of feet in width $\times$ number of inches in thickness.

Hence, if B represents the "number of board feet," l the "length in feet," w the "width in feet," and t the "thickness in inches," the number of board feet in a given piece of lumber is represented by the following formula:

$$B = lwt.$$

When a board is less than 1" in thickness, the number of board feet is computed as if the board were 1" thick; but for a board thicker than 1", the exact dimension is used. A fraction of a board foot is always counted as a full foot in billing lumber.

Thus, for a board 12' long, 12" wide, and 1" thick,

$$B = lwt = 12 \times 1 \times 1 = 12.$$

For a board 12' long, 8" wide, and $\frac{7}{8}$ " thick,

$$B = lwt = 12 \times 1\frac{2}{3} \times 1 = 8.$$

For a plank 14' long, 16" wide, and $2\frac{1}{2}$ " thick,

$$B = lwt = 14 \times \frac{16}{12} \times \frac{5}{2} = \frac{140}{3} = 46\frac{2}{3}.$$

Hence the pieces contain 12 bd. ft., 8 bd. ft., and $46\frac{2}{3}$ bd. ft. respectively.

EXERCISES

Find the number of board feet in the following :

1. A plank 16' long, 10" wide, and 2" thick.
2. 3 boards 12' long, 16" wide, and 1" thick.

In this case, $B = 3 \text{ lw}$.

3. 25 joists 12' long, 4" wide, and 4" thick.
4. 100 pieces of scantling, 14' long, 4" wide, and 3" thick.
5. 50 planks 14' long, 10" wide, and $2\frac{1}{2}$ " thick.

The terms **joist** and **scantling** are applied to the smaller sticks of lumber used in framework or similar construction, as distinguished from **timber**, which is the term generally applied to pieces more than 4" square.

6. 24 timbers 16' long, 8" wide, and 8" thick.
7. 25 joists 18' long, 2" wide, and 4" thick.
8. 100 boards 14' long, 9" wide, and $\frac{3}{4}$ " thick.
9. In Ex. 4 find the cost of the lumber at \$76 per M (per 1000 bd. ft.).

10. In Ex. 6 find the cost of the lumber at \$84 per M.

11. In Ex. 8 find the cost of the boards at \$65 per M.

12. A board is 12' long and 9" wide. How many square feet are there in one side of the board? If the board is 1" thick, how many board feet does it contain?

13. The number of board feet of floor lumber is computed as if the boards were 1" thick. In flooring a gymnasium $80' \times 40'$, how many square feet of floor space is to be covered? If $\frac{1}{3}$ extra is allowed for waste in matching and laying, how many board feet of flooring is required?

An allowance of $\frac{1}{3}$ for waste is made only on flooring which is less than 3" in width.

14. When flooring is 3" wide, or wider, an allowance of $\frac{1}{4}$ is generally made for waste. How many board feet of 4-inch flooring is needed for a room $12' \times 14'$? Find the cost of the lumber at \$98 per M.

15. Find the cost of $2\frac{1}{2}$ -inch flooring for a room $22' \times 26'$ at \$150 per M; of $4\frac{1}{2}$ -inch flooring (see Ex. 14) at \$96 per M.

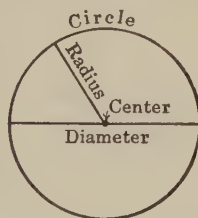
16. A two-story town hall is to be refloored with $2\frac{1}{2}$ -inch hardwood flooring. The lower floor is $40' \times 60'$, and the upper floor is 10' shorter on account of the stairway. Find the cost of the flooring at \$175 per M.

17. Find the cost of laying the floor in Ex. 14 at \$1 per square (100 sq. ft.).

18. Find the total cost of laying the two floors in Ex. 16 at \$1.75 per square.

CIRCLES AND CYLINDERS

Circle. A circle is a closed curved line every point of which lies in the same plane and is equally distant from a point within the curve which is called the **center** of the circle. The length of the curved line is called the **circumference** of the circle. Any straight line which passes through the center and terminates on the circle at both ends is called a **diameter**. Any line from the center to a point on the circle is called a **radius**, and is obviously equal to half the diameter.



Circumference. By careful measurements of the circumferences and the diameters of circles, it has been found that the circumference of any circle is just a little more than three times the diameter of the circle. This ratio of circumference to diameter is indicated by the Greek letter π (pronounced pī). The common value of π is $3\frac{1}{7}$; but in work where a high degree of accuracy is necessary, the value 3.1416 is generally used. Hence

$$\begin{aligned} \text{Number of units in circumference} \\ = 3\frac{1}{7} \times \text{number of units in diameter,} \end{aligned}$$

or

$$\begin{aligned} \text{Number of units in circumference} \\ = 3.1416 \times \text{number of units in diameter.} \end{aligned}$$

If c represents the "number of units in the circumference," d the "number of units in the diameter," and r the "number

of units in the radius," the above facts may be stated by the following formulas:

$$c = \pi d \quad \text{or} \quad c = 2\pi r.$$

All the facts in regard to circles and cylinders are proved in geometry, but the proofs will not be given here since these familiar facts may safely be assumed.

EXAMPLE. Find the circumference of a wheel which has a diameter of 10.6 in.

Solution. In this case, d in the formula is 10.6 in.

$$\text{Hence} \quad c = \pi d = 3\frac{1}{7} \times 10.6 = 33.314+.$$

Since the diameter is given to tenths of an inch, the circumference is given as 33.3 in.

In any exercise which involves the use of the ratio π , the student should use the value $3\frac{1}{7}$, unless otherwise directed. The student should also consult the section on accuracy of measurement on page 111.

EXERCISES

1. Find the circumference of a circle which has a radius of 10 in.; 14 in.; 21 in.; 15 in.; 14.7 in.; 1 ft. 6 in.

2. Find the circumference of a circle whose diameter is 12 in.; 16 in.; 1 ft. 4 in.; 2 ft. 3 in.; 17.9 in.; 27.83 ft.

3. Find the diameter of a circle which has a circumference of 44 in.

Since $c = \pi d$, it is obvious that $d = \frac{c}{\pi}$.

4. Find the diameter of a circle whose circumference is 15 in.; $28\frac{1}{2}$ in.; 3 ft. 9 in.

5. Find the radius of a circle whose circumference is 16 in.; 24.8 in.; 16 ft. 4 in.

6. Using 3.1416 as the value of π , find the outside diameter and the radius of a steam pipe if the distance around the pipe is known to be 12.623''.

The results in this case may be given to the nearest 0.001''.

7. The iron hoops which band a circular silo are 62' 3.4'' in length. Find the diameter and the radius of the base.

8. A circular bicycle track with eight laps to the mile is to be laid out. What radius should be used in laying out the inside circle of the track, which is the one on which the distance around the track is measured?

Use 3.1416 as the value of π , and give the result to the nearest inch.

Area of a circle. If a large number of equally spaced points are taken on a circle, and the radii from these points are drawn, it is obvious that the area of the circle is the sum of the areas of the several sections thus formed, as shown in the first figure below. These sections are approximate triangles, each altitude



of which is practically a radius of the circle, as is seen in the second figure where the sections are spread out in a line. The sum of their bases is obviously the circumference of the circle. Hence, regarding the sections as triangles,

The area of the circle is one half the product of the circumference (sum of the bases) and the radius (altitude); that is,

Area of a circle = $\frac{1}{2} \times (\text{number of units in circumference}) \times (\text{number of units in radius})$ square units.

It is proved in geometry that, if a large enough number of triangles are taken, the altitude and the radius are actually equal, and that the above rule gives the actual area of a circle.

If A represents the "area of the circle in square units," c the "number of units in the circumference," and r the "number of units in the radius," the above fact is stated by the following formula:

$$A = \frac{1}{2} cr.$$

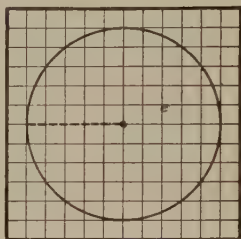
But, from page 124, $c = 2\pi r$,

and hence $A = \frac{1}{2} \times 2\pi r \times r$,

or

$$A = \pi r^2.$$

In this figure the radius of the circle is 5 squares in length, and if each square is taken to represent 1 sq. in., it is seen by counting that there are 60 whole squares and 28 fractional parts of squares. These fractional parts are apparently equal to 18 whole squares, so that the area of the circle is approximately 78 sq. in. This approximate area should be compared with the area found by the formula in the following example.



EXAMPLE. Find the area of a circle whose radius is 5 in.

Solution. In this case, r in the formula is 5 in.

Hence $A = \pi r^2 = 3\frac{1}{7} \times 5^2 = 2\frac{2}{7} \times 25 = 78.57+$

Since r is in inches, A is in square inches; and hence, to the nearest 0.1 sq. in., the area of the circle is 78.6 sq. in.

EXERCISES

1. Find the area of a circle whose radius is 6 in.; 10 in.; 12.4 in.; 26 ft.; 32.2 ft.; 64.7 yd.

2. Find the area of a circle whose diameter is 14 in.; 41 in.; 54.8 in.; 1 ft. 3 in.; 15.64 ft.

3. Find the area of a circle whose diameter is twice that of another circle which has a diameter of 18 in. What fractional part is the area of the second circle of that of the first?

4. The radius of one circle is 8 in.; that of another circle is 24 in. How much greater is the area of the second circle than that of the first? how many times as great?

5. How do the areas of two circles compare when the radius of one is twice that of the other? three times? four times?

6. The distance around a circular tank is 125 ft. Find the area of the base of the tank.

First find the diameter or radius, and then find the area.

7. Write in your own words the method of finding the area of a circle.

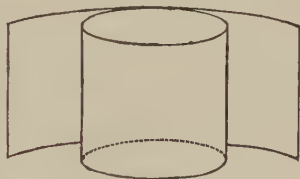
8. Measure the diameter of some circular object and compute its area.

9. By winding a piece of string about some circular object and measuring the length of the string when unwound, find the circumference of the object; then compute its area.

Avoid stretching the string when measuring it.

Cylinder. A **cylinder** is a solid the side of which is a uniformly curved surface and the ends of which are equal circles which are parallel to each other. The curved surface is called the **lateral surface** of the cylinder and the circles are called the **bases** of the cylinder.

Lateral area of a cylinder. If the curved surface of the cylinder at the right is covered with paper, and if the paper is then unrolled, it is seen that the lateral surface is a rectangle. The length of this rectangle is the circumference of the base, and the height is the height of the cylinder. Hence



Lateral area of a cylinder = (number of units in circumference) $\times$ (number of units in height) square units.

Then if A represents the "area of the lateral surface in square units," d the "number of units in the diameter," and h the "number of units in the height,"

$$A = \pi dh.$$

The truth of this formula is obvious since $c = \pi d$.

Volume of a cylinder. Just as the volume of a rectangular solid is the product of the area of the base and the height, it follows that

Volume of a cylinder = (number of square units in area of base) $\times$ (number of units in height) cubic units.

Letting V represent the "volume of a cylinder in cubic units," it is evident that

$$V = \pi r^2 h.$$

This is proved in geometry to be the true volume of a cylinder.

EXAMPLES. 1. The radius of a cylindrical tank is 6 ft. and the height of the tank is 20 ft. Find the area of the lateral surface.

Solution. Since the radius is 6 ft., the diameter, or d in the formula, is 12 ft.; h in the formula is 20 ft.

$$\text{Hence } A = \pi dh = 2\frac{2}{7} \times 12 \times 20 = 754.28+.$$

Since the dimensions are in feet, A is in square feet; and hence, to the nearest 0.1 sq. ft., the lateral area of the tank is 754.3 sq. ft.

2. Find the volume of the tank in Ex. 1.

Solution. In this case, r in the formula is 6 ft. and h is 20 ft.

$$\text{Hence } V = \pi r^2 h = 2\frac{2}{7} \times 6 \times 6 \times 20 = 2262.85+.$$

Since the dimensions are in feet, V is in cubic feet; and hence, to the nearest 0.1 cu. ft., the volume of the tank is 2262.9 cu. ft.

EXERCISES

Find the lateral area of each of the following cylinders, giving the results to an appropriate degree of accuracy:

1. Radius, 5"; height, 3". 3. Radius, 4.7"; height, 3.5".
2. Radius, 11"; height, 9". 4. Radius, 8'; height, 12½'.
5. Radius, 8.24'; height, 14.64'.
6. Radius, 12.76'; height, 24.86'.

The student should use $3\frac{1}{7}$ for π , unless otherwise directed.

7. Find the area of the base of the cylinder in Ex. 1.
8. Find the total surface area of the cylinders in Exs. 2-6.
9. How many square inches of tin are needed to make a closed can in the form of a cylinder 8" in height and 6" in diameter, if $\frac{1}{10}$ is allowed for the overlapping seams?
10. At 45¢ per square yard, find the cost of painting the lateral surface of a cylindrical silo 16' in diameter and 24' in height.

Find the volume of each of the following cylinders, giving each result to an appropriate degree of accuracy:

11. Diameter, 6"; height, 5". 12. Radius, 7.9"; height, 11.4".
13. Diameter, 15' 6"; height, 20' 9".
14. Radius, 48.8'; height, 58.2'.

15. A gallon contains 231 cu. in. Find the capacity of a cylindrical tank of the dimensions given in Ex. 13.

16. The approximate number of gallons contained in a cubic foot is $7\frac{1}{2}$. Using this value, find the number of gallons contained in a tank of the dimensions given in Ex. 14.

17. Find the number of quarts contained in a cylindrical jar which is 20" in diameter and 30" in height.

Use 231 cu. in. as the volume of 1 gal.

18. Using 3.1416 for π , find the number of gallons which it will take to fill a water main 25.6" in diameter and $2\frac{1}{2}$ mi. in length.

Use $7\frac{1}{2}$ gal. as the equivalent of 1 cu. ft.

SQUARE ROOT

Square of a number. The square of a number is the product of the number multiplied by itself. It is often called the **second power** of a number.

For example, 25 is the square of 5, because $25 = 5 \times 5$. The expression 5×5 is generally written 5^2 (read "five-square").

The third power of 3 is 27, or $3 \times 3 \times 3$; the fourth power of 2 is 16, or $2 \times 2 \times 2 \times 2$; and so on.

Square root of a number. The square root of a number is one of the two equal factors whose product is the given number.

Thus 7 is the square root of 49 since $7 \times 7 = 49$; $\frac{3}{5}$ is the square root of $\frac{9}{25}$, since $\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$. The symbol $\sqrt{\quad}$ is used to indicate "the square root of"; that is, $\sqrt{81}$ means the square root of 81, which is 9.

Approximate square roots. When a number is not an exact square, an approximate value of its square root may be obtained.

Thus 2 is not an exact square, but its square root is said to be 1.414, correct to three decimal places. An approximate square root may be found to as many places as required.

Consider the following table of squares and square roots:

SQUARE	1	81	1 00	98 01	1 00 00	99 80 01	1 00 00 00
SQUARE ROOT	1	9	1 0	9 9	1 0 0	9 9 9	1 0 0 0

It should be noticed that the figures in the squares are grouped by pairs. It is at once obvious that a number of one or two figures has a square root of one figure, a number of three or four figures has a square root of two figures, a number of five or six figures has a square root of three figures, and so on.

The number of figures in the integral part of the square root of a number (the part to the left of the decimal point) is at once apparent if the number is separated into periods of two figures each, beginning at the right or at the decimal point in the case of a decimal; that is, there is one figure in the square root for each period in the number.

An incomplete period (of one figure) at the left is counted as a period.

The square root of the greatest square which is contained in the first period at the left of a given number is the first figure of the square root of the number, as is evident in the following example.

EXAMPLE. Find the number of figures and the first figure in the integral part of the square root of 2907, and determine between what numbers the square root lies.

Solution. Separating the number into periods of two figures each, beginning at the right, gives 29 07.

Since there are two periods, the integral part of the square root contains two figures.

The greatest square that is contained in 29 is 25. Hence 5, the square root of 25, is the first figure of the square root.

The square of 6 is 36, which obviously is not contained in 29. Hence, since the square root of 2907 has two figures, the square root lies between 50 and 60.

ORAL EXERCISES

1. What power of 5 is 25? of 7 is 49? of 10 is 1000?
2. State the square of 5; of 7; of 8; of 9; of 10; of 11; of 12; of 16.
3. State the square root of 36; of 25; of 4; of 16; of 81; of $\frac{1}{9}$; of $\frac{16}{25}$; of 4×9 ; of $\frac{36}{100}$; of $\frac{81}{121}$.

4. In each of the following numbers state the number of figures in the integral part of the square root, the first figure of the square root, and the two numbers between which the square root lies:

76	2704	761.76	5730.49	89621.5
134	9216	964.42	7398.4	7328.96
679	4489	125.68	9040.2	9988.52

In the numbers which involve decimals, consider only the integral part of the number.

Finding the square root of a number. The student should study the following examples carefully, as skill and speed in finding square roots can come only through a thorough knowledge of the steps taken.

EXAMPLES. 1. Find the square root of 5776.

Solution. Beginning at the right, separate the number into periods of two figures each, as here shown. The greatest square in the first period at the left

is 49. Hence 7, the square root of 49, is the first figure of the required root. Write 49, the square of the first figure, under the first period at the left and sub-

	Number	Root
	57 76	<u>76</u>
Greatest square	<u>49</u>	
Trial divisor	140) 8 76	Remainder
	6)	
Complete divisor	<u>146</u>) 8 76	

tract. The remainder is 8, to which is annexed 76, the figures in the second period of the number, thus giving 876 for the next number to be divided.

The 7 already found in the root is now doubled, making 14. This 14 is considered as tens, thus making 140, which is used as the trial divisor. It is seen that 140 is contained 6 times in 876, and so 6 is put down as the second figure of the root. The 6 is also added to 140, the trial divisor, making 146. Then 146, the complete divisor, times 6, the second figure of the root, gives 876. Subtracting 876 from the number above it gives no remainder.

Since there are no more figures in the number to be brought down, the exact square root, which in this case is 76, has been found.

2. Find the square root of 182,329.

Solution. Beginning at the right, separate the number into periods of two figures each, as here shown. Then proceeding as before, the process of finding the square root is carried out until there is no remainder.

In finding the second trial divisor, 840, it should be noticed that the part of the root already found is doubled and considered as tens; that is, 42 doubled is 84, and considered as tens is 840.

The exact square root of 182,329 is thus found to be 427.

$$\begin{array}{r}
 18 \quad 23 \quad 29 \quad \overline{)427} \\
 \underline{16} \\
 80 \overline{) \quad 2} \quad 23 \\
 \underline{2} \quad) \\
 82 \overline{) \quad 1} \quad 64 \\
 840 \overline{) \quad \quad 59} \quad 29 \\
 \underline{7} \quad) \\
 847 \overline{) \quad \quad 59} \quad 29
 \end{array}$$

In Example 1 the full explanation of all the steps was given and the different parts of the solution were indicated. In Examples 2 and 3, only the work which it is necessary for the pupil to write down is shown at the right.

3. Find the square root of 2563.1627 correct to the nearest tenth.

Solution. Beginning at the decimal point, separate the number into periods of two figures each both to the right and left, as here shown. Then proceeding as before, the process of finding the square root is carried on until there are two decimal places in the required root.

$$\begin{array}{r}
 25 \quad 63. \quad 16 \quad 27 \quad \overline{)50.62} \\
 \underline{25} \\
 1000 \overline{) \quad 63} \quad 16 \\
 \underline{6} \quad) \\
 1006 \overline{) \quad 60} \quad 36 \\
 10120 \overline{) \quad \quad 2} \quad 80 \quad 27 \\
 \underline{2} \quad) \\
 10122 \overline{) \quad \quad 2} \quad 02 \quad 44 \\
 \underline{77} \quad 83
 \end{array}$$

It should be noticed that 100, the first trial divisor, is not contained in 63, the first remainder. Hence 0 is the second figure of the root. Then 16, the next period, is brought down, the 0 is annexed to the trial divisor, thus making 1000, and the work is continued as before.

Since the hundredths' figure of the root is less than 5, the square root of 2563.1627, correct to the nearest tenth, is 50.6.

In finding a square root correct to a desired number of places, the root should be found to one more place than required, and then corrected as above. Zeros may be annexed to the right of the decimal point if necessary.

Checking square roots. A square root may be checked by squaring the root found and adding the last remainder (if any). If the work is correct, the result is the given number.

EXERCISES

1. Write a statement in your own words of the steps that must be taken in finding, correct to the nearest tenth, the approximate square root of a number which is a decimal.

Find the square root of each of the following, and check each result:

- | | | | | |
|----------|----------|------------|-------------|--------------|
| 2. 841. | 5. 2809. | 8. 51.84. | 11. 96.04. | 14. 412.09. |
| 3. 1764. | 6. 3969. | 9. 62.41. | 12. 102.01. | 15. 1909.69. |
| 4. 2401. | 7. 3249. | 10. 79.21. | 13. 182.25. | 16. 7430.44. |

Find, correct to the nearest tenth, the square root of:

- | | | |
|--------------|--------------|-----------|
| 17. 2.1563. | 19. 82.9600. | 21. 3.00. |
| 18. 15.4500. | 20. 3.5420. | 22. 2.00. |

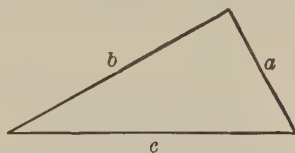
23. Find the side of the square whose area is 1156 sq. in.; 2116 sq. ft.; 2916 sq. yd.

24. Find the side of the square which is equal in area to a rectangle 24 ft. by 6 ft.; to a rectangle 36 ft. by 9 ft.

25. Find, to the nearest 0.1", the side of the square which is equal in area to a rectangle 22" $\times$ 18".

26. The formula for the area A (in square units) of a triangle when the lengths of the three sides are known is

$$A = \sqrt{s(s-a)(s-b)(s-c)},$$



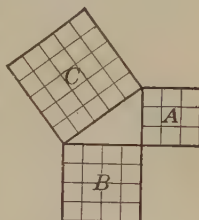
in which a , b , and c are the sides of the triangle, and $s = \frac{1}{2}(a+b+c)$. Find, to the nearest 0.1 sq. in., the area of a triangle whose sides are 15", 21", and 24" respectively.

That is, $a = 15''$, $b = 21''$, $c = 24''$, and $s = \frac{1}{2}(15'' + 21'' + 24'')$, or 30". Then $(s-a) = 15''$, $(s-b) = 9''$, and so on.

27. Find the area of a triangle whose sides are 12', 20', and 30'; 15 yd., 18 yd., and 25 yd.; 27", 36", and 45".

The right triangle. By counting the small squares in the squares on the sides of the right triangle here shown, it is seen that the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides of the triangle. That is,

*Square of the hypotenuse = square of one side
+ square of other side.*



Representing a side of square *A* by *a*, a side of square *B* by *b*, and the side of square *C* on the hypotenuse by *c*, the above fact may be stated by the formula

$$a^2 + b^2 = c^2.$$

This important relation is proved in geometry. In the above figure, since $a = 3$, $b = 4$, and $c = 5$, then $3^2 + 4^2 = 5^2$, or $9 + 16 = 25$, which is readily verified by counting and adding the small squares.

EXAMPLE. Find the hypotenuse of a right triangle whose sides are 12" and 16" respectively.

Solution. In this case, $a = 12''$ and $b = 16''$.

Hence $c^2 = a^2 + b^2 = 12^2 + 16^2 = 144 + 256 = 400$.

Then $c = \sqrt{400} = 20$.

Since a and b are dimensions in inches, the hypotenuse is 20".

EXERCISES

In the following exercises approximate square roots need be found only to the nearest tenth of the unit employed.

1. Find the hypotenuse of a right triangle whose sides are 5 in. and 12 in.
2. The sides of a right triangle are 9.1 ft. and 12.3 ft. Find the hypotenuse.
3. Find the length of the diagonal of a rectangle which is 16 ft. by 24 ft.

The **diagonal** is the line which joins two opposite vertices.

4. Find the length of the other side of a right triangle if one side is 13 ft. and the hypotenuse is 17 ft.

Since $a^2 + b^2 = c^2$, then $b^2 = c^2 - a^2$, and $b = \sqrt{c^2 - a^2}$.

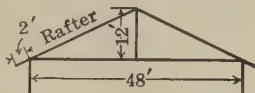
5. A baseball diamond is a square which is 90 ft. on a side. Find the distance across the diamond from first base to third base.

6. A ladder 24 ft. long rests against the side of a building with its foot 6 ft. 3 in. from the building. How high on the wall of the building does the ladder reach?

7. A smoke stack 90 ft. high is to be steadied by three guy wires, which are to run from a point 10 ft. from the top of the stack to three anchors, each 40 ft. from the foot of the stack. If 3 ft. is needed for anchoring each wire, how much wire is required?

8. The rope which passes through a pulley at the top of a flagpole can be stretched out in a double strand to touch the ground at a point 30.5' from the foot of the pole. If the rope is 200' long, how high is the flagpole?

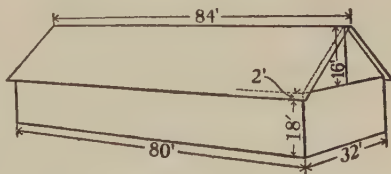
9. The accompanying figure shows the gable end of a roof. Using the dimensions given in the plan, find the length of each rafter.



10. In planning a roof a width of 40' for the building and a height of 10' for the gable are decided upon. If the rafters are to extend 2' 6'' beyond the building, find the length of a rafter.

11. The figure at the right shows the dimensions for a barn. Find the following:

- The area of the gable.
- The length of the rafters.
- The total area of the roof.



- The cost of a composition roof at \$12.50 per square.
- The cost of painting the roof at 40¢ per square yard.
- The cost of painting the sides and ends of the barn at 50¢ per square yard if no allowance is made for windows.

12. In order to have a sufficient capacity it is necessary that a water main should have a cross-section area of 9.4 sq. ft. What is the diameter of such a pipe?

Since $A = \pi r^2$, it is obvious that $r^2 = A \div \pi$, or $r = \sqrt{A \div \pi}$.

13. What must be the diameter of an iron supporting column to give a cross-section area of 78 sq. in.?

14. A drain pipe which carries water from two branch pipes should have its cross-section area equal to the combined area of the branch pipes. If the branch pipes are 5 in. and 8 in. in diameter, what should be the diameter of the drain pipe?

METRIC SYSTEM

Metric system. This system of weights and measures originated in France about 1800. It is in general use in scientific research and in foreign trade with a large part of the world. It is therefore essential that those who are planning to enter the business world should gain a working knowledge of this system of measurement.

Metric units. The principal units of the system are the **meter**, the unit of length; the **liter**, the unit of capacity; and the **gram**, the unit of weight.

The meter, which is the length of one forty-millionth part of the earth's circumference passing through its poles, is equivalent



to a length of 39.37 in. The liter is the capacity of a cube which is 0.1 meter (3.937 in., or approximately 4 in.) on an edge, and the gram is the weight of a cube of distilled water which is 0.01 meter (0.3937 in., or approximately 0.4 in.) on an edge. It should be noted that the unit of capacity (the liter) and the unit of weight (the gram) are thus defined in terms of the unit of length (the meter).

In the measurement of length the relationship of the metric system to our common system is shown in the above diagram, which shows a ruler marked in centimeters (hundredths of a meter)

and millimeters (thousandths of a meter) on one edge, and in inches and sixteenths of an inch on the other.

The whole metric system is built upon a scale of 10, and hence the different measures are either decimal divisions or multiples of the principal units. These are easily learned when the significance of the prefixes which indicate the measures are learned. The fractional parts of the principal unit are indicated by the following Latin prefixes:

milli-	which means 0.001
centi-	which means 0.01
deci-	which means 0.1

The multiples of the principal unit are indicated by the following Greek prefixes:

deka-	which means 10
hekto-	which means 100
kilo-	which means 1000
myria-	which means 10,000

Table of length. The meter (m.) is the unit of length, and the other measures are either decimal divisions or multiples. The following table shows those measures and their abbreviations which are in common use, and should be memorized:

1 millimeter (mm.)	= 0.001 meter (m.)
1 centimeter (cm.)	= 0.01 meter
1 decimeter (dm.)	= 0.1 meter
1 kilometer (Km.)	= 1000 meters

The meter is equivalent to a length of 39.37 in., the centimeter to 0.39 in., and the millimeter to 0.039 in. The kilometer is equivalent to 0.62 mi.

The dekameter (Dm.), which is 10 m., the hektometer (Hm.), which is 100 m., and the myriameter (Mm.), which is 10,000 m., are also used.

Table of square measure. The square meter (sq. m.) is the unit in the following table:

1 square millimeter (sq. mm.)	= 0.000001 square meter (sq. m.)
1 square centimeter (sq. cm.)	= 0.0001 square meter
1 square decimeter (sq. dm.)	= 0.01 square meter
1 sq. kilometer (sq. Km.)	= 1,000,000 square meters

The square dekameter (sq. Dm.), which is 100 sq. m., is called an **are**, and the square hektometer (sq. Hm.), which is 10,000 sq. m., is called a **hectare**, terms which are common in land measurement. A square myriameter (sq. Mm.) would be 100,000,000 sq. m.

Table of cubic measure. The cubic meter (cu. m.) is the unit in the following table:

1 cubic millimeter (cu. mm.) = 0.000000001 cubic meter (cu. m.)

1 cubic centimeter (cu. cm.) = 0.000001 cubic meter

1 cubic decimeter (cu. dm.) = 0.001 cubic meter

The table may be extended to include the cubic dekameter (cu. Dm.), which would be 1000 cu. m., the cubic hektometer (cu. Hm.), which would be 1,000,000 cu. m., the cubic kilometer (cu. Km.), which would be 1,000,000,000 cu. m., and the cubic myriameter (cu. Mm.), which would be 1,000,000,000,000 cu. m.

The cubic meter is also called a **stere** when used in measuring wood.

Table of capacity. The liter (l.) is the unit of capacity, as shown in the following table:

1 centiliter (cl.) = 0.01 liter (l.)

1 deciliter (dl.) = 0.1 liter

1 hektoliter (Hl.) = 100 liters

The liter is the same as the cubic decimeter, and is approximately equivalent to 1 qt.

There is also the milliliter (ml.), which is 0.001 l., the dekaliter (Dl.), which is 10 l., and the kiloliter (Kl.), which is 1000 l., but these measures are not commonly used.

Table of weight. The gram (g.) is the unit of weight, as shown in the following table:

1 centigram (cg.) = 0.01 gram (g.)

1 decigram (dg.) = 0.1 gram

1 kilogram (Kg.) = 1000 grams

1 quintal (Q.) = 100 kilograms

1 tonneau (T.) = 1000 kilograms

The gram is equivalent to a weight of 15.5 grains, or 0.002 lb. The kilogram, the most common measure of weight, is equivalent to 2.2 lb. The tonneau, more frequently called the **metric ton**, is equivalent to 2200 lb.

The other measures, the milligram (mg.), the dekagram (Dg.), the hektogram (Hg.), and the myriagram (Mg.) are not sufficiently used to be studied here.

ORAL EXERCISES

1. State the meaning of the following prefixes: centi-; kilo-; deka-; milli-; deci-; hekto-; and myria-.

2. Name the prefixes for the following: 100; 0.001; 10; 0.01; 10,000; 0.1; 1000.

3. Which prefixes are used to indicate the multiples of the principal units in the metric system?

4. In the abbreviations some begin with capital letters and others with small letters. What use can be made of this distinction in memorizing the measures?

Read the following:

5. 4 cm.; 18 m.; 12 Km.; 4 dm.; 45 mm.

6. 15 l.; 8 cl.; 29 dl.; 75 Hl.; 7 Kg.; 38 g.; 6 cg.; 5 Q.; 11 T.

7. 324 cu. m.; 114 sq. mm.; 96 cu. cm.; 45 sq. dm.; 16 cu. mm.; 12 sq. m.; 75 sq. cm.

8. Read each expression in Exs. 5-7 as a decimal fraction or multiple of its principal unit.

Thus, 4 cm. should be read 0.04 m., and so on. Quintals and metric tons may be expressed in kilograms.

9. From the figure on page 136, state the approximate equivalents of the following: 0.1 cm.; 4 cm.; 3 cm.; 10 cm.; 6.3 cm.; 1 dm.; 1 in.; 3 in.; 1.5 in.; $2\frac{1}{4}$ in.; $1\frac{3}{8}$ in.; $3\frac{3}{16}$ in.

10. With a meter stick measure the length and width of your classroom; the width of your desk; the height of one of your classmates.

EXERCISES

1. The Washington monument is 555 ft. high. Express this height in meters.

2. The height of Niagara Falls is 167 ft. Express this height in meters.

3. The weight of water which passes over Niagara Falls in 1 min. is approximately 700,000 tons. Express this weight in metric tons.

4. Find the weight of this book in pounds. Express this weight in kilograms. Check your work by weighing the book with metric weights, if possible.

5. Express in kilometers the height of a balloon which was observed to be 1500 yd. above the earth.

6. Since an acre contains 160 sq. rd., what approximate part of an acre are 10 ares?

An are is equivalent to 4 sq. rd., and a hectare to 2.5 A.

7. How many hectares are there in 50 A.? in 640 A.?

8. The distance by rail from Paris to Berlin is 1084 Km., from Paris to Rome is 1459 Km., and from Paris to Constantinople is 3055 Km. Express these distances in miles.

9. Measure the length and width of your school grounds in feet. Find its area in ares. Check your work by making the measurements in meters and computing the area.

10. Express a gallon (231 cu. in.) as cubic centimeters.

11. If a gasoline tank has a capacity of 15 gal., how many liters does it hold?

12. A rate of 40 mi. per hour is how many meters per second?

13. An importer buys 1200 m. of silk for \$3000. What should be the selling price per yard if the merchant plans for a gross profit of 0.45 of the cost price?

14. If a hektoliter is approximately equivalent to 2.2 bu., how many bushels of wheat are needed to fill an order for 5000 Hl.?

15. At 50¢ per kilogram, how many pounds can be bought for \$1? for \$4.75? for \$9.50?

16. A dirigible balloon with a capacity of 1000 cu. m. is filled with helium gas from small tanks, each containing 60 cu. ft. How many tanks are required to fill the balloon?

17. Express a cubic inch in cubic centimeters. What is the weight in grams of 1 cu. in. of distilled water?

For additional problems see page 402.

CHAPTER IV

COMPARING QUANTITIES

PERCENTAGE

Per cent. The expression **per cent** means hundredths; that is, a number given as a per cent indicates a number of hundredths.

It is evident that there are three ways of expressing a number of hundredths: (*a*) by a common fraction, as $\frac{72}{100}$; (*b*) by a decimal fraction, as 0.72; and (*c*) by the per-cent symbol, as 72%.

The three forms all have the same meaning, but the last form, that of a per cent, is the common business form.

ORAL EXERCISES

1. Express each of the following as an equivalent fraction with the denominator 100: $\frac{1}{2}$; $\frac{1}{4}$; $\frac{7}{20}$; $\frac{3}{20}$; $\frac{6}{25}$; $\frac{3}{4}$; $\frac{3}{2}$; $\frac{5}{4}$; $1\frac{7}{20}$.

2. Using the results of Ex. 1, express each fraction as a decimal; as a per cent.

3. Express each of the following as a per cent: 0.32; 0.45; 0.5; $0.33\frac{1}{3}$; $0.12\frac{1}{2}$; $0.62\frac{1}{2}$; 1.00; 2.50.

4. Express each of the following as a decimal: 15%; 56%; 6%; $12\frac{1}{2}\%$; $6\frac{1}{4}\%$; $8\frac{1}{3}\%$; 200%; 250%.

5. Express each of the following as a common fraction in lowest terms: 0.50; 0.73; $66\frac{2}{3}\%$; 1.25; 3%; 80%; $12\frac{1}{2}\%$; $87\frac{1}{2}\%$; $33\frac{1}{3}\%$; 125%; $133\frac{1}{3}\%$; $\frac{1}{2}\%$.

Percentage. The process of computing by per cents is called **percentage**; a term also applied outside of business to the result of finding a per cent of a number.

Per cent of a number. The process of finding a per cent of a number is that of finding a given number of hundredths of the number; namely, multiplying the number by a fraction or a decimal which represents the given number of hundredths.

Thus 25% of \$100 means $\$100 \times \frac{1}{4}$, and 17% of \$125 means $\$125 \times 0.17$.

Fractional values of certain per cents. Certain per cents are readily expressed as equivalent common fractions, and in such cases the process of finding a per cent becomes that of finding a fractional part. The following list of fractional equivalents of certain per cents should be memorized, since these per cents are used in many business computations:

PER CENT	FRACTIONAL EQUIVALENT	PER CENT	FRACTIONAL EQUIVALENT
$12\frac{1}{2}\%$	$\frac{1}{8}$	$33\frac{1}{3}\%$	$\frac{1}{3}$
25%	$\frac{1}{4}$	$66\frac{2}{3}\%$	$\frac{2}{3}$
$37\frac{1}{2}\%$	$\frac{3}{8}$	$83\frac{1}{3}\%$	$\frac{5}{6}$
50%	$\frac{1}{2}$	20%	$\frac{1}{5}$
$62\frac{1}{2}\%$	$\frac{5}{8}$	40%	$\frac{2}{5}$
75%	$\frac{3}{4}$	60%	$\frac{3}{5}$
$87\frac{1}{2}\%$	$\frac{7}{8}$	80%	$\frac{4}{5}$
$16\frac{2}{3}\%$	$\frac{1}{6}$	$8\frac{1}{3}\%$	$\frac{1}{12}$

EXAMPLE. Find $12\frac{1}{2}\%$ of 120.

Solution. Since $12\frac{1}{2}\%$ is $\frac{1}{8}$ of the number, then $12\frac{1}{2}\%$ of 120 is $120 \times \frac{1}{8}$, or 15.

ORAL EXERCISES

Using short methods, find the following:

1. 50% of \$400; of \$500.
2. 25% of \$60; of \$80; of \$120.
3. $12\frac{1}{2}\%$ of \$880; of \$1200.
4. $33\frac{1}{3}\%$ of \$360; of \$45.
5. $16\frac{2}{3}\%$ of \$900; of \$1200.
6. $37\frac{1}{2}\%$ of \$240; of \$360.
7. $62\frac{1}{2}\%$ of \$640; of \$720.
8. 20% of \$510; of \$105.

9. 60% of \$220; of \$300.
10. $66\frac{2}{3}\%$ of \$300; of \$105.
11. $83\frac{1}{3}\%$ of \$1080; of \$60.
12. $8\frac{1}{3}\%$ of \$960; of \$720.
13. 80% of \$150; of \$175; of \$210.
14. 75% of \$160; of \$80; of \$12.60; of \$8.28.
15. 100% of \$525; of any number of dollars.
16. $87\frac{1}{2}\%$ of \$824; of \$728; of \$648.

Finding a per cent of a number. When a per cent of a number such as 3%, 4%, 5%, 7%, and so on is to be found, the following method of multiplying mentally should be used.

EXAMPLE. Find 7% of \$200.

Solution. Here 7% of \$200 means $\$200 \times 0.07$. It is seen at once that 1% of \$200, or $\$200 \times 0.01$, can be found by simply moving the decimal point two places to the left, giving \$2. Then 7% of \$200 is $\$2 \times 7$, or \$14.

ORAL EXERCISES

Find the following:

- | | | |
|------------------|------------------|--------------------------------|
| 1. 2% of \$100. | 11. 4% of \$450. | 21. 8% of \$125. |
| 2. 2% of \$400. | 12. 4% of \$60. | 22. 8% of \$75. |
| 3. 2% of \$150. | 13. 5% of \$300. | 23. 10% of \$240. |
| 4. 2% of \$50. | 14. 5% of \$240. | 24. 10% of \$575. |
| 5. 3% of \$100. | 15. 5% of \$90. | 25. 12% of \$400. |
| 6. 3% of \$500. | 16. 6% of \$100. | 26. 15% of \$200. |
| 7. 3% of \$120. | 17. 6% of \$200. | 27. $2\frac{1}{2}\%$ of \$100. |
| 8. 3% of \$80. | 18. 6% of \$250. | 28. $2\frac{1}{2}\%$ of \$500. |
| 9. 3% of \$20. | 19. 6% of \$50. | 29. $3\frac{1}{2}\%$ of \$400. |
| 10. 4% of \$100. | 20. 7% of \$100. | 30. $4\frac{1}{4}\%$ of \$200. |

General method of finding a per cent of a number. When neither of the above methods can be readily applied, the student should see that finding a per cent of a number is only the familiar operation, expressed in different language, of multiplying by a decimal.

Emphasis should be placed upon the relation of percentage to the decimal processes with which the student is familiar. In this way many of the difficulties of the student may be removed.

EXAMPLE. Find 15% of \$725.

Solution. By the definition of a per cent, 15% of \$725 means $\$725 \times 0.15$.

The work is set down as shown at the right. Then multiplying as with decimals, the result is found to be \$108.75.

\$725
0.15
3625
725
\$108.75

EXERCISES

1. Find 7% of \$225; of \$310; of \$165.25; of \$535.75.

Unless otherwise directed, the student should find such per cents only to the nearest cent.

2. Find 12% of \$115; of \$75.25; of \$96.15; of \$210.15.

3. Find 28% of \$540; of 930 bu.; of \$458; of 36 gal.

4. Find 17% of \$375; of \$1240; of 25 T.; of 4250 lb.

5. Find 106% of \$650; of \$775; of \$1240; of \$2465.

The student should see that 106% may be written as 1.06.

6. Find 112% of \$880; of \$1200; of \$535.75; of \$476.95.

7. Find 27% of \$450; 2.7% of \$450; 0.27% of \$450; 0.027% of \$450.

8. Find the difference between 1.4% of \$760 and 0.14% of it.

9. Find $66\frac{2}{3}\%$ of \$720; of \$2400; of \$180; of \$324.

Remember to use a short method.

10. Find $83\frac{1}{3}\%$ of 1200 pupils; of 1620 lb.; of 4500 lb.

11. Find 125% of \$4.88; of \$525; of \$42.75; of \$563.30.

Since $125\% = 1\frac{1}{4} = \frac{5}{4}$, then 125% of \$4.88 is $5 \times \frac{1}{4}$ of \$4.88, or $5 \times \$1.22$, which is \$6.10.

12. Find $66\frac{2}{3}\%$ of 640 A.; of \$1260; of 5610 sq. ft.

13. Find the difference between $16\frac{2}{3}\%$ and $33\frac{1}{3}\%$ of \$420.

14. How much greater is $108\frac{1}{3}\%$ of 240 than $10.8\frac{1}{3}\%$ of 2400 ?
15. A farmer sold $37\frac{1}{2}\%$ of his corn crop of 800 bu. at 60¢ per bushel. How much did he receive for the part sold ? At the same price, how much would the remainder bring ?
16. In paying a bill of \$720, a man is allowed a reduction of $12\frac{1}{2}\%$. Find the amount paid.
17. In moving 2000 bu. of potatoes from a warehouse, it was found that 18% had decayed. How many bushels could be sold ?
18. The price of an automobile was \$2400. If the buyer of the car spent $8\frac{1}{3}\%$ more for extra equipment, how much was the total cost ?
19. In paying several bills, the following reductions were allowed: 2% on a bill of \$450, $2\frac{1}{2}\%$ on a bill of \$600, 15% on one bill of \$250, and $3\frac{1}{2}\%$ on a bill of \$125. Find the total amount paid.

Finding the rate per cent. The process of **finding the per cent** (or rate per cent) **which one number is of another** is illustrated in the following example.

EXAMPLE. What per cent of \$390 is \$78 ?

Solution. It is evident that 1% of \$390 is $\$390 \times 0.01$, or \$3.90 (which is found by moving the decimal point two places to the left). Then \$78 is as many per cent of \$390 as \$3.90 is contained in \$78; that is, $\$78 \div \$3.90 = 20$. Hence \$78 is 20% of \$390.

Check. $\$390 \times 0.20 = \78 .

From the above example the student should see that the process of finding the per cent which one number is of another is a case of dividing the first number by the second, and that the *number of hundredths* in the result is the required per cent.

EXAMPLE. What per cent of 325 is 45.5 ?

Solution. This is a case of finding what part 45.5 is of 325; that is, of dividing 45.5 by 325. Dividing as shown at the right, the student should see that 45.5 is 0.14 of 325; that is, since per cent means hundredths, it is 14% of 325.

Check. $325 \times 0.14 = 45.5$.

$ \begin{array}{r} 0.14 \\ 325 \overline{)45.5} \\ \underline{325} \\ 1300 \\ \underline{1300} \\ 0 \end{array} $

ORAL EXERCISES

In these exercises and those on the following pages many of the results may be found by the use of the fractional equivalents of per cents. Thus in Ex. 4, below, the student should see at once that 25 is $\frac{1}{8}$ of 200, and hence that it is $12\frac{1}{2}\%$ of 200.

In each of the following find what per cent the first number is of the second:

- | | | |
|---------------|--------------------|------------------------|
| 1. 20 ; 100. | 11. \$75 ; \$150. | 21. 72 in. ; 120 in. |
| 2. 25 ; 100. | 12. \$45 ; \$300. | 22. 36 ft. ; 180 ft. |
| 3. 20 ; 200. | 13. \$69 ; \$230. | 23. 50 bu. ; 125 bu. |
| 4. 25 ; 200. | 14. \$85 ; \$170. | 24. 99 rd. ; 90 rd. |
| 5. 15 ; 300. | 15. \$110 ; \$100. | 25. 165 yd. ; 150 yd. |
| 6. 12 ; 200. | 16. \$112 ; \$200. | 26. 640 mi. ; 320 mi. |
| 7. 16 ; 400. | 17. \$125 ; \$500. | 27. 198 cm. ; 180 cm. |
| 8. 28 ; 700. | 18. \$125 ; \$100. | 28. 10 qt. ; 50 qt. |
| 9. 42 ; 600. | 19. \$175 ; \$200. | 29. 4.5 gal. ; 15 gal. |
| 10. 54 ; 900. | 20. \$156 ; \$400. | 30. 7.5 in. ; 25 in. |

31. What per cent is the second number of the first in Ex. 17 ?
Ex. 22 ? Ex. 26 ? Ex. 28 ?

EXERCISES

1. What per cent is 45 of 75 ?
2. What per cent of \$425 is \$385 ?

Such answers may be found to the nearest 0.1% (tenth of one per cent). Except as otherwise directed, the student should not give results involving per cents beyond tenths.

3. \$6.25 is what per cent of \$420 ?
4. A reduction of \$23.50 was allowed on an invoice of \$627.50. By what per cent was the invoice reduced ?
5. If a man gains \$12.50 on an investment of \$250, find the per cent of gain.
6. If a man loses \$37.50 on an investment of \$500, find the per cent of loss.

7. A clerk had his salary increased from \$125 to \$135 per month. What was the per cent of increase?

8. Express a reduction in salary from \$175 to \$160 per month as a per cent.

9. If 415 pupils out of a total enrollment of 425 are present, find the per cent of attendance.

10. In a recent year 8818 pupils were graduated from the high schools in a certain state out of a group of 17,878 pupils who had entered the schools four years before. What per cent of the group which entered was graduated?

11. If the population of a city increased from 45,246 to 71,440 within a ten-year period, what was the average per cent of increase per year?

12. If an article which cost \$137.50 is sold for \$125, find the per cent of loss on the cost price.

13. In a county where the total taxes for a year were \$630,298 the amount allotted to school purposes was \$244,280. What per cent of the total taxes was spent for the schools?

14. The following table shows the purposes for which a year's taxes in one of the states were levied:

State	\$7,930,480
County	11,019,058
School	22,620,967
Township	2,428,349
City and Village	8,374,065

What per cent of the total was the amount levied for each purpose?

Finding a number when a certain per cent of it is known. The process of finding a number when a certain per cent of it is known is illustrated in the following example.

EXAMPLE. If 12% of a number is 156, what is the number?

Solution. Since 12% of the number is 156,

then 1% of the number is $156 \div 12$, or 13.

Hence 100% of the number is 13×100 , or 1300.

The student should thus see that if he divides the given per cent of the number by the given per cent *expressed as hundredths*, the result is the required number.

EXAMPLE. If \$282.03 is 17% of an amount, what is the amount?

Solution. Since \$282.03 is 17% of the amount, then \$282.03 divided by 0.17 is the required amount. Dividing as shown at the right, the result is found to be \$1659.

Check. $\$1659 \times 0.17 = \282.03 .

This case of percentage is used in business in problems of determining the selling price of articles so that their sale shall yield a desired per cent of profit. Problems of this type are considered in the following chapter.

	\$1659
0.17)	\$282.03
	17
	112
	102
	100
	85
	153
	153

ORAL EXERCISES

1. 48 is 2% of what number? 20% of what number?
2. 80 is 5% of what number? 0.5% of what number?
3. 30 is 3% of what number? 30% of what number?
4. 279 is 9% of what number? 90% of what number?
5. 65 is 13% of what number? 130% of what number?
6. \$90 is $12\frac{1}{2}\%$ of what amount? $37\frac{1}{2}\%$ of what amount?

Remember to use the fractional equivalent.

7. \$630 is 25% of what amount?
8. \$840 is $37\frac{1}{2}\%$ of what amount?
9. \$525 is 50% of what amount?
10. \$305 is $62\frac{1}{2}\%$ of what amount?
11. \$96 is 75% of what amount?
12. \$725 is 100% of what amount?
13. \$1250 is $16\frac{2}{3}\%$ of what amount?
14. \$562 is $33\frac{1}{3}\%$ of what amount?
15. \$138 is $16\frac{2}{3}\%$ of what amount?
16. \$235 is $8\frac{1}{3}\%$ of what amount?

EXERCISES

1. The yearly return at $4\frac{1}{4}\%$ from an investment is \$191.25. Find the amount invested.

2. If an investment offers a yearly return at the rate of 9.05%, what sum should be invested to yield an annual income of \$66.97?

3. The yearly return from an investment was \$199.50. If the rate of return was $4\frac{3}{4}\%$, find the amount invested.

4. A man who purchased a piece of property paid down \$1900 in cash. If the cash payment was 38% of the total value, find the value of the property.

5. In settling a claim for loss by fire, an insurance company agreed to pay 65% of the value for which the building was insured. If they paid the owner \$4885, for how much was the building insured?

6. At one time the number of girls enrolled in the commercial courses in the high schools of the United States was 62.5% of the combined enrollment of boys and girls. If the number of girls was 173,855, find the number of boys enrolled in these courses.

7. When a dealer took advantage of a reduction, for prompt payment, of $2\frac{1}{2}\%$ on the amount of a bill, he saved \$143.75. Find the amount of the bill.

8. The property in a certain city is assessed for 45% of its full value. Find the full value of a piece of property which is assessed at \$2262.60.

REVIEW EXERCISES

Since the use of per cents enters into almost every type of business computation the student will find an application of these topics throughout the remaining chapters of the book. The following exercises show general applications of percentage for the purpose of fixing the principles just learned.

1. The total amount of taxes levied in a county was \$48,171, of which 19% was for state purposes. Find the amount of the taxes levied for state purposes.

2. If \$125.20 was saved by taking advantage of a 2% discount for cash in paying an invoice, find the amount of the invoice.

3. Owing to shrinkage, a shipment of potatoes weighing 40,000 lb. lost 750 lb. in weight. Find the per cent of loss.

4. The yearly return from an investment which pays 8% is \$122. Find the amount invested.

5. If a clerk is allowed a sales commission of 15%, find the amount of his commission on the sale of a bill of goods amounting to \$87.50.

6. If 17 of the 255 employees in a factory were absent, what was the per cent of workers present that day?

7. An article which cost \$50 was sold for \$62.50. What per cent was gained on the cost? on the selling price?

8. The selling price in Ex. 7 was what per cent of the cost? The cost was what per cent of the selling price?

9. A grocery store spent \$1275.50 for advertising during a year in which their sales amounted to \$20,126.17. What per cent of the sales was spent for advertising?

10. During January the sales of a store amounted to \$5367.56, while the sales for the corresponding month of the previous year were \$5245. Find the per cent of increase.

11. The expenses of the Department of Trade and Commerce in one state were \$320,855 for a two-year period, while the total expenses of the state were \$25,792,256.33. The expenses of the department were what per cent of the total expenses?

For additional problems see page 405.

GRAPHICAL REPRESENTATION

Use of graphs. When statistics are presented in long columns of figures, the reader will very often fail to grasp all the facts which a comparison of the figures would give. But when these statistics are presented in the form of a diagram or picture, the facts set forth by the figures stand out in a clear and striking

manner. Such a pictorial representation for the purpose of comparing numbers or quantities is called a **graph**.

The average reader today finds graphs used frequently in trade and scientific journals, in magazines, in the daily newspapers, in textbooks, and in statistical reports of all kinds. The ability to grasp quickly the important points presented by a graph, called **interpreting the graph**, is of great importance, since the modern business executive is depending more and more upon the graph in analyzing business conditions.

Types of graphs. There are many ways of comparing quantities graphically, but only three types will be considered here. The following types illustrate most clearly the principle of pictorial comparison and are simple enough so that the student can produce graphs of his own from the data given in the exercises:

1. The **bar graph**, in which the comparison is shown by parallel bars of the same width but with their lengths proportional to the quantities which they represent.

2. The **curved-line graph** (or broken-line graph) in which a curved line is drawn through points located at different distances from a base line according to the quantities which they represent.

3. The **circle graph** in which the relative sizes of the quantities compared are represented by sectors of a circle.

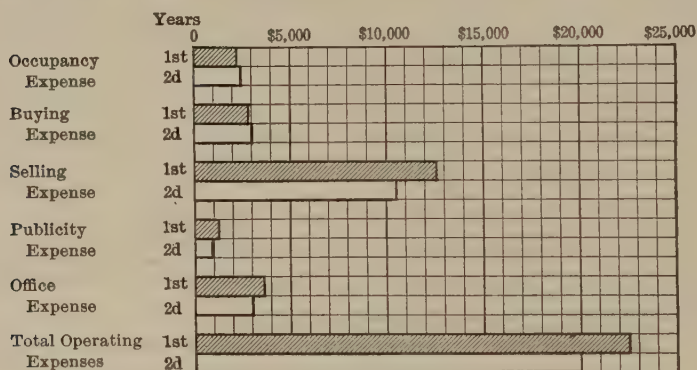
EXAMPLES. 1. For two consecutive years in which the receipts from sales and the cost of the merchandise sold for the second year were practically the same as the corresponding items for the first year, a firm finds that the operating expenses of the business were as follows:

	FIRST YEAR	SECOND YEAR
Occupancy Expense	\$2,275	\$2,400
Buying Expense	2,800	3,000
Selling Expense	12,600	10,540
Publicity Expense	1,225	985
Office Expense	3,600	3,075
Totals	\$22,500	\$20,000

Represent the data graphically by means of a bar graph.

Solution. On a piece of squared paper let 1 square represent \$1000 as shown below. Then a bar 2 squares in length represents \$2000, a bar 2.275 squares (2.3 approximately) in length represents \$2275, and so on. The length of the bar for each item is then determined in this way, and the bars for the items of the first year are shaded to distinguish them from those of the second year.

Interpretation. The graph shows clearly that the occupancy and buying expenses have increased, while the selling, publicity, and office expenses have decreased, and that the total cost of doing the



ANALYSIS OF OPERATING EXPENSES FOR A TWO-YEAR PERIOD

business necessary to bring in equal receipts from the sale of merchandise of equal cost has decreased. This fact indicates better business management for the second year.

The graph also shows roughly the relation which each item of expense bears to the others; for example, that the occupancy, buying, publicity, and office expenses are small in comparison with the selling expenses. Assuming that the business is one in which the selling expenses are always the chief item of the operating expenses, if the length of the bar of any other of the expense items had been nearly that of the selling-expense bar, then this expense item should be investigated by the business manager with a view to reducing its amount, if possible, another year.

The bar graph can, of course, be used in comparing only a single group of figures. For example, a single graph might have been drawn for the operating expenses for each of the years in the above case. The graph for the first year would then have only the shaded bars in the above graph, and that for the second year would have the white bars

2. The yearly sales summary of a department store shows that for two consecutive years the monthly sales in Department C were as follows :

MONTH	THIS YEAR	LAST YEAR
January	\$4965.65	\$4167.30
February	4734.65	4130.70
March	4700.10	4120.90
April	4803.15	3644.45
May	3641.95	3517.10
June	3965.05	3089.20
July	3754.55	3140.50
August	3945.40	3867.60
September	4841.75	3905.75
October	5017.35	4201.25
November	5015.50	4351.40
December	6174.25	4651.95

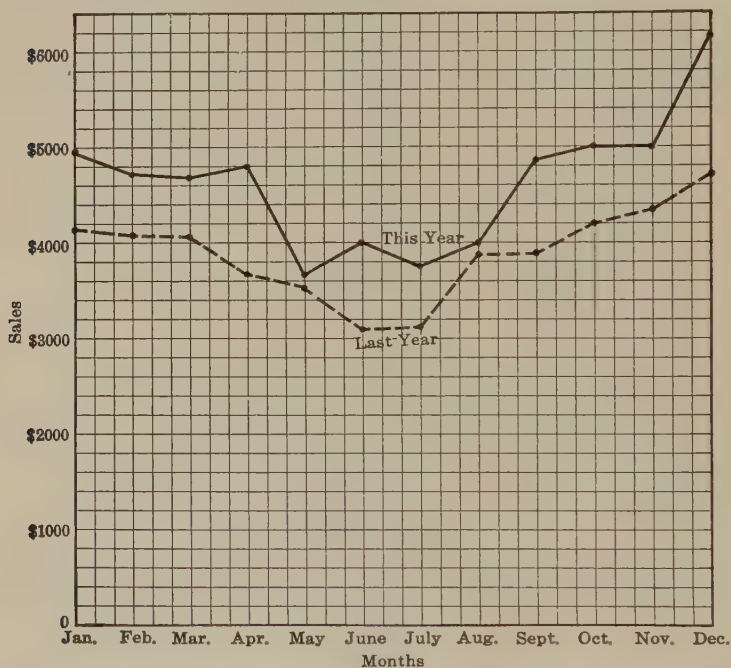
Represent these facts graphically by means of a curved-line graph.

Solution. On a horizontal base line let 3 squares represent 1 mo., and on the vertical lines take 1 square to represent \$200, as in the graph shown on page 154. Then the point for the first month's sales (\$4965.56 in January of this year) is located by taking a point approximately 24.8 squares above the horizontal base line on the first vertical line. The point for the February sales of this year is located by taking a point approximately 23.7 squares above the horizontal base line on the third vertical line, and so on. After all the points are located, the points for each year are connected by the broken lines as shown, the curve which represents the sales for last year being distinguished by the dotted line.

Interpretation. The graph shows clearly the fluctuations of the sales for each month. The rise of the curve for this year between the third and fourth points shows that the April sales were greater than those of March, while the drop in the curve from March to April of last year shows a decrease in the sales for that period. There was a greater falling off during May this year than last year, and the business for August of this year does not show as great a picking up after the midsummer season as it did the year before. The rise of the curves toward the end of each year is explained by the holiday season, the holiday business in December of this year showing a satisfactory increase as compared with the year before.

It is evident that such a graph is a very effective method of showing statistics of this type, as any marked change in the amounts is reflected by a corresponding sudden rise or fall in the curve.

Sometimes a smooth curve is drawn through the points instead of using the broken lines as in the graph below. As with the bar graph, the curved-line graph may be used to make comparisons in a single set of figures.



ANALYSIS OF THE MONTHLY SALES OF DEPARTMENT C FOR A TWO-YEAR PERIOD

3. It was found that the operating expenses for a group of retail shoe stores was distributed as follows:

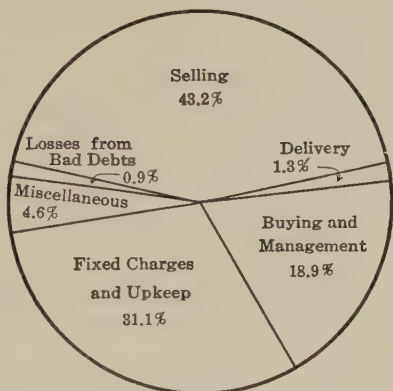
Selling	43.2%
Delivery	1.3%
Buying and Management	18.9%
Fixed Charges and Upkeep	31.1%
Miscellaneous	4.6%
Losses from Bad Debts	0.9%

Represent the above facts by means of a circle graph.

Solution. Since there are 360° in the entire angle about the center of a circle, each item should be represented by a sector whose angle at the center bears the same relation to 360° that the item does to the total expense. Hence the angle for the Selling sector is 43.2% of 360° , or 155.5° ; for Delivery, 1.3% of 360° , or 4.7° ; for Buying and Management, 18.9% of 360° , or 68.0° ; for Fixed Charges and Upkeep, 31.1% of 360° , or 112.0° ; for Miscellaneous, 4.6% of 360° , or 16.6° ; and for Losses from Bad Debts, 0.9% of 360° , or 3.2° . These angles are then laid off with a protractor around the center of the circle (to the nearest degree is generally sufficient) as in the graph here shown.

The first angle at the center may be laid off on any part of the circle that is desired.

Interpretation. The graph shows that the outstanding items of the operating expenses are in order of size, Selling, Fixed Charges and Upkeep, and Buying and Management.



ANALYSIS OF OPERATING EXPENSES

From the above examples the student should see that each type of graph is most efficient in a different type of comparison.

For example, the bar graph is most effective when it is desired to make a simple comparison of figures. Thus the graph in Example 1 compares most effectively the size of the different expenses in the two years and at the same time shows roughly the relative size of each expense.

The curved-line graph is most effective for showing fluctuations in prices, in the amounts of sales, production and demand of goods, and the like. Thus the graph in Example 2 shows most clearly the fluctuations in sales during each year, and at the same time by the two lines compares the sales by months for the two years.

The circle graph is most effective in showing the relationship of one item in a list of figures to the total. It is particularly effective when the quantities to be compared are expressed as per cents of a total. Thus the graph in Example 3 shows most strikingly the proportion which each item bears to the total of the operating expenses, and at the same time compares the different items relatively according to the size of the sectors.

EXERCISES

1. A recent report gives the following estimates (in millions) for the population of the ten largest cities: New York, 7.50; London, 6.72; Paris, 4.15; Berlin, 3.80; Chicago, 2.54; Tokyo, 2.24; Vienna, 2.00; Philadelphia, 1.73; Buenos Aires, 1.58; Osaka, 1.46. Represent these estimates by means of a bar graph.

2. The report of a telephone company shows that at ten-year intervals since 1880 the number of stations (in millions) in operation was as follows:

YEAR	1880	1890	1900	1910	1920
Millions of Stations	0.1	0.25	0.9	3.9	8.3

Represent the data by means of a bar graph.

3. An insurance company reported that for a period of twelve consecutive years the number of policies in force were as follows:

YEAR	POLICIES	YEAR	POLICIES	YEAR	POLICIES
1	1,571	5	23,918	9	68,453
2	7,975	6	31,487	10	74,296
3	14,449	7	44,717	11	97,147
4	18,030	8	55,490	12	122,746

Represent the data by means of a curved-line graph. Was the rate of growth of their business constant or fluctuating? Point out any marked changes which appear.

4. The census reports show that the population of Milwaukee at ten-year intervals was as follows:

YEAR	1860	1870	1880	1890	1900	1910	1920
Population	45,246	71,440	115,587	204,468	285,315	383,900	457,147

Represent these figures by means of a curved-line graph. Compare the general tendency of the increase in population from 1890 to 1920 with that from 1860 to 1890.

5. An annual state report shows that for each dollar expended by the state as a whole, the various departments spent the following amounts for the purposes shown: education, 33.3¢; roads, bridges, and paving, 36.6¢; state institutions, 10¢; state government, 13.3¢; state capitol, 6.8¢. Represent these statistics by means of a circle graph.

6. The Bureau of Immigration reports the immigration by months in a recent year as follows:

MONTH	JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEPT.	OCT.	NOV.	DEC.
Immigrants	6356	7388	6510	9541	15,217	14,247	7780	7862	9997	11,771	8499	10,748

Represent the above data by means of a bar graph in which the bars are vertical. Which months stand out as those of the greatest immigration?

The principle of such a graph is the same as that of the horizontal graph shown in Example 1, p. 152. The vertical form is a very common type of bar graph.

7. The following table shows the imports of sugar in thousands of long tons (2240 lb.) during a period of 11 yr.:

YEAR	1ST	2D	3D	4TH	5TH	6TH	7TH	8TH	9TH	10TH	11TH
Long tons (thousands)	130	238	85	20	42	130	175	122	30	60	885

Represent these statistics by means of a curved-line graph. During the last year reported in the table a large drop in the cost of sugar took place. Is this fact connected with the rapid rise for that year of the curve in the graph? Explain your answer.

8. In a recent year the appropriations of the United States government were distributed as follows:

Past Wars	68%
Future Wars	25%
Civil Departments	3%
Public Works	3%
Education and Science	1%

Represent these figures by means of a circle graph.

9. A firm finds that its sales and selling expenses during the different quarters of a year were as follows:

QUARTER	1ST	2D	3D	4TH
Sales	\$250,000	\$325,000	\$300,000	\$480,000
Selling Expense	\$22,500	\$27,625	\$29,500	\$32,000

Represent these figures by a bar graph, using pairs of horizontal bars, one for sales and the other for selling expense. Does the relation of sales to selling expense remain constant? How can you tell?

10. The following table shows for ten articles the per cent which the cost of doing business and the net profit on each article is of the selling price:

ARTICLE	1	2	3	4	5	6	7	8	9	10
Cost of Doing Business }	20%	25%	22%	20%	18%	19.5%	22%	25%	23%	20%
Net Profit	10%	15%	10%	12.5%	11%	15%	15%	12.5%	10%	15%

Represent these figures by a bar graph with pairs of vertical bars. Which articles yield the largest per cents of profit? Which articles have the highest costs of doing business? How can you tell?

11. The sales summary of a business showed that in two consecutive years one of the clerks had the following record of monthly sales:

MONTH	THIS YEAR	LAST YEAR	MONTH	THIS YEAR	LAST YEAR
January	\$5367.56	\$5245.80	July	\$5390.45	\$4850.55
February	5642.50	5313.35	August	5475.00	4875.00
March	6180.20	5449.45	September	5791.20	5067.30
April	6094.80	5390.75	October	5850.25	5237.25
May	5420.30	5000.20	November	5984.15	5428.00
June	5385.95	5910.95	December	6235.20	5835.65

Represent the sales by a curved-line graph, using a full line for the sales of this year and a dotted line for the sales of last

year. Does the graph show a substantial increase in this year's sales month by month as compared with those of last year? Point out the exceptions.

12. By means of the following table the gas company of A—— compared its rates per thousand cubic feet with the average of twenty-four other cities for a period of 20 yr.:

YEAR	AVERAGE IN OTHER CITIES	RATE IN A——	YEAR	AVERAGE IN OTHER CITIES	RATE IN A——
1	\$1.17	\$0.89	11	\$0.90	\$0.75
2	1.12	.88	12	.89	.72
3	1.08	.87	13	.88	.72
4	1.07	.86	14	.86	.72
5	1.04	.85	15	.86	.72
6	.98	.78	16	.85	.71
7	.96	.78	17	.85	.71
8	.95	.78	18	.93	.68
9	.94	.78	19	.97	.68
10	.92	.77	20	1.18	.68

Represent these statistics by means of a curved-line graph. Outline the reasons that the officials of the gas company in A—— might advance in support of a proposal to raise the price of gas from 68¢ per thousand feet.

13. Two departments of a store showed the following record of net sales during a 12-day period:

DAY	DEPT. A	DEPT. B	DAY	DEPT. A	DEPT. B
1	\$541.20	\$891.25	7	\$482.25	\$701.00
2	486.30	701.55	8	502.50	723.45
3	475.95	683.50	9	513.95	696.60
4	515.70	769.20	10	476.80	684.90
5	513.85	695.60	11	597.30	703.65
6	738.40	873.80	12	648.00	867.35

Represent the net sales by means of a curved-line graph. Compare the sales of the two departments for the period.

14. The receipts of sheep at a stock yard for fourteen consecutive days were as follows:

DAY	NUMBER RECEIVED	DAY	NUMBER RECEIVED
1st	24,396	8th	18,920
2d	17,896	9th	14,320
3d	14,971	10th	10,052
4th	9,657	11th	7,485
5th	8,559	12th	7,592
6th	2,185	13th	2,841
7th	0	14th	0

Represent the above data by means of a curved-line graph. Why does the curve fall to zero on the seventh and fourteenth days?

15. A dry-goods firm finds that its operating expense during a recent year was distributed as follows: Fuel, \$311.25; Advertising, \$1060.50; Light, \$255.17; Delivery Expense, \$2476.24; Rent, \$1000; Salesmen's Salaries, \$1119.02. Represent this distribution of expense by a circle graph. Write a statement of the facts set forth by the graph.

The student should first compute the per cent to the nearest tenth which each item is of the total expense.

16. Mr. Frank found that he apportioned his income as follows:

Savings	10%
Household	62%
Clothing	12%
Amusement	5%
Insurance	3%
Education	5%
Luxuries	3%

Represent these figures by means of a circle graph.

For additional problems see page 409.

CHAPTER V

BUYING AND SELLING

TRADE AND CASH DISCOUNT

Discount. When buying or selling merchandise a deduction is often made from the catalogue, or list, price of an article and, under certain conditions, from the amount of an invoice. Such a deduction, which is generally expressed as a per cent, is called a **discount**.

For example, an article which is listed to sell at \$5 may be sold at a discount of 20%, which means that the selling price is \$5 less 20% of \$5, or \$4.

Trade discount. Trade discount is a deduction, generally allowed only to dealers, from a published list price. The use of trade discount permits a manufacturer, for example, to issue only one catalogue to serve both the wholesale and retail trade. The prices listed are generally the retail prices and the dealer is supplied with a separate sheet, called a **discount sheet**, which shows the trade discounts for the various articles.

More important, however, is the opportunity thus afforded of changing the selling price of an article to meet fluctuations in market prices by altering the trade discount instead of changing the list price. It is thus unnecessary to reprint an entire catalogue, but instead new discount sheets are mailed to the dealer who inserts them in his catalogue, or files them for reference.

EXAMPLE. A trade discount of 30 % is allowed a retailer on an article which is listed in the catalogue at \$75. What is the selling price (invoice cost) to the retailer?

Solution. $\$75 \times 0.30 = \22.50 , the amount of the discount.

$\$75 - \$22.50 = \$52.50$, the selling price.

In case conditions changed so that the above article could be sold for less than \$52.50, an additional trade discount would be announced.

ORAL EXERCISES

State the invoice cost in each of the following :

LIST PRICE	TRADE DISCOUNT	LIST PRICE	TRADE DISCOUNT
1. \$25	20 %	6. \$60	12½ %
2. \$50	25 %	7. \$90	40 %
3. \$120	33⅓ %	8. \$45	6⅔ %
4. \$9	16⅔ %	9. \$30	60 %
5. \$75	10 %	10. \$84	5 %

Remember to use short methods when possible.

11. What is the selling price to the retailer of a table listed in a catalogue at \$125 with a trade discount of 40 % ?

12. A music dealer buys a phonograph listed at \$200 with a trade discount of 35 %. What is the invoice cost ?

13. A trade discount of 30 % is given on a stove listed at \$175. What is the selling price to the retailer ?

14. A hardware dealer receives a trade discount of 33⅓ % on a sled listed at \$7.50. What is the invoice cost ?

15. A trade discount of 40 % is allowed on a dresser the selling price of which is \$60. What is the catalogue price ?

16. The selling price of a davenport listed at \$150 is \$100. What per cent of trade discount is allowed ?

Discount series. Sometimes a trade discount consists of more than a single per cent, in which case it is called a **discount series**. The first per cent quoted indicates a discount from the list price, the second per cent a discount from the remainder, and so on.

Thus, a manufacturer may quote an article in his catalogue at \$100 subject to discounts of 20 % and 10 %. The selling price of this article is then found as follows :

$$\$100 \times 0.20 = \$20, \text{ the amount of the first discount.}$$

$$\$100 - \$20 = \$80, \text{ the first remainder.}$$

$$\$80 \times 0.10 = \$8, \text{ the amount of the second discount.}$$

$$\$80 - \$8 = \$72, \text{ the selling price.}$$

From the illustration given at the foot of page 162, it is obvious that a discount series of 20% and 10% is not equivalent to a single discount of 30%, since the selling price of an article listed at \$100, with a trade discount of 30%, is \$100 - \$30, or \$70.

In quoting discount series the larger per cents are usually stated first, but the order in which the discounts are calculated makes no difference in the selling price.

Thus, a series of 20%, 10%, and 5% is the same as one of 10%, 5%, and 20%, or one of 5%, 20%, and 10%. The truth of this statement may be seen by reversing the order of applying the discounts in any of the following exercises.

ORAL EXERCISES

State the invoice cost in each of the following :

LIST PRICE	TRADE DISCOUNT	LIST PRICE	TRADE DISCOUNT
1. \$40	25% and $33\frac{1}{3}\%$	6. \$100	40% and $16\frac{2}{3}\%$
2. \$50	20% and 10%	7. \$84	$16\frac{2}{3}\%$ and 20%
3. \$60	$33\frac{1}{3}\%$ and 20%	8. \$48	$12\frac{1}{2}\%$ and 10%
4. \$60	$16\frac{2}{3}\%$ and 20%	9. \$36	$33\frac{1}{3}\%$ and 25%
5. \$75	20% and 5%	10. \$10	50% and 10%

EXERCISES

1. On meat choppers which are listed at \$42 per dozen a trade discount of 40% and 10% is given. What is the selling price of each chopper?

2. A merchant buys 150 gross No. 11 Flat-head bright wood screws, $1\frac{1}{4}"$, listed at \$1.40 per gross and subject to a trade discount of 70%, 30%, and 15%. What is the total invoice cost? the invoice cost per gross?

3. Round-head brass screws, No. 10, $1"$, are listed at \$3.20 per gross, with a trade discount of 50% and 10%. Flat-head brass screws, No. 20, $1"$, are listed at \$3.50 per gross with a trade discount of 60% and 10%. What is the total cost to a dealer of 200 gross of each kind?

4. What is the total cost to a hardware dealer of the following invoice:

STOCK NO.	QUANTITY	DESCRIPTION	LIST PRICE	TRADE DISCOUNT
408	8 doz.	Wrought Steel Butts	\$3.00	40%, 10%, 5%
244	10 "	" " "	4.50	40%
16	100 lb.	Wire Brads, $1\frac{1}{4}$ "	.40	60%, 25%
17	300 "	Flat-head Wire Nails	.40	50%, 20%
457	4 doz.	Flush Bolts, 12"	6.50	Plus 40%

The stock number in the above invoice refers to the manufacturer's catalogue number, and the list prices are the prices per unit (dozen or pound) as listed in the column headed "Quantity." In the last item the "Plus 40%" given as the trade discount indicates that the price has advanced beyond the published price, and that instead of a deduction of 40% an addition of 40% must be made to the list price to find the selling price.

5. A tire dealer gives a trade discount of $12\frac{1}{2}\%$ and 10% on a casing listed at \$40. What is the selling price of the casing?

6. The list price of a lot of goods sold with a trade discount of 30% and 10% was \$2850. The goods were resold for \$3000 subject to a trade discount of 15% and 10%. What gross profit was made on the transaction?

7. What is the cost to a dealer for the following order of automatic pencils if he is allowed a trade discount of 40%:

STOCK NO.	QUANTITY	LIST PRICE
232	8	\$1.00
457	5	3.00
462	12	3.50
475	15	4.00

Supply the missing items in the following:

	LIST PRICE	TRADE DISCOUNT	INVOICE COST
8.	\$50	20% and 5%	?
9.	\$30	$62\frac{1}{2}\%$ and 10%	?
10.	\$40	?	\$16
11.	?	40%	\$24

	LIST PRICE	TRADE DISCOUNT	INVOICE COST
12.	\$85	40%, 20%, 10%	?
13.	\$110	70%, 20%, 10%, 5%	?
14.	\$125	?	\$85
15.	?	35%	\$130
16.	\$225	45%, 25%, and 5%	?
17.	\$350	62½%, 20%	?
18.	\$420	?	\$252
19.	\$480	33⅓%, 20%, and 5%	?
20.	\$500	40%, 33⅓%, and 10%	?

Equivalent single discount. In finding the selling price of an article sold with two or more discounts, it is often convenient to know the single discount which is equivalent to the discount series.

For example, in the case on page 162, it is readily seen that a discount of 20% and 10% is equivalent to a single discount of 28%. Since the selling price in that case is \$72, the discount is \$100 - \$72, or \$28. A discount of \$28 on \$100 is obviously a discount of 28%.

An examination of the above example shows the truth of the following simple rule for finding a single discount equivalent to two discounts:

To find the single equivalent discount, subtract the product of the two discounts from their sum.

That is, in the above case $0.20 + 0.10 = 0.30$, $0.20 \times 0.10 = 0.02$, and $0.30 - 0.02 = 0.28$; hence the single equivalent discount is 28%.

In finding the selling price of an article, the equivalent single discount may be found first and then the problem becomes simply a case of finding the selling price with a single discount.

For example, in the case on page 162, after determining that the single discount was 28%, by the above rule the student would proceed as follows:

$$\$100 \times 0.28 = \$28, \text{ the amount of the discount.}$$

$$\$100 - \$28 = \$72, \text{ the selling price.}$$

Where it is not necessary to know the amount of the discount, the student may find the selling price by the method shown in the following example.

EXAMPLE. Find the selling price of an article listed at \$125 and sold with a trade discount of 20% and 10%.

Solution. $1.00 - 0.20 = 0.80$ and $1.00 - 0.10 = 0.90$.

$\$125 \times 0.80 \times 0.90 = \$125 \times 0.72 = \$90$, the selling price.

In this method the discounts are first subtracted from 100% (1.00) and then the list price is multiplied by the product of the remainders. This method may be applied to a series of any number of discounts.

ORAL EXERCISES

State the single rate of discount which is equivalent to each of the following series:

- | | | |
|--------------------------------|---------------------------------|---------------------------------|
| 1. 10% and 5%. | 8. 30% and 20%. | 15. 50% and 20%. |
| 2. 20% and 20%. | 9. 30% and 25%. | 16. 60% and $12\frac{1}{2}\%$. |
| 3. 20% and $12\frac{1}{2}\%$. | 10. 35% and 5%. | 17. 60% and 25%. |
| 4. 20% and 15%. | 11. 35% and 20%. | 18. $33\frac{1}{3}\%$ and 15%. |
| 5. 25% and 5%. | 12. 40% and 5%. | 19. $33\frac{1}{3}\%$ and 25%. |
| 6. 25% and 20%. | 13. 40% and 30%. | 20. 45% and $33\frac{1}{3}\%$. |
| 7. 30% and 5%. | 14. 40% and $12\frac{1}{2}\%$. | 21. 60% and 15%. |

State the selling price of an article listed at:

- | | |
|------------------------------|-------------------------------|
| 22. \$300, less 20% and 10%. | 27. \$600, less 10% and 5%. |
| 23. \$400, less 20% and 10%. | 28. \$400, less 10% and 5%. |
| 24. \$200, less 40% and 20%. | 29. \$1000, less 50% and 10%. |
| 25. \$300, less 30% and 10%. | 30. \$1000, less 50% and 20%. |
| 26. \$500, less 40% and 10%. | 31. \$1000, less 40% and 30%. |

Equivalent single discount with three rates. When a discount series consists of three rates, the single equivalent discount may be found by combining the first two by the rule given on page 165, and then this result and the third discount are combined by the same rule to give the single discount equivalent to the three rates.

EXAMPLE. What single rate of discount is equivalent to a discount series of 20%, 15%, and 10%?

Solution. By the rule given on page 165, $0.20 + 0.15 = 0.35$, $0.20 \times 0.15 = 0.03$, and $0.35 - 0.03 = 0.32$.

Then, as before, $0.32 + 0.10 = 0.42$, $0.32 \times 0.10 = 0.032$, and $0.42 - 0.032 = 0.388$; that is, the single discount equivalent to a series of 20%, 15%, and 10% is 38.8%.

In finding the selling price of an article sold with three (or more) discounts, the method illustrated in the example on page 166 may be followed.

EXERCISES

Find a single rate of discount equivalent to each of the following discount series:

- | | |
|------------------------|---------------------------------------|
| 1. 30%, 20%, and 10%. | 11. $33\frac{1}{3}\%$, 25%, and 10%. |
| 2. 50%, 20%, and 10%. | 12. 25%, 15%, and 10%. |
| 3. 20%, 10%, and 5%. | 13. 25%, 20%, and 10%. |
| 4. 40%, 20%, and 10%. | 14. 20%, 20%, and 10%. |
| 5. 30%, 25%, and 10%. | 15. 40%, 10%, and 5%. |
| 6. 20%, 20%, and 20%. | 16. 20%, 10%, and 10%. |
| 7. 40%, 30%, and 10%. | 17. 30%, 10%, and 10%. |
| 8. 20%, 15%, and 5%. | 18. 40%, 10%, and 10%. |
| 9. 30%, 10%, and 5%. | 19. 50%, 25%, and 10%. |
| 10. 50%, 30%, and 20%. | 20. 40%, 25%, and 10%. |

Find the selling price of an article listed at:

21. \$325, less 25%, 10%, and 5%.
22. \$450, less 30%, 20%, and 10%.
23. \$500, less 40%, 25%, and 5%.
24. \$600, less $33\frac{1}{3}\%$, $12\frac{1}{2}\%$, and 10%.
25. \$750, less 20%, $12\frac{1}{2}\%$, and 5%.

Cash discount. A deduction allowed for the payment of an invoice within a specified time is called **cash discount**. The use of such a discount offers an incentive to the purchaser to save

money by paying the debt within the time allowed and makes it possible for the seller to receive his money promptly for the goods.

Business houses generally state on their order blanks and invoices the **terms** under which goods are sold ; that is, the conditions and time of payment which attend each sale. If the goods are to be paid for at the time of delivery, the statement "Terms: Cash " will appear on the invoice.

In many establishments the payment of an invoice within 10 da. is considered the same as cash.

If an invoice is due within 30 da. without discount, the expression "Terms: net 30 da." may appear, but on account of the limited space on invoices such expressions are usually abbreviated. Thus, "net 30 da." is often written "n/30."

If a cash discount is allowed, the amount and time within which it may be applied are indicated in the terms. Thus, if a cash discount of 2% for payment within 10 da. is allowed on the above invoice, the terms would be shown as 2/10, n/30 ; which means that if the invoice is paid within 10 da. *from the date of the invoice* the purchaser may deduct 2%. If the invoice is not paid until after the 10 da. have elapsed, the whole amount of the invoice must be paid within 30 da.

The date of the invoice is generally the date of shipment of the goods, and cash-discount periods are always calculated from the date of the invoice.

EXAMPLE. What amount is needed to pay an invoice dated June 1 with terms of 5/10, 2/30, n/60, if the invoice amounts to \$500 and payment is made on June 10 ? on June 25 ? on Aug. 20 ?

Solution. In the first case the payment is within the time allowed for the cash discount of 5%, and hence \$500 — \$25, or \$475, would pay the bill.

On June 25 the first cash-discount period has lapsed, but the purchaser may still deduct 2%. Hence \$500 — \$10, or \$490, is needed to pay the invoice on this date.

On Aug. 20 both cash-discount periods have lapsed, and the whole amount, or face, of the invoice, \$500, must be paid.

ORAL EXERCISES

State the amounts which are needed to pay the following invoices if settlement is made within the discount periods:

AMOUNT OF INVOICE	TERMS	AMOUNT OF INVOICE	TERMS
1. \$200	3/10, n/30	5. \$250	2/10, n/30
2. \$200	5/10, n/30	6. \$350	3/10, n/30
3. \$300	3/30, n/60	7. \$450	5/10, n/30
4. \$400	2/10, n/30	8. \$500	1/10, n/30

EXERCISES

Supply the missing items in the following:

	DATE OF INVOICE	AMOUNT OF INVOICE	TERMS	DATE PAID	AMOUNT PAID
1.	April 10	\$175 .	2/10, n/30	April 19	?
2.	April 12	\$325	3/10, 1/30, n/60	April 30	?
3.	April 15	?	5/10, n/30	April 24	\$399
4.	April 17	\$125	?	April 26	\$120
5.	May 3	?	5/10, 2/30, n/60	May 29	\$735

Trade and cash discounts. When both trade and cash discounts are allowed on the same bill, the trade discount is deducted first, and then the cash discount is figured on the invoice cost.

Thus, if an article listed at \$85 is sold subject to a trade discount of 40% and 25% with terms of 3/10, n/30, the invoice cost is $\$85 \times 0.60 \times 0.75$, or \$38.25. The amount of the cash discount is then $\$38.25 \times 0.03$, or \$1.1475, which to the nearest cent is \$1.15. Then $\$38.25 - \$1.15 = \$37.10$, the selling price.

EXERCISES

1. The terms of sale of an article listed at \$35 with a trade discount of 20% and 10% are 2/10, n/30. What is the cost of the article if it is paid for within the discount period?

2. A music dealer orders phonograph records as follows:

96	10" D. D.	\$1.00, less 40%
32	12" X. Sym.	\$1.50, less 40%
146	12" Single	\$2.00, less 40% and 10%

What is the amount of the invoice? If the dealer is allowed a discount of 2% for payment within the first 10 da. of the month following purchase, how much does he save by taking advantage of the cash discount? How much does he pay?

3. Under terms of 2/10, n/30, a stationer who is allowed a trade discount of 40% orders loose-leaf supplies amounting at list prices to \$975. What is the amount necessary to pay the invoice within the discount period?

Find the amount needed to pay each of the following invoices within the discount period:

	AMOUNT OF INVOICE	TRADE DISCOUNT	TERMS
4.	\$250	10% and 5%	2/10, n/30
5.	\$300	20% and 10%	3/30, n/60
6.	\$325	40% and 15%	3/10, n/30
7.	\$375	25% and 10%	5/10, n/30
8.	\$410	30% and 10%	1/10, n/30
9.	\$445	60% and 5%	2/10, n/30
10.	\$480	25% and 20%	2/30, n/60
11.	\$525	30% and 20%	5/10, n/30
12.	\$560	50% and 25%	3/10, n/30

For additional problems see page 411.

TRANSPORTATION CHARGES

Transportation charges. Before an article can be manufactured, the raw materials which enter into its making must be transported to the factory. Similarly, before a retailer can offer an article for sale, it must be transported to his store. **Transportation charges** include the different items which enter into the cost of transporting goods.

Freight rates. Large quantities of goods are generally sent by freight, and the railroads which move the goods charge for their services according to the quantity and character of the shipment.

These charges, called **freight rates**, are determined by the Interstate Commerce Commission if the freight is sent from one state to another.

Freight is divided into different classes depending upon the type of goods, the method of packing used, and the risk involved in transporting them. The first-class rates are the highest regular rates, although on some types of freight double first-class rates are charged where the goods cannot be packed in a car without a large waste of space. The rates also depend upon the quantity shipped, as large shipments generally have a lower rate.

The following table shows the local rates per 100 lb. for five different classes of freight from Chicago to the stations designated:

STATION	CLASSES				
	1	2	3	4	5
Crystal Lake, Ill. .	48½¢	41½¢	32¢	24½¢	17¢
Harvard, Ill. . . .	53½¢	45½¢	35½¢	26½¢	19¢
Janesville, Wis. . .	59½¢	50½¢	40¢	30¢	21¢
Evansville, Wis. . .	62½¢	53¢	42¢	31½¢	21½¢
Madison, Wis. . . .	66½¢	56½¢	45¢	33½¢	23¢
Waunakee, Wis. . .	79½¢	66¢	47½¢	34¢	27¢
Merrimac, Wis. . .	84½¢	71¢	56¢	37¢	27¢
Baraboo, Wis. . . .	84½¢	71¢	56¢	37¢	27¢

Any shipment weighing less than 100 lb. is charged as 100 lb. Special taxes due to war conditions are not considered here.

EXERCISES

From the above table determine the freight charges on each of the following shipments:

1. 16,210 lb., 1st class, from Chicago to Madison.
2. 2780 lb., 4th class, from Chicago to Harvard.
3. 8275 lb., 5th class, from Chicago to Baraboo.
4. 4386 lb., 2d class, from Chicago to Crystal Lake.

5. 18,753 lb., 3d class, from Chicago to Janesville.
6. 780 lb., 1st class, from Chicago to Merrimac.
7. 1758 lb., 1st class, from Chicago to Madison.
8. 10,109 lb., 1st class, from Chicago to Waunakee.
9. 5435 lb., 2d class, from Chicago to Baraboo.
10. 11,256 lb., 3d class, from Chicago to Janesville.
11. 1008 lb., 4th class, from Chicago to Harvard.
12. 15,635 lb., 5th class, from Chicago to Evansville.
13. Find the total freight charges on the following shipments:
 - 19,756 lb., 4th class, from Chicago to Baraboo.
 - 3643 lb., 1st class, from Chicago to Merrimac.
 - 13,564 lb., 3d class, from Chicago to Crystal Lake.
 - 10,358 lb., 2d class, from Chicago to Madison.
 - 6283 lb., 5th class, from Chicago to Harvard.
14. Find the total freight charges on the following shipments:
 - 150 lb., 1st class, from Chicago to Madison.
 - 85 lb., 2d class, from Chicago to Madison.
 - 13,210 lb., 4th class, from Chicago to Merrimac.
 - 2346 lb., 5th class, from Chicago to Merrimac.
 - 14,236 lb., 2d class, from Chicago to Janesville.
 - 7235 lb., 3d class, from Chicago to Janesville.

Express rates. Goods are generally sent by **express** when shipped in small quantities or when rapid delivery is essential. The rates charged are fixed by the Interstate Commerce Commission and depend upon the distance and the weight.

Express shipments are divided into two classes, although most articles come under the first classification. Items of food and drink are classified as second-class matter under detailed instructions issued to express agents. On small packages the difference between the rates is slight (often no difference), but for large shipments over long distances the saving is considerable.

Special war taxes are not considered in connection with express charges.

The following tables show the rates between certain points:

POUNDS	BETWEEN NEW YORK AND CHICAGO		BETWEEN NEW YORK AND NEW ORLEANS		BETWEEN NEW YORK AND DENVER	
	First Class	Second Class	First Class	Second Class	First Class	Second Class
5	\$0.50	\$0.50	\$0.66	\$0.66	\$0.73	\$0.73
6	.50	.50	.71	.71	.82	.82
7	.57	.50	.78	.72	.88	.84
8	.59	.50	.83	.72	.97	.84
9	.64	.50	.89	.72	1.03	.84
10	.67	.50	.96	.72	1.12	.84
15	.82	.62	1.26	.95	1.51	1.13
20	.98	.74	1.56	1.17	1.99	1.43
25	1.13	.85	1.86	1.40	2.28	1.71

POUNDS	BETWEEN CHICAGO AND BOSTON		BETWEEN CHICAGO AND GALVESTON		BETWEEN CHICAGO AND PORTLAND, OREGON	
	First Class	Second Class	First Class	Second Class	First Class	Second Class
5	\$0.52	\$0.51	\$0.62	\$0.62	\$0.96	\$0.96
6	.54	.51	.68	.67	1.07	1.07
7	.58	.51	.73	.67	1.20	1.16
8	.60	.51	.79	.67	1.31	1.16
9	.66	.51	.84	.67	1.44	1.16
10	.68	.51	.89	.67	1.55	1.16
15	.84	.63	1.17	.88	2.17	1.63
20	1.01	.76	1.45	1.09	2.77	2.08
25	1.17	.88	1.73	1.30	3.38	2.54

EXERCISES

Find the cost of sending each of the following packages by express :

1. 15 lb., first class, from Boston to Chicago.
2. 8 lb., second class, from Boston to Chicago.
3. 20 lb., first class, from Chicago to Galveston.

4. 6 lb., first class, from Chicago to Portland, Ore.
5. 5 lb., second class, from Portland, Ore., to Chicago.
6. 25 lb., first class, from New York to Denver.
7. Find the total express charges on the following packages :
 - 9 lb., second class, from New Orleans to New York.
 - 10 lb., first class, from New York to Chicago.
 - 7 lb., first class, from New Orleans to New York.
 - 25 lb., first class, from Chicago to Boston.
 - 15 lb., second class, from Denver to New York.
 - 5 lb., first class, from New York to Denver.
8. Find the total express charges on the following packages :
 - 6 lb., first class, from Galveston to Chicago.
 - 10 lb., second class, from New York to New Orleans.
 - 15 lb., first class, from Boston to Chicago.
 - 8 lb., second class, from Chicago to New York.
 - 20 lb., first class, from New York to Denver.
 - 25 lb., second class, from Portland, Ore., to Chicago.

Parcel post. Within certain limits as to the size of packages and their weight, goods may also be sent by **parcel post**, which is part of the postal service.

The rates of postage depend upon the weight and the distance which the package is to go. Regarding the point of mailing as a center, the United States is divided into eight zones, the first zone covering points of delivery within 50 mi., the second zone covering points distant 50 mi. to 150 mi., and so on. The following table shows the rates for packages of different weights in the eight zones, and also the local rate which applies to packages to be delivered within the area covered by the post office at which the parcel is mailed.

In the table on page 175 only a few rates for weights in excess of 11 lb. have been given. Complete information as to rates together with a parcel-post map which shows the regions included in the different zones can be obtained at the local post office.

A parcel may be insured against loss to the amount of its actual value, not exceeding \$100, by the payment of a small fee. Special delivery of a parcel at its point of delivery will be made upon the payment of an additional fee of 10¢ when the parcel is mailed.

WEIGHT IN POUNDS	LOCAL ZONE	50 mi.	50 mi. to 150 mi.	150 mi. to 300 mi.	300 mi. to 600 mi.	600 mi. to 1000 mi.	1000 mi. to 1400 mi.	1400 mi. to 1800 mi.	Over 1800 mi.
		FIRST ZONE	SECOND ZONE	THIRD ZONE	FOURTH ZONE	FIFTH ZONE	SIXTH ZONE	SEVENTH ZONE	EIGHTH ZONE
1	\$0.05	\$0.05	\$0.05	\$0.06	\$0.07	\$0.07	\$0.09	\$0.11	\$0.12
2	.06	.06	.06	.08	.11	.14	.17	.21	.24
3	.06	.07	.07	.10	.15	.20	.25	.31	.36
4	.07	.08	.08	.12	.19	.26	.33	.41	.48
5	.07	.09	.09	.14	.23	.32	.41	.51	.60
6	.08	.10	.10	.16	.27	.38	.49	.61	.72
7	.08	.11	.11	.18	.31	.44	.57	.71	.84
8	.09	.12	.12	.20	.35	.50	.65	.81	.96
9	.09	.13	.13	.22	.39	.56	.73	.91	1.08
10	.10	.14	.14	.24	.43	.62	.81	1.01	1.20
11	.10	.15	.15	.26	.47	.68	.89	1.11	1.32
15	.12	.19	.19	.34	.63	.92	1.21	1.51	1.80
20	.15	.24	.24	.44	.83	1.22	1.61	2.01	2.40
30	.20	.34	.34						
40	.25	.44	.44						
50	.30	.54	.54						

The limit of weight for packages to be delivered in the local and first three zones is 70 lb.; for all other zones the limit is 50 lb. Parcels weighing 4 oz. or less, except books, seeds, and so on, are mailable at the rate of 1¢ for each ounce or fraction thereof, regardless of distance. Books and catalogues weighing over 8 oz. are sent at regular parcel-post rates.

EXERCISES

1. What is the cost of mailing a package weighing 2 lb. to a point in zone 4? in zone 8? in zone 2? in zone 6?

2. Find the cost of mailing a package weighing $3\frac{1}{2}$ lb. to a point in zone 7; in zone 3; in zone 5.

The rate for a full pound is charged for a fraction of a pound.

3. What is the cost of mailing a package weighing 20 lb. to zone 2? to zone 1?

4. Find the total cost of mailing the following packages, if the first four are sent special delivery:

WEIGHT	TO ZONE	WEIGHT	TO ZONE
6 lb.	3	$8\frac{1}{2}$ lb.	6
$7\frac{1}{4}$ lb.	5	$3\frac{1}{16}$ lb.	Local
11 lb.	8	$7\frac{3}{4}$ lb.	5
50 lb.	2	10 lb.	4

Additional postal information. All mailable matter for transmission through the mails to points within the United States, to Cuba, Mexico, Hawaii, Porto Rico, Canada, the Republic of Panama, the Philippine Islands, and certain foreign countries is divided into four classes.

First-class matter includes letters, postal cards, and any sealed matter. The rate for this class is 2¢ per ounce or fraction thereof; for postal cards, 1¢. The rates to foreign countries not included in the above rates are 5¢ for the first ounce and 3¢ for each additional ounce.

Any written or typewritten matter, whether sealed or unsealed, is considered as first-class matter.

Second-class matter includes newspapers and periodicals which bear a notice of entry at a post office as second-class matter. Publishers of such matter now pay postage on a scale which is determined by the distance which the publications travel. The postal rates on this class of matter when sent unsealed by a person other than the publisher is 1¢ for each 4 oz., or fraction thereof, on each separately addressed copy.

Publications of a religious, scientific, or educational nature are mailable at a flat rate of $1\frac{1}{4}$ ¢ per pound, regardless of distance.

Third-class matter includes books and catalogues weighing 8 oz. or less, circulars, pamphlets, proof sheets and the manuscript copy, and engravings. The rate is 1¢ for each 2 oz. or fraction thereof, with a limit of weight of 4 lb.

Printed matter weighing over 4 lb. is sent at parcel-post rates.

Fourth-class matter comprises the domestic parcel post, as described on page 174.

Any piece of first-class, second-class, or third-class matter may be **registered** by the payment of an additional fee of 10¢. Such a letter or package is in this way insured by the government, and in case of loss the sender will be reimbursed for the actual value of the contents, not to exceed \$50 for first-class matter or \$25 for third-class matter. By paying a fee of 20¢, a package may be insured for \$100.

Any piece of mail will be given **special delivery** from the post office to which it is sent upon payment of an additional fee of 10¢ at the point of mailing.

While there are special-delivery stamps, ordinary stamps may be used if the words "Special Delivery" are written on the envelope or wrapper. There are also certain limits as to the distances within which special delivery is made.

THE COST OF A MANUFACTURED ARTICLE

Factory cost. The **factory cost** of an article is the cost of that article completed and ready for the market. It consists of three elements — the cost of material, the cost of labor, and factory expense.

Material. The commodities and substances which become a part of the manufactured article are called **material**.

For example, lumber, glue, hardware, and varnish are some of the materials which enter into the manufacture of furniture.

The cost of the material used in manufacturing is the cost of the material when ready for use. It therefore includes the invoice cost and any transportation charges on the material up to the time that it is received at the factory.

The cost of freight, express, or drayage may be charged directly against the material received or the total amount of these items may be distributed over all materials on a per-cent basis.

Thus, where a factory estimates its transportation charges as 10% of the invoice cost, the cost of material billed at \$33.50 would be \$33.50 + \$3.35, or \$36.85.

EXAMPLE. The elements of the factory cost of a machine are as follows: material, \$180; labor, \$108; factory expense, \$72. What is the factory cost of the machine, and what per cent is each element of the factory cost?

Solution. The factory cost is $\$180 + \$108 + \$72$, or $\$360$. The cost of material is $\$180 \div \360 , or 50% of the factory cost. The cost of labor is $\$108 \div \360 , or 30% of the factory cost. The factory expense is $\$72 \div \360 , or 20% of the factory cost.

The above results may be tabulated as follows:

ITEM	AMOUNT	PER CENT OF FACTORY COST
Material	\$180	50
Labor	108	30
Factory Expense	72	20
Factory Cost	\$360	100

EXERCISES

1. On five different jobs the cost records of a factory showed the following charges for material, labor, and factory expense:

JOB NO.	MATERIAL	LABOR	FACTORY EXPENSE
1	\$120	\$90	\$90
2	216	156	108
3	90	72	63
4	108	63	54
5	68	60	32

Find the factory cost of each job.

2. Find the per cent which each element of the factory cost in Ex. 1 is of the total cost.

3. A factory received an order of rough castings on which the invoice cost was \$1475, the freight was \$78.45, and the drayage was \$39.20. What was the total cost of the material?

4. If the factory in Ex. 3 had calculated its transportation charges as 9% of the invoice cost, what amount would have been charged to the cost of material on the castings?

Labor. The labor employed in a factory is of two kinds, **productive** and **nonproductive**. Productive labor is labor spent directly upon the different jobs or processes as they pass through the factory; that is, the labor of the workmen at bench or machine. Nonproductive labor is labor essential to the operation of the plant, but which is not expended directly upon the article being manufactured, such as the work of the superintendent, the foreman, or the janitor.

A record of the amount paid for each type of labor is kept. The cost of productive labor is charged directly against the job or process upon which it was expended, but nonproductive labor is charged as a part of factory expense.

EXERCISE

In ten successive years a factory spent the following amounts for productive and nonproductive labor. Find, for each of the ten years, the total amount spent for labor and the per cent which each type of labor is of the total labor cost:

YEAR	PRODUCTIVE LABOR	NONPRODUCTIVE LABOR	YEAR	PRODUCTIVE LABOR	NONPRODUCTIVE LABOR
1	\$25,000	\$5000	6	\$42,150	\$7850
2	27,720	5280	7	44,500	8500
3	30,600	5400	8	38,250	6750
4	29,750	4250	9	40,140	6860
5	39,100	6900	10	37,610	5890

Factory expense. The various items of expense which cannot be charged directly against the different jobs or processes, such as nonproductive labor, factory rent, heat, light, power, depreciation, insurance, property taxes, and repairs, are included in **factory expense**.

One method of determining the proper share of factory expense to be included in the factory cost of an article is on the basis of the relation of the total factory expense to the total amount spent for productive labor during a given period.

Thus, if during a year the cost of productive labor was \$285,000 and the total factory expense was \$114,000, then $\$114,000 \div \$285,000$, or 40%, would be used in determining the share of factory expense to be added to the factory cost of an article. Then for every dollar of productive labor charged to a job, 40¢ would be added for factory expense; that is, if the productive labor on a job amounted to \$10, the job would also be charged with \$4 for factory expense.

EXERCISES

1. The cost records of a factory show a total of \$150,000 for productive labor and \$50,000 for factory expense during the same period. How much should be figured for factory expense on a job upon which \$45 has been spent for productive labor?

2. The manufacturing costs of a machine were as follows: material, \$200; labor, \$150; factory expense, 60% of the labor charge. What is the factory cost of the machine?

3. Sometimes the total factory expense is distributed by departments and then the per cent of the productive-labor charge to be added to cover factory expense is determined for each department. Find this per cent for each of the following departments:

TYPE OF EXPENSE	DEPT. A	DEPT. B	DEPT. C
Productive Labor	\$3200	\$2500	\$3500
Factory Expense	4000	2500	2800

4. Using the per cents found in Ex. 3, find the factory cost of Job #152 on which the following material and productive-labor charges have been made:

CHARGES	DEPT. A	DEPT. B	DEPT. C
Material	\$15.00	\$23.00	\$42.00
Productive Labor	22.50	36.80	27.00

The factory cost of the job is the sum of the cost totals for the several departments.

5. The costs on Job #218 were as follows: material, \$500; productive labor, \$1000; factory expense, 25% of the labor cost. What per cent was each element of the factory cost?

6. The pay roll of a company for a year amounted to \$280,000 for productive labor and 20% of this amount for nonproductive labor. How much was paid for nonproductive labor? What was the total amount spent for labor?

7. On Order #750 the following charges were entered: material, \$360; productive labor, \$480; factory expense, \$420. Factory expense is what per cent of the charge for productive labor? of the factory cost?

8. In a certain year the factory cost of a machine was \$864, of which 40% was for material, 40% for productive labor, and 20% for factory expense. The factory expense was what per cent of the labor cost?

9. If for the next year the material in Ex. 8 increased 25%, the labor 20%, and the factory expense was 50% of the labor cost, what was the factory cost of the machine?

SELLING THE MANUFACTURED ARTICLE

Selling expense. Before a manufactured article reaches the wholesaler or retailer, **selling expenses** are incurred. All the manufacturer's expenses of marketing the finished product are classed as selling expense: in particular, such items as salesmen's salaries or commissions; traveling expenses; salesroom rent, heat, and light; depreciation and repairs of salesroom equipment; advertising expenses; credit-department expenses; delivery expenses; a share of the general office expenses; and so on.

In determining the selling price of the product, allowance is made for selling expense, generally by a per cent which is determined from the relation of the total selling expense of a period to the total sales of that period.

Thus, if in a given year the selling expenses are \$30,000 and the sales amount to \$200,000, then the selling expenses are $\$30,000 \div \$200,000$, or 15% of the sales.

EXERCISE

A manufacturer's records show that for five successive years the amounts of sales and selling expense were as follows:

YEAR	SALES	SELLING EXPENSES
1	\$575,000	\$57,500
2	400,000	56,000
3	560,000	67,200
4	480,000	72,000
5	640,000	96,000

For each year determine the per cent which the selling expense was of the total sales for that year.

Net profit. In addition to the allowance for selling expense when determining the selling price, allowance must also be made for the **net profit** which the manufacturer desires to make.

Thus, if a manufacturer desires to make a net profit of 10% on an article, the selling price must be such an amount as will yield this per cent after the factory cost and the selling expense are deducted.

Determining the selling price. The different elements which enter into determining the selling price of the manufactured article have now been considered; namely, factory cost, selling expense, and net profit. The relationship of these elements to the selling price is shown in the following outline:

$$\text{Selling price (100\%)} = \left\{ \begin{array}{l} 1. \text{ Factory cost} = \left\{ \begin{array}{l} \text{a. Material} \\ \text{b. Productive labor} \\ \text{c. Factory expense} \end{array} \right. \\ 2. \text{ Selling expense} \\ 3. \text{ Net profit} \end{array} \right.$$

The factory cost is an item which can be figured in dollars and cents, as was seen on page 178, but the allowances for selling expense and profit are generally calculated as *per cents of the selling price*. In this text, unless the contrary is stated, all profits and expenses in connection with selling are figured on the selling price as a basis and not upon the cost of an article.

Figuring profit on the cost basis is discussed on page 193.

That is, unless otherwise directed, *the selling price is considered as 100%*. Hence determining the selling price is a case of finding a number when a per cent of it (in this case, the factory cost) is known. First, however, the per cent which the factory cost is of the selling price must be determined. From page 182,

selling price = factory cost + selling expense + net profit,
and two of these elements, selling expense and net profit, are known as per cents of the selling price. Hence, if the sum of these two per cents is subtracted from 100%, the result is the per cent which the factory cost is of the selling price.

EXAMPLE. What should be the selling price of a manufactured article on which the factory cost is \$112.50, the selling expense is calculated as 15% of the selling price, and a net profit of 10% of the selling price is desired?

Solution. In this case, using the above relation,

$$100\% \text{ (selling price)} = \$112.50 + 15\% + 10\%,$$

where the per cents at the right are calculated on the selling price.

Now $100\% - 25\% = 75\%$,
and hence $75\% \times \text{selling price} = \112.50 .

Then $\text{selling price} = \$112.50 \div 0.75 = \150 .

After the student thoroughly understands the above example he should see that the analysis here given is not necessary each time, but that he should simply combine the per cents for selling expense and net profit, subtract the sum from 100%, and divide the factory cost by this result.

EXERCISES

1. The sales and selling expenses of a company for the four quarters of a year are as follows:

QUARTER	SALES	SELLING EXPENSES
1st	\$250,000	\$22,500
2d	325,000	27,625
3d	300,000	29,500
4th	480,000	32,000

Determine the per cent which this company should charge for selling expense for each quarter, and for the year.

2. In two consecutive years a factory finds that the amounts of the elements of the selling price of an article which it manufactures are as follows:

	FIRST YEAR	SECOND YEAR
Material	\$5.25	\$6.00
Productive labor	3.50	5.00
Factory expense	1.75	3.00
Factory cost	<u>\$10.50</u>	<u>\$14.00</u>
Selling expense	2.10	2.50
Net profit	1.40	1.80
Selling price	<u>\$14.00</u>	<u>\$18.30</u>

Find the per cent of increase for each element.

3. In each of the years in Ex. 2, what per cent of the selling price is for factory cost? for selling expense? for net profit?

4. For each of the years in Ex. 2, what per cent of the factory cost is for material? for productive labor? for factory expense?

5. The factory expense for each year in Ex. 2 is what per cent of the charge for productive labor?

6. In the example on page 183 determine the amount of the selling expense on the article; of the net profit on the article.

7. The factory cost in the example on page 183 was distributed as follows: material, \$35.25; productive labor, \$51.50; factory expense, 50% of productive-labor charge. Determine the amount charged to factory expense in fixing the factory cost and make an outline similar to the one on page 182, showing the amount of each element of the selling price of this article.

8. The factory cost of an order for ten tools is \$139.50. Selling expense in this company is calculated as 10% and a net profit of $12\frac{1}{2}\%$ is desired. Determine the selling price which should be placed on each tool.

9. The factory cost of an automobile was \$1155, distributing expense (selling expense) was calculated as 8%, and a net profit of 15% was desired. What price should the factory quote the dealer on this automobile?

10. The selling price of a machine is \$300, the selling expenses are 8%, and a net profit of $12\frac{1}{2}\%$ is made. Find the factory cost of the machine.

11. For the machine in Ex. 10, the cost of material was 46% and that of productive labor was 40% of the factory cost. Find the amount of factory expense charged to the machine and the per cent which this item was of the productive-labor charge.

12. The cost of material on an order for 50 articles was \$715 and the charge for productive labor was \$525. If factory expense was calculated as 40% of the productive-labor charge, find the factory cost of the order. If selling expense is calculated as $12\frac{1}{2}\%$ and a net profit of 15% is desired, what selling price should be quoted on each article?

THE MERCHANT'S PROFIT AND LOSS

Profit and loss applied to the retail business. In the preceding pages the cost of manufacturing an article and of placing it on the market in such a manner as to insure a profit to the manufacturer have been discussed. Since the factory rarely sells to the individual consumer, the manufactured article now passes — often through various other steps which will not be discussed here — to the retailer's store to be sold by him to the consumer.

What was the manufacturer's selling price now becomes the invoice cost to the retailer, and in order that the latter may make a profit on the sale of the article he must know how to determine the proper selling price.

The principles in the following sections apply to any type of business where goods are bought and sold, but for purposes of illustration the business of the retail store is taken as an example.

Gross profit. If the price at which merchandise is sold at retail exceeds the cost price, the difference is **gross profit**. Gross profit is frequently expressed as a per cent, in which case it is based on the selling price, as stated on page 182.

Thus, if during a year merchandise costing \$56,000 is sold for \$80,000, the gross profit for the year is \$80,000 — \$56,000, or \$24,000. The per cent of gross profit is $\$24,000 \div \$80,000$, or 30%.

Operating expenses. The expenses incurred in conducting a business are called **operating expenses**. In a store such expenses include rent, heat, light, salaries, advertising, depreciation, property taxes, and so on.

The total of the operating expenses is called the **cost of doing business**, which is generally expressed as a per cent of the total sales for a period.

Thus, if in the above case the merchant's expenses are \$16,000 for the year, then his cost of doing business is $\$16,000 \div \$80,000$, or 20%.

Net profit. If the operating expenses are subtracted from the gross profit, the result is the **net profit**. When net profit is expressed as a per cent it is based on the selling price.

Thus, in the above case the merchant's net profit for the year is $\$24,000 - \$16,000$, or \$8000. The per cent of net profit is $\$8000 \div \$80,000$, or 10%.

The different items of the year's operations of the business mentioned above may be shown in tabular form as follows:

	AMOUNT	PER CENT OF SALES
Sales	\$80,000	100
Cost of merchandise	56,000	70
Gross profit	<u>\$24,000</u>	30
Operating expenses	16,000	20
Net profit	<u>\$8,000</u>	10

The student should notice that the figures for the gross profit shown in heavy type are obtained by subtracting the cost of merchandise from the total sales, and that the figures for the net profit are obtained by subtracting the operating expenses from the gross profit.

Gross and net loss. Although a merchant generally plans to sell merchandise so that he will receive a net profit from the sale, losses are sometimes incurred. If the selling price of merchandise is less than its cost, the difference is the **gross loss**. The **net loss** is the sum of the gross loss and the operating expenses. When gross and net loss are expressed as per cents they are based on the selling price.

Thus, if during a year merchandise costing \$84,000 is sold for \$80,000, the gross loss is $\$84,000 - \$80,000$, or \$4000. The per cent of gross loss is $\$4000 \div \$80,000$, or 5%.

If the operating expenses for the year are \$16,000, the net loss is \$1000 + \$16,000, or \$20,000. The per cent of net loss is $\$20,000 \div \$80,000$, or 25%.

If during a year in which a gross profit is made on sales the operating expenses are greater than this profit, the year's operations still show a net loss. But in this case the net loss is the result of subtracting the gross profit from the operating expenses.

Thus, if during a year merchandise costing \$51,000 is sold for \$60,000, a gross profit of $\$60,000 - \$51,000$, or \$9000, is made. The per cent of gross profit is $\$9000 \div \$60,000$, or 15%.

If the operating expenses are \$12,000, the net loss is $\$12,000 - \9000 , or \$3000. The per cent of net loss is $\$3000 \div \$60,000$, or 5%.

EXERCISES

1. Arrange the items in the two illustrations above in the tabular form given on page 186.

The student should notice that in the case of losses, reverse subtraction (p. 34) is necessary; that is, the larger item appears below the number which is to be subtracted.

Arrange the following data in tabular form as shown on page 186, and determine the amounts of gross and net profit or loss, and the per cent which each item is of the total sales:

	SALES	COST OF MERCHANDISE	OPERATING EXPENSES
2.	\$124,000	\$115,320	\$12,400
3.	\$95,000	\$97,375	\$7125
4.	\$101,000	\$84,436	\$12,423
5.	\$76,500	\$77,647.50	\$13,770
6.	\$51,460	\$49,916.20	\$10,292

7. A shoe merchant's sales for a year amounted to \$115,000 and his gross profit was \$34,500. What per cent of gross profit did he make? What was the cost of the merchandise sold?

8. The total operating expenses of the merchant in Ex. 7 were \$25,300. What per cent was his cost of doing business?

9. What amount and what per cent of net profit did the merchant in Exs. 7 and 8 make?

10. The sales of the T. B. Jayne Co. for the year are \$175,000, the gross profit is \$52,500, and the operating expenses are \$35,000. Find the per cent of gross profit; the cost of doing business; the per cent of net profit.

11. A merchant sold goods amounting to \$22,500, which included 18% as his cost of doing business and a net profit of \$1800. What was the cost of the merchandise sold?

Supply the missing items in the following:

	SALES	COST OF GOODS SOLD	GROSS PROFIT Amount	Per Cent	OPERATING EXPENSES	NET PROFIT Amount	Per Cent
12.	\$4200	\$3108	?	?	\$840	?	?
* 13.	\$15,000	\$10,000	?	?	?	?	8 $\frac{1}{3}$
14.	\$20,000	?	?	30	\$4000	?	?
15.	\$22,000	\$15,000	\$7000	?	?	\$2200	?
16.	?	?	\$1250	25	?	?	5
17.	\$24,000	?	?	25	?	?	10
18.	?	\$7000	?	30	\$2000	?	?
19.	\$56,000	?	?	25	\$10,080	?	?
20.	\$7450	?	\$1490	?	?	?	2

Determining the selling price of an article. With an accurate knowledge of the cost of doing business a merchant is able to decide intelligently upon the selling price of an article so that it shall return a net profit.

The merchant determines his selling price in exactly the same way as the manufacturer, as described on page 182. In this case the known item is the cost of the article, which in addition to the invoice cost includes any transportation charges or other expenses incurred in making the article ready for sale, and the known per cents of the selling price are the cost of doing business and the desired net profit. In outline form,

selling price (100%) = cost of article + cost of doing business
+ net profit.

The sum of the per cents which represent the cost of doing business and the net profit is the per cent of gross profit. Hence

$$\text{selling price (100\%)} = \text{cost of article} + \text{gross profit,}$$

where the per cent of gross profit is based on the selling price. Then if this per cent is subtracted from 100% (selling price), the result is the per cent which the cost of the article is of the selling price.

EXAMPLE. A clothier knows that his cost of doing business is 25% of his sales and desires a net profit of 10% of the selling price. What should be the selling price of a suit costing \$32.50?

Solution. $25\% + 10\% = 35\%$, the per cent of gross profit,
and $100\% - 35\% = 65\%$, the per-cent cost of the suit.

Then $0.65 \times \text{selling price} = \32.50 ;
whence $\text{selling price} = \$32.50 \div 0.65 = \50 .

Check. $\$50 \times 0.35 = \17.50 , the amount of gross profit,
and $\$50 - \$17.50 = \$32.50$, the cost of the suit.

ORAL EXERCISES

State the selling price of each of the following articles :

	COST OF ARTICLE	COST OF DOING BUSINESS	DESIRED NET PROFIT
1.	\$5.60	20%	10%
2.	\$3.00	18%	7%
3.	\$4.20	22%	8%
4.	\$2.25	15%	10%
5.	\$6.50	27%	8%

State the per cent of gross profit on each of the following articles :

	SELLING PRICE	GROSS PROFIT		SELLING PRICE	GROSS PROFIT
6.	\$5	\$2	11.	\$150	\$30
7.	\$8	\$2	12.	\$100	\$25
8.	\$9	\$3	13.	\$180	\$60
9.	\$12	\$2	14.	\$200	\$50
10.	\$18	\$4	15.	\$250	\$50

State the cost of each of the following articles :

	SELLING PRICE	GROSS PROFIT		SELLING PRICE	GROSS PROFIT
16.	\$8	25%	21.	\$50	30%
17.	\$10	30%	22.	\$60	$33\frac{1}{3}\%$
18.	\$12	$33\frac{1}{3}\%$	23.	\$75	20%
19.	\$15	20%	24.	\$90	40%
20.	\$18	$16\frac{2}{3}\%$	25.	\$120	25%

Determining loss on an article. The method of determining loss on an individual article is illustrated in the following examples. The terms **gross loss** and **net loss** are applied in the same way as when applied to the total operations of the business (p. 186).

EXAMPLES. 1. What is the per cent of gross loss when an article which cost \$10 is sold for \$8?

Solution. $\$10 - \$8 = \$2$, the amount of gross loss.

$$\$2 \div \$8 = \frac{1}{4}.$$

Hence the gross loss, expressed as a per cent of the selling price, is 25%.

2. In a store where the cost of doing business is 20% of the selling price, an article costing \$20 is sold for \$18. What is the amount of the net loss?

Solution. $\$20 - \$18 = \$2$, the amount of gross loss.

$\$18 \times 0.20 = \3.60 , the share of cost of doing business.

$\$2 + \$3.60 = \$5.60$, the amount of net loss.

3. In a store where the cost of doing business is 28% of the selling price, an article costing \$45 is sold for \$52. Determine whether the sale of the article returned a profit or a loss to the merchant, and find the per cent of net profit or loss.

Solution. $\$52 - \$45 = \$7$, the amount of gross profit.

$$\$7 \div \$52 = 0.1346 +.$$

Hence the per cent of gross profit is approximately 13.5% of the selling price. But the cost of doing business is 28% of the selling price, so that the article was sold at a net loss.

Then $28\% - 13.5\% = 14.5\%$, the per cent of net loss.

ORAL EXERCISES

State the per cent of gross loss on each of the following articles :

	COST	SELLING PRICE		COST	SELLING PRICE
1.	\$12	\$10	4.	\$20	\$16
2.	\$16	\$12	5.	\$24	\$24
3.	\$18	\$15	6.	\$36	\$24

State the selling price of each of the following articles :

	COST	GROSS LOSS		COST	GROSS LOSS
7.	\$210	5%	10.	\$330	10%
8.	\$110	10%	11.	\$315	5%
9.	\$240	20%	12.	\$72	12½%

The student should see that in case of a loss,

selling price (100%) + gross loss = cost,

and hence that in Ex. 7, 100% + 5%, or 105% of the selling price, is the cost.

EXERCISES

1. What per cent of gross profit is made by a merchant who sells for \$9 a sweater which cost \$6 ?

2. If the cost of doing business is 15% and a net profit of 10% is desired, what selling price should be placed on an article which costs \$35 ?

3. The selling price of an article which cost \$21 was \$30. If the cost of doing business was 20%, what was the amount of the net profit ? the per cent of net profit ?

4. If the cost of a pair of shoes is \$3.50, what should be the selling price in order to return a net profit of 10% when the cost of doing business is known to be 20% ?

5. What should be the selling price of an article costing \$56 if the cost of doing business is 20% and a net profit of 10% is desired ?

6. A merchant's gross profits totaled \$99,750 in a year when his sales amounted to \$285,000. At what price should he sell a coat which cost \$12 in order to yield the same per cent of gross profit ?

7. If a merchant desires a gross profit of 32%, for how much should he sell an invoice of merchandise which cost him \$1224?

8. If the merchant in Ex. 7 desires a net profit of 9%, at what per cent should he estimate his cost of doing business?

9. What is the amount of net profit made by the merchant in Exs. 7 and 8 on the sale of the merchandise?

10. In deciding to close out a line of suits, a clothier sells one for \$35 which cost him \$45. What is the per cent of gross loss?

11. For how much did a hardware dealer sell a stove which cost him \$27.60 if he made a gross profit of 40%?

12. A clothier bought two dozen caps at \$18 a dozen and retailed them at \$2 each. What was the per cent of gross profit?

13. If the clothier in Ex. 12 estimated his cost of doing business at 23%, what was the amount of net profit on the whole purchase?

14. What was the cost of an automobile which a dealer sold for \$1800 if he made a gross profit of 20%?

15. A dealer buys automobile tires listed at \$40 at a trade discount of $12\frac{1}{2}\%$ and 10%, and the manufacturer pays the transportation charges. What is his per cent of gross profit on the sale of a tire at the list price?

16. A dealer made a gross profit of \$150 on a piano which was sold for \$500. What per cent of gross profit did he make?

17. A merchant reduced the prices on a group of articles as shown in the following list:

MARKED PRICE	REDUCED PRICE	MARKED PRICE	REDUCED PRICE
\$12	\$10.00	\$9	\$5.00
18	17.50	12	6.00
20	14.00	16	11.50

What was the per cent of reduction in each case?

18. If the cost of the articles in Ex. 17 averaged 60% of the marked price, find the amount of gross profit or of gross loss on the sale of each article.

Supply the missing items in the following:

	SELLING PRICE	COST PRICE	GROSS PROFIT	
			Amount	Per Cent
19.	\$150	?	\$50	?
20.	?	\$75	?	25
21.	\$45	?	?	35
22.	\$120	\$100	?	?
23.	?	?	\$60	40
24.	\$60	?	\$15	?

Profit and loss figured on the cost. As stated on page 182, in this text the per cents of profit and loss are figured with the selling price as the basis. This method is rapidly coming into general use and is distinctly in accord with modern business practice.

An older method, which is still in use in some places, is to figure the per cents of profit and loss upon the cost price. In this case, the selling price is determined by adding the amount of gross profit to the cost price, or in case of a loss by subtracting the amount of gross loss from the cost price. The gross profit is expected to cover the cost of doing business and the net profit.

Thus, if a merchant who figures his profits on the cost-price basis desires to make a gross profit of 25% on an article which costs \$2, the selling price is $\$2 + (\$2 \times 0.25)$, or \$2.50.

If an article costing \$5 is sold for \$4.50, then the gross loss is \$0.50, and the per cent of gross loss on the cost is $\$0.50 \div \5 , or 10%.

For a comparison of the two methods of figuring profits see page 195.

ORAL EXERCISES

In the following exercises use the cost-price basis.

State the per cent of gross profit on each of the following articles:

COST PRICE		GROSS PROFIT	
1.	\$6	\$2	
2.	\$8	\$2	
3.	\$4	\$2	
4.	\$10	\$4	
5.	\$12	\$3	
6.	\$20	\$4	
7.	\$25	\$10	
8.	\$30	\$10	
9.	\$40	\$8	
10.	\$48	\$12	

State the amount of gross profit on each of the following articles :

	COST	GROSS PROFIT		COST	GROSS PROFIT
11.	\$2.70	$33\frac{1}{3}\%$	16.	\$2.50	10%
12.	\$2.80	25%	17.	\$3.00	25%
13.	\$3.50	20%	18.	\$4.50	20%
14.	\$4.00	50%	19.	\$4.00	5%
15.	\$4.20	$16\frac{2}{3}\%$	20.	\$4.80	$8\frac{1}{3}\%$

21. In Exs. 1-20 determine the selling price of each article.

EXERCISES

In the following exercises use the cost-price basis.

1. A merchant's cost of doing business is 25% of the merchandise cost and he desires a net profit of \$1 on an article which cost him \$5. What selling price should he place on the article, and what per cent of net profit does he make?

2. A farmer sold a tract of land for which he paid \$125 per acre at \$200 per acre. Find his per cent of gross profit.

3. A merchant bought gloves at \$18 per dozen pairs and retailed them at \$2.25 per pair. Find the per cent of gross profit.

4. At what price per pair should the merchant in Ex. 3 have sold the gloves to make a gross profit of $33\frac{1}{3}\%$? of 25%?

5. If a house and lot which cost \$8715 were sold for \$7000, what was the per cent of gross loss?

6. Of five suits which he bought at \$32 each, a merchant sold two at \$45, two at \$42.50, and the other at \$40. What average per cent of gross profit did he make on the sale of these suits?

7. What amount of gross profit would the merchant in Ex. 6 have made if the suits had all been sold at 45% above cost?

8. A man sold two lots for \$3000 each. On one he made a gross profit of 25% and on the other he lost 25%. Find the total amount of profit or loss on the two transactions.

9. The cost of a lot of goods was \$1275. At the end of the first month the sales of these goods amounted to \$803.25 and the value, at cost price, of the goods remaining on hand was \$645. Find the per cent of gross profit made on the sales.

10. A wholesaler sold an article for \$34.20, thereby losing 5%. For how much should he have sold the article to have made a gross profit of $12\frac{1}{2}\%$?

11. A merchant buys the following lots of goods and sells them at the prices shown. Find the amount of gross profit or loss made on each lot and the per cent of gross profit made on the entire purchase:

LOT	COST PRICE	SELLING PRICE
1	\$235	\$310
2	\$372	\$340
3	\$296	\$420
4	\$366	\$482
5	\$408	\$500

Supply the missing items in the following:

	COST PRICE	SELLING PRICE	GROSS PROFIT	
			Amount	Per Cent
12.	\$75	?	\$25	?
13.	\$100	\$140	?	?
14.	?	\$60	\$15	?
15.	?	\$30	?	25
16.	\$125	?	?	20
17.	\$160	\$210	?	?
18.	\$175	?	\$105	?
19.	\$65	?	?	45
20.	\$140	?	\$49	?

Advantages of the selling-price basis in figuring profit. The reasons that the use of the selling price as the basis of figuring profit and loss is coming into more general favor are that the method is based on modern business principles, it is more

accurate, and it offers a more convenient method than the cost-price basis of determining the proper selling prices. A few of its advantages are as follows:

Many expenses of doing business are commonly figured as per cents of the selling price, so that the method makes for a uniform practice in the conduct of the business.

Thus salesmen are often paid a commission based upon the total of their sales, or a merchant estimates the amount which he is willing to spend on advertising as a per cent of past or expected sales. Discounts to customers are also figured on the selling price.

The sales-price basis provides a clearer and more accurate analysis of the operation of the business by the use of per cents.

Thus the selling price of 100% can be separated into its different elements — the cost of merchandise, the cost of doing business, and the net profit. In analyzing these factors the relation of the different elements to each other and to the whole is clearly apparent. With the cost-price basis this is impossible, since the cost — which is actually only a part of the selling price — must itself appear as 100% and the cost of doing business and the net profit must be added to it; that is, the cost-price method furnishes an indefinite basis for a comparison of this sort.

The use of the selling-price basis makes it easier to determine the proper selling price of an article so as to be sure that it will return a gross profit sufficiently large to cover the cost of doing business and the desired net profit.

A merchant's books will always show the total sales during a period but will seldom show the total cost price of merchandise sold unless the goods on hand are counted and their value is subtracted from the total purchases.

If a merchant knows, for example, that his cost of doing business is 20% of his sales, and if he desires a net profit of 10% of the sales, he knows at once that the invoice cost of an article is 70% of the proper selling price. If this merchant based his per cents on the cost-price basis, he would have to make several calculations to determine the proper selling price.

For these reasons, the use of the selling-price basis has been favored in this text, and to avoid confusion the student should always, from this point on, compute profits on the selling price unless a direction to the contrary is given.

It is possible, of course, to secure the same gross profit by a per cent figured on the cost-price basis as on the sales-price basis, provided that a larger per cent of gross profit is used.

Thus, on an article which costs \$1 and sells for \$1.50, the per cent of gross profit on the selling price is $33\frac{1}{3}\%$, while that on the cost price is 50%. Either per cent if used with its proper base will give the same amount of gross profit.

The following table shows certain per cents which give equivalent gross profits when based on the cost price and on the selling price :

$17\frac{1}{3}\%$ based on cost price	=	15% based on selling price
25% based on cost price	=	20% based on selling price
$33\frac{1}{3}\%$ based on cost price	=	25% based on selling price
50% based on cost price	=	$33\frac{1}{3}\%$ based on selling price

Sometimes merchants determine their per cents of expenses by basing them upon the selling price, but erroneously apply these per cents to the cost price of the merchandise. The result is that they do not make the necessary gross profit to cover the cost of doing business and insure a satisfactory net profit.

Return upon investment. At the end of the year, or other business period, the net profit or net loss of the business is also considered in connection with the investment, which is the capital employed in carrying on the business for the period.

The per cent of net profit upon the investment is the result of dividing the amount of the net profit by the amount of the investment. This per cent should not be confused with the net profit on sales which is the amount of the net profit divided by the total of the sales for the period.

At the beginning of a year the merchant generally fixes the per cent of net profit upon the investment — the **return upon investment** — which he desires to make. He then determines the per cent of net profit on sales which he must make to insure this return.

For example, a merchant may desire to make a net profit of 10% upon an investment of \$15,000; that is, he desires a net profit of \$1500 upon this amount. If he estimates that his sales for the coming year will amount to \$30,000, it is evident that he must make a net profit of $\$1500 \div \$30,000$, or 5% on the total sales, in order to return a net profit of 10% upon the investment.

THE SELLING OF MERCHANDISE

Preparing merchandise for sale. When a shipment of merchandise is received by a store, the goods are inspected and checked as to quantity and quality to see that they correspond to what was ordered and to the invoice which covers the shipment. Then the **unit cost** of the goods — the cost of the individual article — is accurately determined so as to include the invoice cost, the transportation charges, and any other expenses incurred in making the merchandise ready for sale. Then, after the selling price is determined, the goods are marked and placed in stock ready for the customer, as described later.

Use of twelfths. When goods are invoiced by the gross and sold by the dozen, the unit cost is the cost per dozen. Similarly, if the goods are invoiced by the dozen and sold by the piece, the unit cost is the cost of each piece. In determining the cost of goods invoiced and sold in this way, it is important that the student be able to divide rapidly and accurately by 12. In making such calculations much time is saved if the decimal equivalents of the 12ths of a dollar, as shown in the following table, are memorized:

TWELFTHS OF \$1	DECIMAL EQUIVALENT	TWELFTHS OF \$1	DECIMAL EQUIVALENT
$\frac{1}{12}$	0.08 $\frac{1}{3}$	$\frac{7}{12}$	0.58 $\frac{1}{3}$
$\frac{2}{12}$	.16 $\frac{2}{3}$	$\frac{8}{12}$	.66 $\frac{2}{3}$
$\frac{3}{12}$	.25	$\frac{9}{12}$	.75
$\frac{4}{12}$	.33 $\frac{1}{3}$	$\frac{10}{12}$	.83 $\frac{1}{3}$
$\frac{5}{12}$	.41 $\frac{2}{3}$	$\frac{11}{12}$	.91 $\frac{2}{3}$
$\frac{6}{12}$	.50	$\frac{12}{12}$	1.00

EXAMPLE. If neckties are invoiced at \$17 per dozen, what is the cost of one tie?

Solution. $\$17 \div 12 = \$1.41\frac{2}{3}$; and from the table, $\$1.41\frac{2}{3} = \$0.41\frac{2}{3}$. Hence to the nearest cent the cost of one tie is \$1.42.

This method is not so readily applied when the invoice price is given in dollars and cents.

ORAL EXERCISES

State the cost per article when the cost per dozen is as follows :

1. \$38.00.	7. \$7.50.	13. \$64.00.	19. \$5.60.
2. \$17.00.	8. \$8.40.	14. \$80.00.	20. \$3.40.
3. \$11.00.	9. \$19.00.	15. \$13.00.	21. \$13.20.
4. \$9.00.	10. \$25.00.	16. \$8.00.	22. \$21.60.
5. \$7.00.	11. \$37.00.	17. \$3.60.	23. \$19.20.
6. \$5.00.	12. \$42.00.	18. \$2.40.	24. \$66.60.

Marking goods. From the unit cost of the goods the selling price is determined, as described on page 188. The goods are then marked before they are placed on sale. Sometimes only the selling price is marked on the goods and sometimes both the selling price and the cost price are indicated.

The prices may be written upon the goods themselves, upon a tag or label attached to the articles, or upon the box or bin which contains them.

When both selling prices and cost prices are indicated, a cipher or code is often used in order to keep the cost prices secret. The selling price is generally given in figures, although in some stores it also is in cipher.

In case an article does not sell, the placing of the cost prices on merchandise enables those in charge to decide readily, and with accurate knowledge of the cost, upon a reduction in price. It also facilitates the taking of inventories.

In making up a cipher or code for use as a private mark for goods, a key word or key phrase may be employed in such a way that its letters represent the figures 0 to 9.

Thus, if the word "regulation" is used as the key, its letters represent figures as follows:

<i>r</i>	<i>e</i>	<i>g</i>	<i>u</i>	<i>l</i>	<i>a</i>	<i>t</i>	<i>i</i>	<i>o</i>	<i>n</i>
1	2	3	4	5	6	7	8	9	0

In this cipher an article which costs \$3.75 would have its cost price indicated by the mark *g.tl.*

To make a cipher more secure against unauthorized persons learning its meaning, an extra letter may be used as a "repeater" where a figure is repeated one or more times in the price.

Thus, if *s* is used as the repeater in the cipher on page 199, the mark *l.sn* indicates a price of \$5.50. Sometimes words or phrases of eleven letters are used as the key, and the eleventh letter serves as a repeater.

When marking the cost and selling prices, the cost is generally written above a horizontal line on the tag or label, and the selling price is placed below the line.

Thus, if both cost and selling prices are indicated by the cipher on page 199, article Stock No. 615, which costs \$3.50 and sells for \$5, would be marked as follows:

Stock No.	<u>615</u>
<i>q.ln</i>	
<i>l.ns</i>	

Sometimes a different cipher is used for the cost and selling prices. In such cases the cost cipher may be known to all the people in the store, or only to the buyers and their assistants who inspect and accept the goods.

Arbitrary characters may also be used for price ciphers, in which case ten different characters represent the figures, and still other characters are used as repeaters. Thus, if the characters

┐	└	┌	┘	└	┐	┘	└	┐	+		Repeaters: □	▷
1	2	3	4	5	6	7	8	9	0			

are used, the mark $\begin{smallmatrix} \text{┐} \\ \text{└} \end{smallmatrix}$ $\begin{smallmatrix} \text{└} \\ \text{┐} \end{smallmatrix}$ or $\begin{smallmatrix} \text{┐} \\ \text{┘} \end{smallmatrix}$ $\begin{smallmatrix} \text{┘} \\ \text{└} \end{smallmatrix}$ + would represent a price of \$5.50.

EXERCISES

1. In a store the phrase "was not blue" with repeaters *r* and *c* was used as a key for the cost-price cipher, and the phrase "your making" with repeaters *x* and *b* was used for the selling-price cipher. Write labels for the following six articles in these ciphers:

STOCK No.	COST OF ARTICLE	GROSS PROFIT ON SALES	STOCK No.	COST OF ARTICLE	GROSS PROFIT ON SALES
146	\$1.40	30%	627	\$3.45	30%
241	\$3.00	25%	821	\$2.72	32%
327	\$4.20	30%	460	\$2.10	33 $\frac{1}{3}$ %

In this and all following exercises, unless otherwise directed, profits are figured on the selling price.

2. With the character cipher given on page 200 for the cost prices and with figures for the selling prices write labels for the following six articles:

STOCK No.	COST OF ARTICLE	GROSS PROFIT ON SALES
112	\$24.00 per dozen	33 $\frac{1}{3}$ %
117	\$129.60 per gross	25%
219	\$224.00 per C	20%
221	\$3.60 per dozen	30%
276	\$14.40 per gross	40%
319	\$27.50 per dozen	25%

The articles should, of course, be marked with the price per piece; that is, the prices per dozen or per 100 (C) should first be changed to the unit prices.

3. In a store the character cipher shown on page 200 was used for cost prices, and the phrase "worthy sale" was the key to the selling-price cipher. Find the per cent of gross profit or loss made on the sale of each article marked as follows:

No.	PRICE TAG	No.	PRICE TAG
1	$\overline{\text{J. } \triangle \text{ J}}$ <i>y.te</i>	3	$\overline{\text{J. L. +}}$ <i>h.ye</i>
2	$\overline{\text{L. } \vdash \text{ L}}$ <i>r.he</i>	4	$\overline{\text{L. L. +}}$ <i>s.he</i>

Preparing catalogue price lists. Manufacturers or wholesale dealers, who sell goods subject to a trade discount, have to determine their list prices — the catalogue prices — in such a way that a profit is realized on the sale of an article after the trade discount has been deducted.

The method of determining the list price is exactly the same as that by which the manufacturer fixes his selling price, as described on page 182. The list price is determined by first finding the selling price which will return the desired profit, and then increasing this price in such a way that after the trade discount is deducted the fixed selling price is realized. In outline form,

$$\text{list price} - \text{trade discount} = \text{selling price},$$

where the trade discount is a per cent based on the list price.

EXAMPLE. What list price should a wholesaler who desires to quote a trade discount of 20% place on an article costing \$10 if he desires to make a gross profit of $37\frac{1}{2}\%$ on the selling price?

Solution. $100\% - 37\frac{1}{2}\% = 1 - \frac{3}{8} = \frac{5}{8}.$

Then $\frac{5}{8} \times \text{selling price} = \$10,$

and $\text{selling price} = \$10 \div \frac{5}{8} = \$10 \times \frac{8}{5} = \$16.$

But this selling price is 20% below the required list price; that is, the selling price is $100\% - 20\%$, or 80% of the list price.

Then $0.80 \times \text{list price} = \$16,$

and $\text{list price} = \$16 \div 0.80 = \$20.$

Check. $\$20 \times 0.20 = \$4,$ the amount of the trade discount.

$\$20 - \$4 = \$16,$ the selling price.

$37\frac{1}{2}\%$ of $\$16 = \$16 \times \frac{3}{8} = \$6,$ the amount of gross profit.

$\$16 - \$6 = \$10,$ the cost of the article.

EXERCISES

Find the list price of each of the following articles:

	COST PRICE	GROSS PROFIT	TRADE DISCOUNT
1.	\$4.20	30%	10%
2.	\$5.00	25%	15%
3.	\$12.00	25%	20%
4.	\$24.00	40%	25% and 10%
5.	\$30.00	35%	10% and 5%

Supply the missing items in the following:

	COST OF ARTICLE	GROSS PROFIT	SELLING PRICE	TRADE DISCOUNT	LIST PRICE
6.	\$17.50	30%	?	10%	?
7.	\$15.00	?	\$22.50	?	\$25.00
8.	?	40%	\$35.00	30%	?
9.	\$7.00	35%	?	15%	?
10.	?	40%	\$20.00	?	\$25.00

REVIEW EXERCISES

Find the selling price of each of the following articles :

	INVOICE COST	TRANSPORTA- TION CHARGES	COST OF DOING BUSINESS	NET PROFIT
1.	\$2.00	\$0.10	20%	10%
2.	\$4.75	\$0.35	25%	15%
3.	\$5.60	\$0.18	22%	10%
4.	\$5.55	\$0.12	20%	12½%
5.	\$17.10	\$0.65	18%	11%

The student should remember that transportation charges are an element of the cost of an article.

Supply the missing items in the following :

	COST OF ARTICLE	COST OF DOING BUSINESS	NET PROFIT	SELLING PRICE
6.	?	18%	12½%	\$4.00
7.	\$7.37	?	15%	\$11.00
8.	\$5.60	24%	11%	?
9.	\$5.00	21%	?	\$7.58
10.	\$6.50	20%	?	\$10.00

11. The manufacturer's selling price of a machine is \$15. If he increases his price 25%, at what price will a dealer have to sell the machine in order to realize a gross profit of 30%?

12. If the machine in Ex. 11 is listed in the manufacturer's catalogue at \$25 subject to a single trade discount, what per cent of trade discount was quoted before the increase in price? after the increase in price?

13. On an article which cost \$1 and sold for \$1.20 a merchant sold the following quantities at the discounts shown in order to move the stock rapidly:

QUANTITY	DISCOUNT	QUANTITY	DISCOUNT
40	10%	20	20%
50	12½%	32	25%

What per cent of gross profit or loss did he make on each of these sales? on the total sales?

14. On a lot of merchandise which cost \$575 a merchant made a gross profit of 50%, on a second lot which cost \$450 he made a gross profit of 20%, and on a third lot which cost \$300 he made a gross profit of 40%. What per cent of gross profit did he make on the total sales?

15. A real-estate dealer bought two city lots at \$900 each, and upon one lot he built a house costing \$5000. The house and lot were sold for \$7500 and the second lot was sold for \$1200. If the profit is figured on the cost, what per cent of gross profit was made upon each sale? upon the entire transaction?

16. If the real-estate dealer in Ex. 15 had figured his profit on the selling price, what per cent of gross profit would he have made on the entire transaction?

17. The following table shows the list price and the trade discount at which an office-supplies dealer buys certain articles, together with the retail price at which he sells them:

STOCK NO.	DESCRIPTION	LIST PRICE	TRADE DISCOUNT	RETAIL SALES PRICE
112	Wooden Filing Cabinet	\$60	50%	\$50
276	Steel Cabinet	\$85	40%	\$80
381	Special Label Steel Safe	\$255	40%	\$250
447	Commercial Steel Safe	\$104	40%	\$110
701	Steel Desk	\$165	40%	\$140
723	Steel Desk	\$192	45%	\$175

Find for each article the invoice cost and the per cent of gross profit made by the dealer.

18. A hardware dealer sells for \$132 a refrigerator which is listed at \$240 with a trade discount of 60%. Find his per cent of gross profit.

19. A stationer buys 4 gross bottles of ink at \$21.60 per gross less a trade discount of 40% and 10%. If he sells the ink at 15¢ a bottle, what amount of gross profit does he make on the whole lot?

20. A manufacturer of phonographs fixes his list prices so that he can quote a trade discount of 40%. What list price should he place on a machine for which the factory cost is \$100.80 if his selling expense is estimated at 20% and he desires a net profit of 10%?

Recording retail sales. When a customer makes a purchase in a retail store, the clerk making the sale generally fills out a printed form, called a **sales slip** or **sales ticket**, which has spaces for inserting the date, the clerk's number, the quantity and description of the articles sold, and the amount of the sale.

Some stores use elaborate forms which provide spaces for directions as to shipping and charging the articles or for other details in connection with the sale.

These sales slips are generally put up in the form of a pad and are numbered consecutively. By means of carbon paper, one or more copies of the sales slip are made at the time the original form is filled out. One copy is given to the customer, and the original ticket or a copy is retained by the store as its record of the sale. These forms furnish the source for compiling different statistics which enable those in charge to follow the progress of the store's sales.

Much information of value to the management of a business can be obtained by a study of sales slips which may be sorted and totaled according to cash and on-account sales, according to clerks and to departments, and so on. From the data thus secured the management can plan for the cash requirements of the business, the compensation of clerks, the securing of the greatest profit from each department, and so on.

Daily sales summary by clerks. The salaries of many clerks are dependent upon the amount of their sales and the gross profits which result therefrom. If the sales slips for each day are sorted out by clerks, a tabulation, or **sales summary**, can be made from them which will show the total sales made by each clerk. If on this summary the cost prices of the goods sold are also entered, the gross profits realized from their sale can be readily determined. The following summary shows a form which furnishes this information:

DAILY SALES SUMMARY					
CLERK No. 10					
Jan. 5, 19—					
SLIP No.	SALES PRICE		COST PRICE		GROSS PROFIT
251	30	00	18	00	12 00
252	37	75	22	65	15 10
253	24	75	16	00	8 75
254	27	50	17	50	10 00
255	48	00	32	50	15 50
256	52	00	34	00	18 00
257	21	00	15	50	5 50
258	25	00	16	75	8 25
TOTALS	266	00	172	90	93 10

Weekly or monthly sales summaries by clerks. From the daily sales summaries by clerks, weekly summaries may be made up, and these in turn consolidated into a monthly sales summary.

In some stores the weekly summary is omitted and the daily summaries are consolidated directly into a monthly summary.

WEEKLY SALES SUMMARY								
Week ending Feb. 6, 19—								
DAY	CLERK No. 10		CLERK No. 11		CLERK No. 12			
	Net Sales	Gross Profits	Net Sales	Gross Profits	Net Sales	Gross Profits		
1	186 00	56 00	162 50	50 80	205 00	79 50		
2	241 00	64 00	180 00	56 25	169 75	63 00		
3	124 00	43 00	154 00	59 00	180 50	65 50		
4	213 00	58 00	163 00	50 50	194 25	62 00		
5	233 00	84 10	194 00	61 70	202 00	80 00		
6	211 00	54 50	188 75	58 75	200 50	79 25		
TOTALS	1208 00	359 60	1042 25	337 00	1152 00	429 25		

The above form shows a weekly sales summary which consolidates the daily sales summaries of several clerks.

Yearly sales summary by clerks. The total sales and gross profits as shown by the monthly sales summaries may be transferred to a yearly summary showing the sales of each clerk. Such a form usually provides spaces for recording the same information for the preceding year so that the management may compare the sales and profits for the two years. Space is also provided for recording the increases in sales, both the amount and the per cent of increase, over the sales of the previous year.

If there is a decrease, this fact is generally indicated by writing the figures in red ink.

The following form shows a yearly sales summary which provides spaces for furnishing this information:

YEARLY SALES SUMMARY											
Harry Bronson, Clerk No. 10											
MONTH	THIS YEAR				LAST YEAR				INCREASES IN SALES		
	Sales		Gross Profits		Sales		Gross Profits		Amount		Per Cent
January	5367	45	1666	20	5245	80	1680	40			
February	5642	50	2026	10	5313	35	1840	75			
March	6180	20	2341	05	5449	45	2036	20			
October	5850	25	1985	20	5237	25	1854	55			
November	5985	15	2086	05	5428	00	1875	50			
December	6235	20	2405	85	5835	65	2130	05			
TOTALS	68818	15	24317	05	62605	75	20790	45			

The figures for the months from April to September have been omitted in the above form, but the totals include the omitted figures.

EXERCISES

1. From the daily sales summary given on page 206 determine the per cent of gross profit upon each sale and upon the total sales.

In these exercises find the results to the nearest 0.1%.

2. From the weekly sales summary given on page 206 determine the per cents of the total gross profits for the week which were earned by each clerk.

3. From the yearly sales summary given on page 207 determine the amount and the per cent of the increase in sales for each of the six months shown and for the whole year.

On account of the omitted figures, the increase for the whole year should be determined from the column totals.

4. In Ex. 3 what per cent of gross profit was made on this year's sales? on last year's sales?

Departmental sales summaries. When a store operates several departments, sales summaries similar to the daily, monthly, and yearly sales summaries by clerks are made by departments.

The information obtained from such summaries enables the management to push the sale of the more profitable lines, and to study why certain departments are less profitable than others with a view to discontinuing them or to applying such measures as will place them upon a profit-producing basis.

Departmental sales summary by clerks. The first step toward obtaining sales information by departments is to summarize the daily sales of each department from the sales slips. In making this summary a form which shows the per cent of the total sales and the per cent of the total gross profits made by each clerk is sometimes used, as shown in the following summary:

DAILY SALES SUMMARY					
DEPT. A					
Sept. 25, 19—					
CLERK NO.	SALES			GROSS PROFITS	
	Amount		Per Cent of Total	Amount	Per Cent of Total
1	151	90		46 80	
2	75	95		28 08	
3	108	50		37 44	
4	86	80		28 08	
5	119	35		46 80	
TOTALS	542	50		187 20	

Daily sales summary by departments. The departmental daily summaries by clerks are consolidated into a daily sales summary by departments for the whole store. This form is similar to the one on page 208, except that it is headed "Daily Sales Summary by Departments" in place of "Daily Sales Summary, Dept. A," and in place of the column "Clerk No., 1, 2, 3, ..." there is a column "Department, A, B, C, ..."

Weekly sales summary by departments. The daily sales summary by departments may be consolidated into a weekly sales summary which shows the departmental sales and the total store sales for each day of the week and for the whole week. Spaces for the per cent of the week's sales made on each day and the per cent of the total weekly sales made by each department may also be provided, as shown in the following form:

WEEKLY SALES SUMMARY BY DEPARTMENTS										
Week ending October 6, 19—										
DAY	DEPT. A		DEPT. B		DEPT. C		TOTAL		PER CENT OF TOTAL SALES	
Monday	541	20	691	25	401	00	1633	45		
Tuesday	486	30	701	55	408	75	1596	60		
Wednesday	475	95	683	50	390	80	1550	25		
Thursday	515	70	679	20	408	70	1603	60		
Friday	513	85	692	60	426	20	1632	65		
Saturday	738	40	873	80	601	50	2213	70		
TOTALS	3271	40	4321	90	2636	95	10230	25		
Per Cent of Total Sales										

The total of the daily per cents and of the departmental per cents should both equal 100%.

Monthly sales summary by departments. The monthly sales summary by departments may be compiled from the weekly sales summaries. If weekly summaries are not used, the daily

store summaries by departments may be consolidated directly into the monthly summary, as shown in the following form:

MONTHLY SALES SUMMARY BY DEPARTMENTS										
Month of October, 19—										
DATE	DEPT. A				DEPT. B				DEPT. C	
	Sales		Gross Profits		Sales		Gross Profits		Sales	Gross Profits
1	541	20	154	60	691	25	244	40	401	00
2	486	30	151	30	701	55	245	30	408	75
3	475	95	151	00	683	50	231	15	390	80
4	515	70	162	25	679	20	232	10	408	70
5	513	85	158	40	692	60	240	90	426	20
6	738	40	216	80	873	80	301	95	601	50

29	452	10	172	90	623	10	189	80	381	45
30	473	60	178	85	715	00	234	15	395	10
31	502	15	183	25	597	35	181	10	403	60
TOTALS	14269	10	4965	65	18921	30	6183	85	11826	55

In the above form the figures from the 8th to the 27th of the month inclusive have been omitted, but the totals include the omitted figures.

Departmental yearly sales summary. The monthly sales summaries by departments furnish the data for the yearly summary of the sales of each department. On this form space is usually provided for a comparison of the sales and gross profits with those of the preceding year, as shown in the following summary:

YEARLY SALES SUMMARY										
Dept. A										
MONTH	THIS YEAR				LAST YEAR				INCREASES IN SALES	
	Sales		Gross Profits		Sales		Gross Profits		Amount	Per Cent
January	14 269	10	4 965	65	12 120	55	4 167	30		
February	13 165	50	4 734	65	11 854	20	4 130	70		
March	12 401	65	4 700	10	11 969	00	4 120	90		

October	14 007	90	5 017	35	12 079	75	4 201	25		
November	14 157	95	5 015	50	12 847	30	4 351	40		
December	15 976	00	6 174	25	13 684	25	4 651	95		
TOTALS	154 653	25	55 559	35	135 654	55	46 788	10		

Yearly sales summary by departments. From the yearly summaries of each department, the total sales may be transferred to a yearly sales summary for the whole store. On this form spaces are often provided for entering the per cent of the total sales made each month and the per cent of the total sales made by each department, as shown in the following summary :

YEARLY SALES SUMMARY									
All Departments, 19—									
	DEPT. A		DEPT. B		DEPT. C		TOTAL		PER CENT
January	6 263	42	3 030	95	2 879	50			
February	27 254	89	3 103	62	7 491	02			
March	8 298	45	3 379	38	4 464	36			
October	14 962	78	10 381	88	9 634	60			
November	7 448	52	10 021	79	4 443	79			
December	7 277	04	6 554	01	6 008	97			
TOTALS	142 521	11	63 366	35	69 895	01			
Per Cents									

The figures for the months from April to September have been omitted, but the totals include the omitted figures.

EXERCISES

1. Copy the daily sales summary by clerks given on page 208 and fill in the two columns headed "Per Cent of Total."

2. Rule a form similar to the one in Ex. 1, and write the main heading "Daily Sales Summary by Departments," the date, the heading "Dept." over the first column, and the headings over the other columns as in Ex. 1. Fill in the form from the following table which shows daily departmental sales and gross profits, and compute the per cents :

DEPT.	SALES	GROSS PROFIT
A	\$438.75	\$158.48
B	303.75	96.22
C	270.00	84.90
D	405.00	130.18

3. Without copying the weekly sales summary given on page 209, determine the per cent of the total weekly sales made on each day, and the per cent of the week's sales made by each department.

4. From the monthly sales summary given on page 210, determine the per cent of gross profit made by each department during the month, and the per cent of gross profit made by the store as a whole during the month.

5. From the departmental yearly sales summary given on page 210, determine the increase in sales, both the amount and the per cent, for each of the six months shown.

6. In Ex. 5 determine from the column totals the amount and per cent of increase for the whole year.

7. In the yearly sales summary given on page 211 determine the total sales for each of the six months shown and the total of the year's sales.

The last result should be found from the column totals.

8. In Ex. 7 determine the per cent of the year's sales made each month, and the per cent made by each department.

For additional problems see page 414.

CONSIGNMENTS AND SHIPMENTS

Commission merchants and brokers. In the preceding pages the question of buying and selling has been treated as if the manufacturer, or producer, sold his goods directly to the retailer. In a country as large as the United States such direct contact is not always practical, and much buying and selling, particularly of agricultural products, is done through **agents**, persons who transact business for another, called the **principal**.

When an agent has actual possession or control of the goods of his principal, he is known as a **commission merchant**. An agent who merely arranges for the sale or purchase of goods without having such control is called a **broker**. The sum charged by an agent for transacting the business of his principal is called

commission or brokerage, and is often determined as a per cent of the amount involved in the transaction.

Account sales. When a commission merchant sells goods for his principal, he renders a statement, known as an **account sales**, which shows the details of the sale, as in the following form:

No. 4395	BOSTON, MASS., Oct. 6, 19--			
THE E. R. WALTON COMPANY				
PRODUCE COMMISSION MERCHANTS				
Sold for Account of Mason Fruit Exchange				
Winthrop, Me.				
Lot No. 3020		Date Rec'd 9/15		
220	bu. pears	.50	110	--
75	" plums	1.50	112	50
275	" "	1.25	343	75
40	" apples	1.25	50	--
10	" "	2.00	20	--
25	" crabs	2.00	50	--
GROSS PROCEEDS			686	25
CHARGES:				
Freight		193.50		
Cartage		38.70		
Commission		68.63	300	83
NET PROCEEDS				385 42

ACCOUNT SALES

In this case the principal, the Mason Fruit Exchange, has made a **shipment** of fruit to their agents, the E. R. Walton Company, to be sold on a commission basis. A commission merchant calls such a shipment a **consignment**.

The shipper of the goods, the Mason Fruit Exchange, is known as the **consignor**, and the person who receives them, the E. R. Walton Company, is known as the **consignee**.

The account sales shows all the details of the sale: the quantity, description, and unit price at which the fruit was sold; the **gross proceeds**, or the amount received by the agent for the lot; the charges connected with the sale, including the commission; and the **net proceeds**, or the difference between the gross proceeds and the sum of the charges, which are sent or credited to the principal.

In the account sales on page 213 the gross proceeds are \$686.25 and the net proceeds are \$385.42.

The amount of the commission is determined by a per cent based on the gross proceeds; in this case 10% on \$686.25.

Account purchase. When goods are bought by an agent, a detailed statement of the transaction, known as an **account purchase**, is rendered to the principal in some such form as here shown:

CHICAGO, ILL., May 20, 19--					
PURCHASE OF MERCHANDISE					
For Account of E. W. Brown & Co.,					
Milwaukee, Wis.					
By E. J. MATTHEWS					
1	car Eggs	@ 30 ¢	3600	--	
CHARGES:					
	Commission	1/2 ¢ per dozen	60	--	
AMOUNT CHARGED TO YOUR ACCOUNT				3660	--

ACCOUNT PURCHASE

In this case a broker, E. J. Matthews, has bought eggs for the principal, E. W. Brown & Co. The account purchase shows the quantity, description, and unit price of the goods purchased; the **prime cost**, or the invoice cost, of the goods; the charges in

connection with the purchase; and the **gross cost**, or the sum of the prime cost and the charges, which is charged to the principal.

In this case the prime cost is \$3600, the charges are \$60, and the gross cost is \$3660. A car of eggs contains 400 cases of 30 doz. each.

When the commission for purchasing goods is determined by a per cent, the charge is based on the prime cost.

EXERCISES

Supply the missing items in the following:

	GROSS PROCEEDS	RATE OF COMMISSION	AMOUNT OF COMMISSION	NET PROCEEDS
1.	\$182.40	5%	?	?
2.	?	?	\$6.60	\$125.40
3.	?	2½%	?	\$251.55
4.	\$255.50	?	\$10.22	?
5.	\$112.75	?	?	\$108.24
6.	\$282.00	?	\$4.23	?
7.	?	?	\$14.84	\$409.16
8.	\$457.75	2%	?	?
9.	\$823.40	3%	?	?

10. On Nov. 15, 19—, P. F. Marshall & Co., commission merchants, sell for the account of F. J. James 557 lb. of dressed geese at the following unit prices:

125 lb. @ 33¢	172 lb. @ 30¢
200 " " 32¢	60 " " 29¢

If Marshall & Co. charge a commission of 10%, what are the gross proceeds of the sale? the net proceeds?

11. Johnson & Co. sell for M. A. Jackson 132 bbl. of apples, of which 69 bbl. were sold at \$5 per barrel and the remainder at \$6 per barrel. The freight was \$80.96; cartage, \$26.40; and the rate of commission, 10%. Find the gross proceeds; the net proceeds.

12. Andrews & Co. sell for the Jordan Fish Co. 10 bbl. of turtles, 1950 lb. @ 7¢ per pound. If the express charges are \$19.20 and the rate of commission is 10%, what are the net proceeds?

13. The Northwestern Produce Co. sells a consignment of poultry for B. A. Barron, as follows:

175 lb. @ 22¢ per pound	324 lb. @ 26¢ per pound
218 " " 24¢ per pound	168 " " 27½¢ per pound

The express charges are \$10.11, and the rate of commission is 12½%. Find the net proceeds.

14. Carter & Co. buy 2 cars (a car holds 400 cases and a case contains 30 doz.) of eggs @ 30¢ per dozen for the account of A. J. Barton and charge a buying commission of ½¢ per dozen. Three days later, upon Barton's order, they sell 18,000 doz. @ 33¢ per dozen and the balance of the purchase at 33½¢ per dozen, and charge a selling commission of ¼¢ per dozen. What is the net amount due A. J. Barton from his agent?

15. On June 15, 19—, Sykes & Co. ship to J. Donald, a commission merchant, 200 cs. (cases) of eggs, 100 bx. (boxes) of cheese, and 200 tb. (tubs) of creamery butter. From this consignment Donald sells to Scovill & Co., on June 22, 19—, 200 tb. (10,000 lb.) of creamery butter @ 35¢ per pound and 100 cs. (3000 doz.) of eggs @ 30¢ per dozen. If the terms of the sale are 3/10, n/30, and Scovill & Co. take advantage of the cash discount, what amount do they remit to Donald?

16. From the consignment of Sykes & Co. mentioned in Ex. 15, Donald sells to Wood & Co. on June 23, 19—, 100 cs. (3000 doz.) of eggs @ 30¢ per dozen and 100 bx. (6000 lb.) of cheese @ 22¢ per pound, under the same terms of sale as given in Ex. 15. If Wood & Co. pay the invoice before the cash-discount period has elapsed, what amount do they remit to Donald?

17. On June 27, 19—, Donald sends Sykes & Co. an account sales covering the consignment mentioned in Exs. 15 and 16. The charges on the shipment to Scovill & Co. were \$22 for freight and \$3.75 for drayage; on the shipment to Wood & Co., \$37.50 for freight and \$6 for cartage. Donald charges a commission of 5% for his services. Make out an account sales similar to the one on page 213 such as Donald would render to Sykes & Co.

The commission is based upon the amount actually received for the goods.

18. If Sykes & Co. in Ex. 15 bought the eggs in the shipment at $28\frac{1}{2}\phi$ per dozen, the cheese at 18ϕ per pound, and the butter at 31ϕ per pound, what amount of gross profit did they make on the transaction?

19. J. C. Langley shipped a car containing 756 bx. of apples from Missoula, Mont., to J. C. Brown & Co., commission merchants, of New York City. The freight charges on the shipment amounted to \$907.20, the apples were sold at \$2.75 per box, and Brown & Co. charged a commission of $12\frac{1}{2}\%$ for selling them. What amount was remitted to Langley?

20. J. C. Langley in Ex. 19 paid a Missoula commission merchant 5ϕ per box for handling the shipment there. What amount should Langley consider that he received from the whole transaction?

Merchandise brokers. Agents who sell merchandise to wholesalers and jobbers are known as **merchandise brokers**. Frequently they have the sole right to distribute certain lines of merchandise throughout a special territory. The usual commission charged by such brokers varies from $1\frac{1}{2}\%$ to 2% of the amount of the order which is sent to the principal.

EXERCISES

1. The J. C. West Co., merchandise brokers of Milwaukee, Wis., sell to J. B. Harkness, of Janesville, Wis., for the Tarrant Canning Co. of Puyallup, Wash., the following order of canned goods:

NO.	SIZE	GRADE	VARIETY	PRICE PER DOZEN
50 doz.	#10	Wtr.	Blackberries	\$10.75
25 "	#10	Wtr.	Loganberries	\$15.75
25 "	#10	Wtr.	Raspberries	\$18.75
25 "	#5	Spec.	Raspberries	\$21.50
50 "	#5	Spec.	Blackberries	\$12.50

If the rate of commission is 2% , what is the amount of the charge made by West & Co. for brokerage?

2. Johnson and Mallow, merchandise brokers for the Holt Canning Co., sell the following order of canned goods, in cases (cs.) of 24 cans each, to J. B. Sparks of Des Moines, Ia. :

NO.	BRAND	DESCRIPTION	PRICE PER DOZEN
10 cs.	Paragon	White Cherries	\$2.50
10 "	Paragon	Black Raspberries	\$2.85
20 "	Paragon	Green Gage Plums	\$1.50
20 "	Indian	Bartlett Pears	\$2.00
25 "	Standard	Red Raspberries	\$11.00

At $1\frac{3}{4}\%$, what amount do the brokers receive as commission on this order ?

3. Wells and Washburn are the Wisconsin agents of the cannery mentioned in Ex. 2. They sell the following order to C. E. Watson of Oshkosh, Wis. :

NO.	BRAND	DESCRIPTION	PRICE PER DOZEN
150 cs.	Paragon	Red Raspberries	\$3.85
50 "	B.S.P.	Cherries in Juice	\$3.25
25 "	Navy	Bartlett Pears	\$3.25
30 "	Standard	Red Raspberries	\$11.00

If Wells and Washburn charge a commission of $1\frac{3}{4}\%$, what amount do they remit to the cannery for this order ?

Selling live stock. In many of the larger cities stockyards are maintained to which cattle, sheep, and hogs are shipped from country points. Commission merchants sell the shipments to packers or other concerns on a commission basis. The commissions which are charged for buying and selling live stock are generally determined at so much per head, but the rates vary according to the method in which the live stock is shipped to the stockyards, and there are also maximum and minimum charges which depend upon the size of the shipment.

When the shipments are sold, the commission merchant renders an account sales to the shipper, in some such form as the one shown on the following page.

WILLIAM WOODMAN & SONS
LIVE-STOCK COMMISSION

CHICAGO, Jan. 17, 19--

Sold for Account of Robert Thompson

P. O. Beresford, S. Dak.

Shipping Station Beresford, S. Dak.

Purchaser		Cattle	Hogs	Sheep	Weight	Price	Extensions	
Wiley & Co.		37			40 720	8 90	3624 08	
Hoffman		5			4 840	7 75	375 10	
Smith & Co.		1			1 130	5 75	64 98	
		43			46 690			4064 16
Car No.	Railroad Weight	Rate						
8093	22 500	44¢	99	—	Freight, including			
90197	"	44¢	99	—	Terminal Charges	204 26		
War Tax				6 12	Yardage	15 05	219 31	
Fire Ins.				14	Hay 400 lb.		8 --	
					Commission		38 70	266 01
					NET PROCEEDS			3798 15
PROCEEDS SENT TO First State Bank, Beresford, S. Dak.								

LIVE-STOCK ACCOUNT SALES

EXERCISES

1. Harris Bros. of Pawnee, Ill., shipped 21 head of cattle to their agents, Weber & Marston, at the Union Stock Yards, Omaha, Neb. According to the account sales rendered, the cattle were sold as follows:

SOLD TO	No.	WEIGHT	PRICE PER HUNDREDWEIGHT
Wells & Co.	19	27,340 lb.	\$8.80
Holt	2	2,410 "	\$7.00

Charges: freight, \$66.79; insurance, \$4.30; yardage, \$7.35; hay, \$7.50; commission, \$18.90. What amount was remitted to Harris Bros.?

2. A. J. Watterson, Union Stock Yards, St. Paul, Minn., sold for W. J. Harmon of Cannon Falls, Minn., 2 cars of sheep as follows:

SOLD TO	No.	WEIGHT	PRICE PER HUNDREDWEIGHT
Winslow & Co.	248	32,860 lb.	\$5.50
"	87	11,580 "	\$5.75
"	21	2,030 "	\$8.50
"	3	390 "	\$4.50

Charges: freight, \$229; yardage, \$28.80; hay, \$4; commission, \$48. What were the net proceeds of this shipment?

3. R. E. Hand of Stanwood, Ia., shipped 66 hogs to Wm. Waterhouse, Union Stock Yards, Chicago, Ill. The whole shipment, which weighed 18,670 lb., was sold to Forrest & Co. at \$9.50 per hundredweight. The charges were as follows: freight, \$60.96; yardage, \$7.92; inspection, \$0.20; corn, \$5; insurance, \$6.60; commission, \$19.80. Find the amount remitted by Waterhouse to Hand.

4. J. B. Woodward, Union Stock Yards, Chicago, sells for H. E. Barnes of Rollo, Ill., a shipment of 20 cattle weighing 24,000 lb. at \$9 per hundredweight. The charges are as follows: freight, including terminal charges, \$30; yardage, \$7; hay, \$3; commission, \$3. Find the net proceeds of this shipment.

PRODUCE EXCHANGES

Produce exchanges. In many of the large cities there are organizations, known as **produce exchanges**, where farm products such as wheat, corn, oats, pork, lard, and so on are bought and sold.

These organizations are sometimes known as boards of trade, as chambers of commerce, or by some similar designations.

The members of these organizations are commission merchants or brokers, who buy and sell the products for their principals under the rules of the exchange, which provides them a place to trade, establishes uniform customs of trading, regulates the commissions which may be charged by members, establishes standards for the products handled, and so on.

The principal produce and grain exchange in the United States is the Chicago Board of Trade. On the floors of this exchange are bought and sold a large part of the cereal and meat products of the Mississippi Valley and the West. In the case of cereals, the commission merchants who sell the grain represent the country elevator men who have bought it from the farmers, while the buyers are the millers, exporters, terminal-market elevator representatives, or other purchasers.

Such exchanges are important accessories of commerce, since they help to supply a regulated market, gather and publish important business information, and by their standards and inspection bureaus protect both buyer and seller against fraud.

Cash market. Products are bought and sold on the exchanges for present or future delivery. When grain consigned to a commission merchant is received, it is sampled and inspected, and samples are delivered to the merchant so that prospective buyers may examine them. Business done in this way, or transactions in grain about to arrive, are called **cash-market**, or **sample-market, business**.

When the grain is sold, the commission merchant renders an account sales to his principal, which shows the details and the charges in connection with the sale. Such charges include those for freight, inspection of the grain, weighing, and commission, together with interest on the money which is often advanced by the broker to the shipper before the grain is actually sold. The form on page 222 illustrates a typical account sales which shows the details of a transaction in yellow corn.

When the grain is shipped to the commission merchant, the shipper will often draw on the consignee for an amount not to exceed 75% or 80% of the estimated value of the grain. It is the interest on such a draft which is shown as part of the charges in the form on page 222. The details of drafts are considered in Chapter VIII, but may be discussed in this connection if desired.

ACCOUNT SALES

CHICAGO, Jan. 22, 19--

SAMPSON BROTHERS & CO.

COMMISSION MERCHANTS

306 S. MAIN STREET

For Account and Risk of

John Doe

Storm Lake, Ill.

Date Sold	Car No.	Weight	Bushels	Grade	Price		Amount	
1-15	113638	85120	1520	5YC		66 $\frac{1}{2}$	1010	80
Freight \$225.57 Freight tax \$6.77					232	34		
Inspection 1.00 Reinspection 1.00					2	--		
Weighing 1.00					1	--		
Interest on draft for \$600 from 1-12 to 1-22					1	--		
Commission @ 1¢ bu.					15	20	251	54
NET PROCEEDS TO YOUR CREDIT							759	26

ACCOUNT SALES

Future market. The members of the exchange also execute orders for buying and selling grain or other products for future delivery, and such business is called the **future market**.

The details of the future market are too involved for this text, but it may be informally explained that millers and elevator men make such future transactions for the purpose of protecting their purchases and sales. Speculative trading in these products is usually in futures. Thus, a speculator may think that the price of grain today is too high, and may therefore sell grain which he does not now own, expecting to buy later at a lower price the amount necessary to fulfill his contract. The prices in the future market depend upon the crop prospects, the demand, and various other factors.

When the order to buy or sell for future delivery is executed, the commission merchant renders his principal a statement which

shows the details and the charges connected with the transaction. The following form shows a typical account purchase and sale for future delivery of 10,000 bu. of corn bought at 68¢ per bushel and sold at 68½¢ per bushel:

CHICAGO, Jan. 22, 19--									
ACCOUNT PURCHASE AND SALE									
By SAMPSON BROS. & CO.									
306 S. MAIN ST.									
For Account and Risk of					John Doe				
Ledger 1					Storm Lake				
No. SK965					Ill.				
Bought	Sold	Delivery	Commodity	Price					
10 M		May	Corn	68	6800	--			
	10 M	"	"	68½				6850	--
								50	--
Tax						1	48		
Commission						25	--	26	48
TO YOUR Credit								23	52

ACCOUNT PURCHASE AND SALE

Brokers' commissions. The produce exchange generally fixes the commissions which its members may charge each other and nonmembers. The rates vary according to the product handled, the quantity handled, and whether the transaction is for cash or for future delivery. Minimum rates are also specified in the case of certain cereals.

Market quotations. The variations in the prices of the products handled are carefully recorded and posted on the floor of the

exchange. The following table shows the opening, the highest, the lowest, and the closing prices for each of several products during a day's business on the Chicago Board of Trade:

PRODUCT	OPENING	HIGH	LOW	CLOSING
Wheat — Dec.	\$1.04 $\frac{1}{2}$	\$1.08 $\frac{1}{2}$	\$1.04 $\frac{1}{2}$	\$1.08 $\frac{1}{4}$
Wheat — May	1.09	1.12 $\frac{3}{4}$	1.09	1.12 $\frac{1}{2}$
Corn — Dec.	.46 $\frac{1}{2}$	.47	.46 $\frac{1}{4}$	.46 $\frac{7}{8}$
Corn — May	.51 $\frac{5}{8}$	.52 $\frac{1}{4}$	.51 $\frac{5}{8}$	.52 $\frac{1}{8}$
Oats — Dec.	.32 $\frac{3}{4}$	.33 $\frac{3}{8}$	.32 $\frac{5}{8}$	.33 $\frac{1}{4}$
Oats — May	.37 $\frac{3}{8}$	.37 $\frac{7}{8}$	.37 $\frac{3}{8}$	.37 $\frac{3}{4}$
Rye — Dec.	.83 $\frac{1}{2}$	.85 $\frac{3}{4}$	.82 $\frac{3}{4}$	.85 $\frac{1}{4}$
Rye — May	.87 $\frac{1}{2}$	.89	.87	.89
Barley — Dec.	.56	.56 $\frac{1}{4}$	.56	.56 $\frac{1}{4}$
Pork — Jan.	15.00	15.00	15.00	15.00
Lard — Jan.	8.80	8.92 $\frac{1}{2}$	8.80	8.92 $\frac{1}{2}$
Lard — March	9.00	9.15	9.00	9.15

In the above table, "Wheat — Dec." refers to certain grades of wheat to be delivered any time during December, and similarly, for "Wheat — May," and so on. The prices given are the prices per bushel in the case of cereals, the price per barrel in the case of pork, and the price per 100 lb. in the case of lard.

EXERCISES

1. If the broker charges a commission of $\frac{1}{4}\%$ per bushel, what will it cost a purchaser to buy 5000 bu. of May wheat at the lowest price given in the above table?

2. If the broker charges a commission of \$12.50 per 5000 bu. bought and sold in one transaction, what is the net loss of a man who buys 10,000 bu. of December oats at 62¢ per bushel and sells them at the highest price shown in the above table?

3. A man bought 15,000 bu. of December corn at the lowest price and sold them at the highest price in the above table. If he paid his broker a single commission on the transaction of \$12.50 per 5000 bu., what was his net profit? his per cent of net profit on the lowest price?

4. If the broker's commission is \$20 per 50 M lb. (50,000 lb.), what will it cost to buy 750,000 lb. of March lard at the opening price given in the above table?

5. If the broker's commission is \$20 per 250 bbl., what net profit did a man make who bought 1500 bbl. of January pork at the closing price in the above table and sold it at \$23.35 per barrel?

6. A man ordered his broker to sell 5000 bu. of May corn at the highest price and to buy 5000 bu. of December corn at the lowest price in the above table. If the May corn cost $70\frac{3}{8}\text{¢}$ per bushel, the December corn was sold later at $52\frac{5}{8}\text{¢}$ per bushel, and the broker's total commission was \$25, what amount would stand to the man's debit on the broker's books?

7. A man bought 10,000 bu. each of May wheat, December corn, and December oats at the opening prices and sold them all at the closing prices given in the above table. If he had to pay \$75 in commissions, what was his net profit or loss?

8. A miller bought the following items in the cash market:

AMOUNT	ITEM	PRICE PER BUSHEL
10,000 bu.	#22 Yellow Corn	$66\frac{1}{2}\text{¢}$
5,000 bu.	#1 Oats	43¢
10,000 bu.	#3 Red Wheat	$\$1.84\frac{3}{4}$

If the commissions were figured as 1¢ per bushel for the corn, $\frac{3}{4}\text{¢}$ per bushel for the oats, and 1% on the selling price for the wheat, what was the total commission charged and the total cost of the purchase?

9. A country elevator ships 1640 bu. of oats to a commission merchant who sells them at $43\frac{3}{4}\text{¢}$ per bushel. If the merchant charges a commission of $\frac{3}{4}\text{¢}$ per bushel and the other charges amount to \$187.52, what amount is placed to the elevator's credit on the merchant's books?

For additional problems see page 419.

CHAPTER VI

SPECIAL BUSINESS EXPENSES

Business expenses. In preceding pages the different items which make up the total operating expenses of a business have been enumerated, but no attempt has been made to treat them separately. A detailed analysis of a few of these expenses will now be considered.

WAGES

Weekly-rate system. In business, and in some types of manufacturing, wages are generally paid at a specified weekly or daily rate. The amount due the employee is readily determined where payment is made each week, since deductions from or additions to the regular wages in accordance with the rules covering the particular type of employment are easily figured.

Hourly-rate system. Under the **hourly-rate system**, which is used in a large number of establishments, the employee is paid according to the number of hours spent in the factory, shop, store, or office. The time at which the employee enters in the morning, leaves at noon, returns at noon, and leaves in the evening is often recorded on cards by means of time clocks, so that an accurate record of his hours is obtained.

This time should not be confused with the time spent by the employee in working on different jobs or processes, and which is recorded on job tickets for the purpose of ascertaining the amount to be charged for productive labor, as described on page 179.

Under this system a day's work is determined by a fixed number of hours, and **overtime**, or time worked in excess of this number, is often paid for at a higher rate per hour. "Time and a half," or one and one-half times the regular hourly rate, is very frequently

paid for such excess time. "Double time," or twice the hourly rate, is sometimes paid for work on Sundays and holidays.

Pay rolls. The employees' time cards, which show the number of hours worked, are used as the basis for making up the weekly **pay roll** which shows the amount due each employee for the period.

Some establishments pay their employees only twice a month, but since weekly payment is the most common, it alone is considered here.

The following form shows a weekly pay roll of an establishment which pays wages under the hourly-rate system based upon an eight-hour day, and gives the number of hours worked by each man per day, the total time for the week, the rate per hour, and the total amount earned by each employee:

PAY ROLL											
Week ending Feb. 15, 19—											
No.	NAME	M	T	W	T	F	S	TOTAL TIME	RATE PER HOUR		AMOUNT EARNED
1	Chas. Adams	8	8	7½	8	8	8	47½	50		23 75
2	Wm. Ray	7	8	8	8	8	6½	45½	42½		19 34
3	John Cross	8	8	8	8	8	8	48	45		21 60
4	James Ross	8	6	7	8	8	8	45	47½		21 38
5	Fred Boyle	8	8	8	8	5	7	44	52½		23 10
6	Robt. Orton	8	8	8	6	8	7	45	55		24 75
7	Wm. Krohn	5	7½	8	8	8	8	44½	50		22 25
8	John Stamm	8	6½	7½	8	8	7½	45½	50		22 75
TOTALS		60	60	62	62	61	60	365			178 92

WEEKLY PAY ROLL

Sometimes pay-roll forms have a column for recording the amount, if any, which has been advanced to an employee during the week. Where such advances have been made it is necessary to record the amount due, the difference between the amount earned and the amount advanced, in a column provided for this purpose.

Overtime is often recorded on the pay roll in red figures directly above the figure for the day.

EXERCISES

1. After a pay roll is made out it is checked to see that the extensions have been properly made. Verify the pay roll on page 227, placing a check mark ($\checkmark$) against each item as it is checked.

If a day's work is 8 hr., time and a half is paid for overtime, and double time is paid for work on Sundays and holidays, make up weekly pay rolls, like the model on page 227, from the following :

2. J. C. Park : Sunday, 4 hr.; Thursday, 11 hr.; all other days, 8 hr.; rate, 50¢.

Sunday's time may be recorded in a separate column before Monday's time. The entry of Park's time for Thursday would show a red 3 (for overtime) above the 8 (for regular time).

In figuring the total time, convert the overtime and double time to hours of regular time; that is, Park's total time would be figured as 48 hr. (regular time) + 3 hr. $\times$ $1\frac{1}{2}$, or $4\frac{1}{2}$ hr. (overtime) + 4 hr. $\times$ 2, or 8 hr. (double time), or $60\frac{1}{2}$ hr. in all.

B. M. Stern : Wednesday, $4\frac{1}{2}$ hr.; all other days, 8 hr.; rate, 55¢.

Sunday time is to be considered only when specifically mentioned.

L. R. Hunt : Monday and Saturday, 4 hr.; all other days, 8 hr.; rate, $52\frac{1}{2}$ ¢.

A. L. Mack : Sunday, $4\frac{1}{2}$ hr.; Tuesday, $10\frac{1}{2}$ hr.; Wednesday, 10 hr.; rate, $57\frac{1}{2}$ ¢.

C. B. Marsh : All days, 8 hr.; rate, $52\frac{1}{2}$ ¢.

C. M. Hart : Monday, Tuesday, Wednesday, 8 hr.; left Wednesday night; rate, $52\frac{1}{2}$ ¢.

3. Wm. Grady : Sunday, 2 hr.; Monday, 10 hr.; all other days, 8 hr.; rate, $57\frac{1}{2}$ ¢.

Robt. Jones : All days, 8 hr.; rate, $47\frac{1}{2}$ ¢.

A. C. Egan : Monday, Tuesday, Wednesday, Thursday, 8 hr.; left Thursday night; rate, 50¢.

T. H. Olwell : Friday, 10 hr.; Saturday, 11 hr.; all other days, 8 hr.; rate, $52\frac{1}{2}$ ¢.

J. B. Cole : Sunday, 4 hr.; all other days, 8 hr.; rate, 55¢.

M. W. Wells : Tuesday, 4 hr.; Saturday, 10 hr.; all other days, 8 hr.; rate, $52\frac{1}{2}$ ¢.

Piece-work system. Under the **piece-work system**, employees are paid a fixed amount for each piece of work produced, so that the amount which is earned in a day depends upon the number of pieces produced or upon the number of operations performed. Different rates are generally paid for work upon different parts of an article or for the various operations connected with producing the article.

The following table shows the rates paid for several operations in the finishing department of a shoe factory which pays wages under the piece-work system :

PIECE RATES FOR HEELS			
Finishing Department		Unit, 72 pr.	
OPERATION NO.	DESCRIPTION	WOMEN'S AND MISSES' SHOES	CHILDREN'S AND INFANTS' SHOES
1	Inking heel and spring heel	\$0.10	\$0.10
2	Inking tan heel and spring heel	.20	.15
3	Inking heel breast	.05	.05
4	Inking black bottoms	.08	.08
5	Inking shanks	.08	.08
6	Inking top pieces	.05	.05
7	Heel, burnish, and wheel	.20	.16
8	Heel tan, burnish, and wheel	.24	.20

In this department a case of 72 pr. of shoes is considered as the **piece**, or **unit**.

Thus, 10¢ is paid for inking the heels or spring heels on 72 pr. of shoes. If only 36 pr. were inked, the amount paid would be 5¢, and so on. The wages of a worker who inked 31 cases during a day would be \$3.10.

Often some employees of a factory work under the piece-work system, while others work under the hourly-rate system where they are engaged on work which cannot be satisfactorily paid for on a piece-work basis.

Piece-rate pay rolls. Under the piece-work system the weekly pay roll is made up from records which show the number of pieces produced per day instead of from time cards. The following

Paying employees. While some concerns pay their employees by means of checks, the **envelope system** in which cash payments are made is generally found to be more convenient and satisfactory.

In using this system, the bookkeeper or clerk who places the money in the envelopes must first determine just what bills and coins are needed to make the correct payment to each employee. This may be easily found by making out a **change memorandum** from the "Amount Earned" column of the pay roll. The following change memorandum shows the denominations which are needed for the model pay roll on page 227:

CHANGE MEMORANDUM										
NO.	BILLS					COINS				
	\$20	\$10	\$5	\$2	\$1	50¢	25¢	10¢	5¢	1¢
1	/			/	/	/	/			
2		/	/	2			/		/	4
3	/				/	/		/		
4	/				/		/	/		3
5	/			/	/			/		
6	/			2		/	/			
7	/			/			/			
8	/			/		/	/			
Totals	7	/	/	8	4	4	6	3	/	7

CHANGE MEMORANDUM

Just as in making change, the largest denominations which meet the requirements should be used.

Since most concerns do not keep on hand enough cash to meet a large pay roll, a **pay-roll memorandum**, which shows the quantities of the different denominations required, is made up and sent to the bank with which the concern does business. The pay-roll memorandum on page 232 is made up from the change memorandum above. The total of this memorandum should, of course, agree

with the total of the pay roll. The bank furnishes the desired currency and charges the amount against the manufacturer's account.

FIRST UNION BANK Madison, Wisconsin PAY-ROLL MEMORANDUM MARTIN & CO. Require the following:			
	Number	Amount	
Pennies	7		07
Nickels	1		05
Dimes	3		30
Quarters	6	1	50
Halves	4	2	--
Dollars	4	4	--
2's	8	16	--
5's	1	5	--
10's	1	10	--
20's	7	140	--
		178	92

PAY-ROLL MEMORANDUM

In some of the larger banks the manufacturer sends a copy of his pay roll to the bank. The bank figures the change required, makes up the envelopes, and sends the money to the factory by bank messengers so as to insure its safety. Such service is generally rendered only to large concerns.

EXERCISES

1. Make out a change memorandum for each of the pay rolls in Exs. 2 and 3 on page 228, the piece-rate pay roll on page 230, and the pay roll in Ex. 2, page 230.

2. Make out a pay-roll memorandum from each of the change memorandums made in Ex. 1. In each case compare the total of the pay-roll memorandum with the total of the pay roll.

FIRE INSURANCE

Nature of insurance. If a merchant's store burned to the ground, he would suffer a serious loss — not only in the actual destruction of valuable property, but in the opportunity to continue his business. Unless he had saved sufficient money, the cost of rebuilding and of restocking his store might prove to be more than he could meet. If, however, in return for a small payment made each year the merchant had had an agreement with a company whereby he would be paid for any loss occasioned by fire, he would have been able to rebuild and to continue his business. That is, he would have been protected against the risk of fire, and obviously the cost of such protection is properly an expense of doing business.

Protection of this sort is called **insurance**, and insurance companies are organized to render such service in return for payments made to them. When many people who live in widely scattered parts of the country enter into such agreements with a company, the chance of loss occurring to all of them—or even to very many of them—is exceedingly small, so that a large amount of protection can be given to each individual for a comparatively small sum.

There are many different types of property insurance, and among them are the following: fire, marine, tornado, automobile, steam-boiler, crop, live-stock, burglary, and plate-glass. The general principle in all these types is the same as that of insurance against fire, the details of which will now be considered.

Fire insurance. Insurance against loss of property or damage to it by fire is called **fire insurance**. Such protection also covers any damage incidental to a fire, such as the damage done by water used in putting out the fire or other damage caused by the firemen's efforts to subdue the fire, and frequently covers loss caused by lightning.

The **insurer**, who is also called the **underwriter**, is the one who agrees to pay the loss in case of fire, and is generally a company organized to transact such business. The **insured** is the person

with whom the agreement is made and to whom payment for the loss is to be made. In consideration of the protection given by the company, the insured agrees to pay a fixed sum which is called the **premium**. All the details covering the conditions under which the property is insured,—the amount that will be paid in case of total loss, the methods of adjusting and paying claims for partial losses, the details in connection with the payment of the premium, and so on,—are all included in a written agreement or contract, which is called the **policy**.

The above terms apply to any type of insurance.

The amount of insurance carried is seldom equal to the full value of the property, since the owner often feels that it is unnecessary to pay for full protection where the chance of loss is small; that is, he is willing to assume some of the risk himself. Furthermore, insurance companies are sometimes unwilling to write, or issue, policies of more than a specified amount on some classes of property. The amount of insurance carried by business establishments is usually from 50% to 90% of the value of the property.

When a loss occurs the company pays the insured only for the actual value of the loss sustained, and in no case is the company liable for more than the sum for which the policy is issued.

Thus, if damage estimated at \$400 is caused by fire in a building insured for \$3000, only \$400 is paid. If fire caused a loss of \$3500 in this building, the company would be liable for only \$3000, the amount for which the policy was issued.

When a large amount of insurance is carried, it is frequently divided among several companies. In such cases each company pays only its proper fractional share of the loss. This fraction is determined for each company by dividing the amount of its policy by the total amount of insurance on the property. The amount which each company pays is then this fractional part of the total loss, provided this loss is not greater than the amount of its insurance on the property.

ORAL EXERCISES

1. A man insures a building worth \$64,000 for $\frac{5}{8}$ of its value. What is the amount of insurance carried?

2. A merchant decides that he should carry insurance equal to $\frac{2}{3}$ of the value of his property. If his store is valued at \$25,000 and his stock at \$14,000, how much insurance should he carry?

3. On account of the extra risk of fire, insurance companies sometimes refuse to issue a policy of more than \$3000 on an ice house. If an ice house is valued at \$9000, for what per cent of its value can it be insured?

4. A farmer insured his barn valued at \$6000 for $\frac{3}{4}$ of its value. If the barn burned to the ground and nothing was saved, what amount would the insurance company pay him?

5. A factory valued at \$30,000 was insured for 80% of its value. A fire in one of the departments caused a loss of \$4000. The loss was what per cent of the amount of insurance?

6. The insurance on property valued at \$100,000 and insured for 80% of its value was divided equally among four insurance companies. How much would each company pay if a fire caused a loss of \$20,000?

Coinsurance. Sometimes in consideration of paying a lower premium, the owner of the property agrees to keep it insured for a certain per cent of its value. If he fails to keep his insurance with the company up to the amount agreed upon, then in case of loss the company is liable for only a share of the loss. This share is determined as that part of the loss which the amount of insurance actually carried is of the amount agreed upon.

Such an arrangement is called **coinsurance**, and the clause in the policy which defines the agreement is called the **coinsurance clause**. One of the rates most frequently used is 80%, and for this reason such a clause is often referred to as the "80-per-cent clause." Any per cent, however, may be agreed upon between the insured and the company.

Such a clause does not, of course, apply when the insured keeps his insurance up to the amount agreed upon. Neither does it apply in case of a total loss of the property, or of a loss which equals the amount of insurance agreed upon, since in no case can the company be liable for more than the value of the policy which is actually carried.

Thus, under an 80-per-cent coinsurance clause, property valued at \$10,000 should be kept insured for $\$10,000 \times 0.80$, or \$8000. If the property is so insured and fire causes a loss of \$5000, then the owner could collect \$5000. If, however, insurance for only \$6000 was carried, then in case of any loss less than \$6000 the company would be liable for only $\$6000 \div \8000 , or $\frac{3}{4}$ of the loss. Hence, if the loss were \$5000, the company would be liable for only $\$5000 \times \frac{3}{4}$, or \$3750, the insured thus losing \$1250 because the conditions of the coinsurance clause were not observed.

EXERCISES

In each of the following assume that an 80-per-cent coinsurance clause is part of the policy, and find the amount of insurance collectible:

	VALUE OF THE PROPERTY	INSURANCE CARRIED	FIRE LOSS
1.	\$10,000	\$4,000	\$2,000
2.	\$10,000	\$6,000	\$5,000
3.	\$5,000	\$4,000	\$2,000
4.	\$10,000	\$5,000	\$4,500
5.	\$20,000	\$12,000	\$8,000
6.	\$25,000	\$18,000	\$6,500
7.	\$12,000	\$9,000	Total
8.	\$20,000	\$12,000	\$16,000

Rates. Fire-insurance companies quote their rates for insurance as so many cents (or dollars and cents) for each \$100 of insurance for the period of 1 yr.

The annual premium is then the product of this rate and the number of hundred dollars' worth of insurance which is issued.

Thus, if a policy for \$8000 is issued on a building for which the rate is 48¢, then the premium for the year is $\$0.48 \times 80$, or \$38.40. That is, to find the premium, point off two decimal places (divide by 100) in the amount of the insurance and multiply the rate by this result.

ORAL EXERCISES

State the annual premium for each of the following policies :

AMOUNT OF INSURANCE	RATE	AMOUNT OF INSURANCE	RATE
1. \$10,000	32¢	5. \$12,000	50¢
2. \$5,000	60¢	6. \$6,000	20¢
3. \$7,000	42¢	7. \$4,500	30¢
4. \$20,000	85¢	8. \$8,000	40¢

Determining the rates. The rates for fire insurance vary according to the nature of the property insured and the probable risk of fire. In many states the activities of insurance companies doing business within the state are supervised by a state insurance department. A licensed **inspection bureau** makes surveys of the different cities and towns to determine just rates for insuring different classes of property in each locality in accordance with the local conditions.

Thus a building may be occupied as a store where the risk of fire is small; another building may be occupied as a garage in which gasoline is stored. The first building may be in a town where the local fire department is inefficient, and the second in a city with an excellent fire department. It is obvious that under such varying conditions a single rate for insurance would be impractical. Again, a building may be constructed of brick or stone with a metal or other noncombustible roof, while another building is constructed entirely of wood (frame construction) with a roof of wooden shingles. In certain cases automatic sprinkler systems are installed within buildings, thus reducing the risk of serious loss. It is such factors as these that the inspection bureau takes into consideration in determining the rates for fire insurance.

The rates established by the inspection bureau are published in schedules of **basic rates** which show the different rates to be used in insuring various types of dwelling houses and private garages according to their construction. The following tables are made up from such a schedule prepared for a city of 40,000 population, and show the rates for such buildings when they are located within the city limits and situated within 1000 ft. of a public hydrant and within 2 mi. of a public fire-department station.

TABLE I. PRIVATE DWELLINGS (ONE FAMILY ONLY)

ITEM	CONSTRUCTION	ANNUAL RATE
1	Brick, metal roof	\$0.14
2	Brick, shingle roof	.18
3	Frame, metal roof	.18
4	Frame, shingle roof	.22

TABLE II. DOUBLE DWELLINGS (TWO FAMILIES ONLY)

ITEM	CONSTRUCTION	ANNUAL RATE
1	Brick, metal roof	\$0.16
2	Brick, shingle roof	.20
3	Frame, metal roof	.20
4	Frame, shingle roof	.24

TABLE III. PRIVATE BOARDING AND ROOMING HOUSES

ITEM	CONSTRUCTION	ANNUAL RATE 1	ANNUAL RATE 2
1	Brick, metal roof	\$0.26	\$0.40
2	Brick, shingle roof	.30	.44
3	Frame, metal roof	.34	.48
4	Frame, shingle roof	.38	.52

Where there are from 6 to 10 boarders or where from 6 to 10 rooms are available for lodgers, rate 1 applies; where there are from 11 to 20 boarders or where from 11 to 20 rooms are available for lodgers, rate 2 applies. Buildings where not over 5 rooms are occupied by boarders or lodgers take the basic rates for private dwellings (Table I). Only those rooms which are occupied by lodgers as sleeping rooms are considered; sleeping rooms occupied by the proprietor's family are not considered in determining the classification.

TABLE IV. PRIVATE GARAGES

ITEM	CONSTRUCTION	ANNUAL RATE
1	Brick, metal roof	\$0.22
2	Brick, shingle roof	.26
3	Frame, metal roof	.32
4	Frame, shingle roof	.36

These rates apply where electric-driven, steam-driven, or gasoline-driven machines are kept in the garages, and not more than 3 machines, other than electric-driven, are so kept.

For certain classes of property no schedules of basic rates are published. When insurance on such property is desired a special rate is determined after an inspection of the conditions. The rates for insuring mercantile and manufacturing properties are determined in this way.

Rates for mercantile and manufacturing properties depend, as with dwelling houses, upon the construction of the property and the fire protection available. The use which is made of the property is also considered in determining the rates, since certain types of business or manufacturing involve greater risk of fire than others.

Term rates. Insurance policies are often made for a longer period than 1 yr. Where insurance is carried in this way, lower rates are given, as shown in the following table:

FOR	RATE
2 yr.	$1\frac{3}{4}$ times the annual rate
3 yr.	$2\frac{1}{2}$ times the annual rate
4 yr.	$3\frac{1}{4}$ times the annual rate
5 yr.	4 times the annual rate

Agent's commission. The insurance agent who writes the policy and collects the premium receives a commission for his services in representing and obtaining the business for the insurance company. Such commissions vary from 15% to 25% of the premiums collected.

The following example illustrates the method of finding the premium for a fire-insurance policy on a building located in the city to which the tables on page 238 apply.

EXAMPLE. A private dwelling of frame construction with a metal roof and valued at \$8750 is to be insured for 60% of its value. If the policy is written for a period of 2 yr., find the premium and the agent's commission at 25%.

Solution. Table I, p. 238, gives the rate for such cases as 18¢. The amount of insurance is to be $\$8750 \times 0.60$, or \$5250.

Hence the premium for 1 yr. is $52.5 \times \$0.18$, or \$9.45; the premium for 2 yr. is $\$9.45 \times 1\frac{3}{4}$, or \$16.54.

The agent's commission is 25% of the premium collected and hence is $\$16.54 \times 0.25$, or \$4.14.

EXERCISES

Find the premium for fire-insurance policies on each of the following buildings:

VALUE OF THE BUILDING	PER CENT INSURED	TYPE OF BUILDING Table No.	Item	TERM OF POLICY
1. \$6000	70	II	2	1 yr.

The type of building refers to the tables on page 238; thus, in this case, the building is a double dwelling of brick construction with a shingle roof, the annual rate on which is 20¢.

2. \$7500	65	I	1	3 yr.
3. \$8400	60	III	4	5 yr.

In this case there are 7 rooms for lodgers.

4. \$800	80	IV	3	3 yr.
5. \$3500	65	I	4	3 yr.
6. \$9000	50	III	1	2 yr.

In this case there are 16 boarders.

7. \$5250	75	II	3	5 yr.
8. \$12,750	70	III	3	3 yr.

In this case there are 12 rooms for lodgers.

9. \$500	80	IV	1	2 yr.
10. \$11,500	65	I	1	2 yr.

11. If all the policies in Exs. 1-10 were written by one agent, what would be his total commission if he received 15% for insuring garages and 25% for other types of property?

12. Mrs. Smith owns a frame house with metal roof in which 15 rooms are rented to students. If the property is valued at \$11,000 and she desires to insure it for 3 yr. at 70% of its value, find the premium. Also find the agent's commission at 25%, and the amount which the agent remits to the insurance company.

13. J. B. Hayes, who owns a store valued at \$25,000, desires to insure it for 1 yr. at 70% of its value. After a survey the inspection bureau fixes the rate at \$1.60. Find the premium and the agent's commission at 15%.

If an 80% coinsurance clause applies to each policy on the following properties, find the amount paid by the insurance company for each loss and the amount of loss borne by the insured :

	VALUE OF PROPERTY	AMOUNT OF INSURANCE	FIRE LOSS
14.	\$10,000	\$7,000	\$3,600
15.	\$20,000	\$16,000	\$5,000
16.	\$20,000	\$16,000	\$16,000
17.	\$20,000	\$12,000	\$16,000
18.	\$20,000	\$12,000	\$5,000
19.	\$18,575	\$10,000	\$11,000
20.	\$18,575	\$10,000	\$5,500

21. A brick building occupied as a store is worth \$20,000, the contents of the first floor are worth \$27,000, and those of the second and third floor together are worth \$40,000. The building and its contents are insured at 80% of their value for a period of 2 yr. under an 80% coinsurance clause. If the rate for the whole term is 70¢, find the premium paid. If a fire which causes damage of \$25,000 occurs, what amount can the insured collect?

22. Suppose that the building in Ex. 21 was insured by Company A for \$16,000 at a 2-year rate of 50¢, and that the contents were insured by Company B for \$50,000 at a 2-year rate of 75¢, with the 80% clause as part of each policy. If fire damaged the building to the extent of \$3000 and the contents to the extent of \$2500, how much would each company have to pay? Including the premium paid, what would be the net loss to the owner of the building? to the owner of the contents?

23. A church is worth \$30,000 and its contents are worth \$3500. The whole property is insured for \$8100 for a period of 1 yr. at a rate of 85¢ and the agent receives a commission of 15%. Deducting the amount of the premium received by the

insurance company, what is the net loss to the company if the property is totally destroyed by fire?

24. A brick school building is worth \$35,000 and its contents are worth \$7500. If the annual rate is 60¢, what will it cost under term rates to insure the property for 80% of its value for a period of 5 yr.?

25. A building occupied as a store is worth \$35,000 and is insured for 80% of its value, the insurance being divided equally among five companies. If the rate is 40¢, what is the total annual premium? If fire causes a damage of \$6000, what amount does each company pay?

Short rates. When policies are written for terms of less than 1 yr., different per cents of the annual rate are used according to the length of the term. These per cents are published in tables called **short-rate tables**, which show the per cent of the annual rate to be charged for any period of days.

For example, a merchant may desire to insure certain goods worth \$10,000 for a period of 75 da. If the annual rate is 50¢, and the short-rate table shows that 37% of this rate is to be charged for 75 da., then the premium is $\$0.50 \times 0.37 \times 100.$, or \$18.50.

EXERCISES

Find the premium for each of the following short-term policies:

	AMOUNT OF INSURANCE	ANNUAL RATE	SHORT-RATE PER CENT		AMOUNT OF INSURANCE	ANNUAL RATE	SHORT-RATE PER CENT
1.	\$2500	32¢	28.5	6.	\$6000	48¢	21
2.	\$2700	28¢	60.	7.	\$6400	50¢	51.67
3.	\$3500	40¢	77.5	8.	\$6800	44¢	90.
4.	\$3000	54¢	25.95	9.	\$7500	70¢	10.
5.	\$4200	36¢	37.	10.	\$3000	85¢	73.34

In the above exercises the periods for which the insurance is written are as follows: Ex. 1, 50 da.; Ex. 2, 150 da.; Ex. 3, 225 da.; Ex. 4, 40 da.; Ex. 5, 75 da.; Ex. 6, 1 da.; Ex. 7, 125 da.; Ex. 8, 300 da.; Ex. 9, 10 da.; Ex. 10, 200 da.

Cancellation of a policy by the insured. A policy for fire insurance may always be canceled at any time by the insured. When a policy is canceled, the company returns to the insured the unearned portion of the premium.

The earned part of the premium which the company retains is determined by the use of tables. The following table shows the per cents of the premium which are retained by the company upon cancellation of a policy by the insured:

TIME IN MONTHS	TWO-YEAR POLICY	THREE-YEAR POLICY	FOUR-YEAR POLICY	FIVE-YEAR POLICY
1	11%	8%	6%	5%
2	17	12	9	8
3	23	16	12	10
4	29	20	15	13
5	34	24	18	15
6	40	28	22	18
7	43	30	23	19
8	46	32	25	20
9	49	34	26	21
10	51	36	28	23
11	54	38	29	24
12	57	40	31	25
18	79	55	42	34
24	100	70	54	44
30	—	85	65	53
36	—	100	77	63
42	—	—	88	72
48	—	—	100	81
54	—	—	—	91
60	—	—	—	100

When a policy has been in force for a period between two periods given in the table, the higher per cent is used in determining the earned portion of the premium.

When one-year policies are canceled, short-rate tables, as described on page 242, are used in determining the earned portion of the premium.

If a policy is canceled by the insured and a portion is rewritten, the amount not rewritten is canceled at short rates.

EXAMPLE. A man who has a 3-year policy on which he paid a premium of \$45 desires to cancel the policy 6 mo. after it was written, as he is moving to another city. What amount of the premium is retained by the company and what amount is returned to the insured?

Solution. The above table shows that when a 3-year policy is canceled after being in force 6 mo., 28% of the premium is considered as earned.

Hence the amount retained by the company is $\$45 \times 0.28$, or \$12.60.

The amount returned to the policyholder is $\$45 - \12.60 , or \$32.40.

Cancellation of a policy by the company. Upon due notice a policy may also be canceled by the insurance company. In such a case the earned portion of the premium is determined *pro rata*; that is, the earned premium is that part of the total premium which the term that the policy has been in force is of the whole term of the policy.

EXAMPLE. A 1-year policy for which a premium of \$30 was paid is canceled by the insurance company 73 da. after it was issued. What amount of the premium is retained by the company and what amount is returned to the insured?

Solution. In this case the expired term, the term which the policy has been in force, is 73 da. The term for which the policy was written is 1 yr., or 365 da.

Hence $\frac{73}{365}$, or $\frac{1}{5}$, of the premium has been earned, and the amount retained by the company is $\$30 \times \frac{1}{5}$, or \$6.

The amount returned to the insured is $\$30 - \6 , or \$24.

EXERCISES

1. What amount of a premium of \$172.50 has been earned on a 5-year policy which is canceled by the company after it has been in force for 42 mo.?

2. What amount is refunded to the holder of a 2-year policy on which a premium of \$65 was paid if it is canceled by the insured 2 mo. after it is issued?

3. What amount is returned to the insured under a 3-year policy on which he has paid a premium of \$210 if the insured cancels the policy after it has been in force for 9 mo. ?

In each of the following canceled policies find the amount of the premium retained by the company and the amount returned to the insured :

	PREMIUM	TERM OF POLICY	EXPIRED TERM	CANCELED BY
4.	\$28	2 yr.	10 mo.	Insured
5.	\$35	1 yr.	100 da.	Company
6.	\$42	3 yr.	11 mo.	Insured
7.	\$84	1 yr.	273 da.	Company
8.	\$120	5 yr.	18 mo.	Insured
9.	\$160	4 yr.	7 mo.	Insured
10.	\$136	1 yr.	187 da.	Company
11.	\$154	2 yr.	5 mo.	Insured
12.	\$178	3 yr.	10 mo.	Insured

Automobile insurance. There are so many different types of automobile insurance, and the conditions and rates under which this sort of insurance is issued vary so greatly in different parts of the country, that detailed reference to this subject is not practical in a student's textbook on business arithmetic.

The instructor may secure excellent class material on this topic through local insurance agents, who can furnish information as to the local rates and conditions which govern this type of insurance.

DEPRECIATION

Nature of depreciation. Fixtures and other equipment used in a store, or machines used in manufacturing, are constantly wearing out and growing less in value, until eventually they have to be replaced with new ones. This lessening of value due to wearing out or to becoming obsolete is called **depreciation**.

In business the term *depreciation* is commonly applied to an allowance made in order to distribute the loss caused by this lessened value over the different years during which the property is usable. If such an allowance were not made, the whole loss due to replacing the property would come in its last year, which is obviously unfair, since each year should bear its share of the decrease in value.

For example, if an automobile which costs \$1800 is worth only \$360 after 4 yr., the loss due to depreciation for that period is $\$1800 - \360 , or \$1440. If depreciation is not recognized each year during which the machine is in use, the entire loss of \$1440 would be borne in the year in which the automobile is replaced by a new one.

Determining depreciation. The three factors which enter into determining the rate at which property depreciates are the **original cost**; the **estimated life of the property**, the probable number of years during which it will be usable; and the **scrap value**, the final value when it can no longer be used in the business. The original cost of the property is the only factor definitely known; the other factors are estimates based upon experience.

The total loss due to depreciation is the difference between the original cost and the scrap value. One method of offsetting this loss is to charge each year as part of the operating expenses the share of the loss determined by distributing the total loss equally over the life of the property.

Thus, on the automobile mentioned above the total loss due to depreciation is \$1440, and hence the annual loss is $\$1440 \div 4$, or \$360. This amount would be charged to "Depreciation" annually as an operating expense, so that each year bears an equal share of the loss in value.

Although there are several other methods of determining depreciation, this method is the one that is used in ordinary cases.

The annual depreciation is generally expressed as a per cent of the original cost, and when expressed in this way is called the **rate of depreciation**.

Thus the rate of depreciation of the automobile mentioned above is $\$360 \div \1800 , or 20%. The automobile may be said to depreciate in value 20% a year.

EXERCISES

Find the annual depreciation charge for each of the following:

TYPE OF PROPERTY	ORIGINAL COST	RATE OF DEPRECIATION
1. Tools	\$5000	10%
2. Automobile	\$2500	20%
3. Factory machinery	\$20,000	12½%
4. Power plant	\$15,000	20%
5. Office building	\$12,500	3%

Find the rate of depreciation for each of the following:

TYPE OF PROPERTY	ORIGINAL COST	ESTIMATED LIFE	ESTIMATED SCRAP VALUE
6. Machine	\$2000	10 yr.	\$400
7. Automobile	\$1000	4 yr.	\$200
8. Factory building	\$50,000	50 yr.	\$1000
9. Residence	\$10,000	30 yr.	\$100
10. Office furniture	\$2000	10 yr.	\$200

11. A machine which cost \$1200 is estimated to be worth \$300 at the end of 10 yr. What is the rate of depreciation and the amount of the annual depreciation charge?

12. The furniture and fixtures in the dry-goods store of J. T. Gibson cost \$4000. He believes that he can use them for 10 yr. and then sell them for \$200. What is the rate of depreciation and the amount which should be charged monthly on this account?

13. A merchant estimates that the different items of his store property depreciate as follows:

TYPE OF PROPERTY	ORIGINAL COST	RATE OF DEPRECIATION
Store building	\$20,000	4%
Store furniture	\$6350	10%
Delivery trucks	\$2875	30%
Office equipment	\$2420	12½%

What is the total annual depreciation charge?

14. In Ex. 13 find the average annual rate of depreciation.

15. The annual depreciation rates frequently used for several types of property are as follows:

Machinery	10%	Harness	20%
Patterns	25%	Wagons	16%
Small tools	25%	Sleighs	10%
Drawings	30%	Automobiles	20%
Horses	16%	Trucks	25%

Using these rates, determine the total annual depreciation charge on the following list of manufacturing property:

TYPE OF PROPERTY	ORIGINAL COST	TYPE OF PROPERTY	ORIGINAL COST
Machinery	\$28,575	Drawings	\$2730
Small tools	\$3840	3 trucks	\$1800, each
Patterns	\$4325	2 wagons	\$175, each

16. Using the rates in Ex. 15, determine the total annual depreciation charge on the following farm property of which the original cost is as shown:

2 horses	@	\$250	3 sets harness	@	\$50
2 horses		\$300	1 set harness		\$85
1 wagon		\$125	1 automobile		\$2000
2 wagons		\$115	1 truck		\$1750

17. A grocer bought a delivery truck for \$1500 and used it for 3 yr. If he charged depreciation each year at 30%, what was the value of the truck on the grocer's books at the end of the third year? If he then sold the truck for \$300, what amount of gross profit or loss did he make on the sale?

18. On Jan. 1 a manufacturer had on hand equipment valued at \$85,000. During the year the following purchases of additional equipment were made: Mar. 1, \$3750; June 1, \$8240; Aug. 1, \$12,375; Dec. 1, \$18,725. If the manufacturer charged depreciation at 12%, what was the total charge for the year?

The new equipment should be charged a partial rate depending upon the month of purchase.

CHAPTER VII

THE CONDITION OF THE BUSINESS

Condition of the business. While a merchant may follow the amount of his sales by means of sales summaries, such as those described on pages 205-211, he cannot arrive at an exact conclusion as to the amount of his profit or loss for a period unless he is able to compare the sales with a record of the expenses and of the cost of goods sold. Accordingly, at regular intervals statements are prepared which set forth the facts in connection with the operations and financial standing of a business; that is, they reveal **the condition of the business.**

The methods explained in this chapter apply to any type of business, but the case of the retail store has been taken for purposes of illustration.

INVENTORIES

Taking inventories. The first step in the preparation of statements setting forth the condition of the business is the taking of **inventories**, which are itemized lists of the quantity and value of unsold or unused goods, in order to determine the total value of the goods on hand. Inventories are usually taken at stated periods, — such as the end of a year or of a quarter, — although they may be taken at any time when it is necessary to determine the value of the unsold stock.

The modern way of recording inventories is upon loose-leaf sheets of uniform size and ruling, which can be bound together and filed away after the desired facts have been obtained.

By using loose-leaf forms the inventories of different departments can be taken together, since a large number of persons can work on them at the same time. One person counts and calls off the quantity, description, and, in some cases, the unit cost price of each kind of merchandise on hand,

and a second person records them on the forms. As the sheets are filled they are sent to the office where the price extensions are made. All the sheets are then assembled by departments, numbered consecutively, and placed in

INVENTORY <u>Dec. 31, 19__</u> Page <u>19</u>							
Sheet No. <u>2</u>		Department <u>F. Hosiery</u>		Priced by <u>E. L. S.</u>			
Called by <u>J. C. W.</u>		Location <u>A 14</u>		Extended by <u>J. L. S.</u>			
Entered by <u>F. R. W.</u>		Examined by <u>M. W. W.</u>					
Stock or List No.	DESCRIPTION	Unit	Quantity	Price	Extension	Total	
784	Brown Mixture	doz.	1/6	54 -	9 -		
384	Assorted	"	1/4	22 -	5 50		
461	Black	"	6	6 -	36 -		
473	Cordovan	"	3	11 25	33 75		
424	Dr. Gray	"	3	9 -	27 -		
432	Black	"	1	9 75	9 75		
657	Black & White	"	1/6	14 -	2 33		
286	Cordovan	"	2	5 -	10 -		
288	Navy	"	1/2	8 -	2 50		
439	Brown	"	1/3	9 75	3 25		
672	Dr. Gray	"	1/2	15 50	23 25		
476	Navy	"	1 2/3	11 25	18 75		
651	Black	"	2	14 -	28 -		
479	Black	"	2	11 25	22 50		
						231 58	

INVENTORY AT COST PRICES

a binder. The inventory is totaled and all prices, extensions, and totals are carefully checked to avoid errors.

Spaces are provided on the forms for inserting the names or initials of each person who helped to prepare the sheet, so that in case a question arises about any item on the sheet it can be referred to the proper person without delay. The above form shows a sheet from an inventory completed in the manner just described.

EXERCISE

Rule and prepare a form similar to the one shown on page 250. Let one student call off the items in the following list while another student records them. The extensions and total may then be made by the first student, and the work checked by the second, each student entering his initials opposite the proper spaces on the form.

Assume that this is sheet No. 1, and that the goods have been priced by E. R. N.

DEPARTMENT C, BATHING SUITS

STOCK No.	DESCRIPTION	UNIT	QUANTITY	PRICE
428	Black & White	Doz.	$\frac{8}{12}$	\$60.00
1342	Sterling & White	"	$\frac{7}{12}$	51.00
342	Navy	"	1	12.50
876	White	"	1	33.00
947	Black	"	$\frac{8}{12}$	26.00
863	White	"	$\frac{6}{12}$	33.00
1016	Navy	"	$\frac{6}{12}$	42.00
1048	Taupe & Sterling	"	$\frac{4}{12}$	45.00
1044	Black & White	"	$\frac{5}{12}$	45.50
1045	Navy & White	"	$\frac{8}{12}$	45.50
1128	Black & White	"	$\frac{4}{12}$	57.00
1291	Taupe & Kelly	"	$\frac{3}{12}$	60.00

Inventory at cost and selling prices. Sometimes the inventory shows both the cost prices and the selling prices of the goods, so that the gross profit which should be made from the sale of the goods is obtainable.

In some department stores the inventory is recorded on the forms at the selling prices only, and the cost prices are inserted in the office when the extensions and totals are made. The first form on page 252 shows an inventory sheet prepared in this manner.

Inventory at cost and market prices. It is sometimes desirable that in addition to the cost price the inventory shall show the **market prices** of the goods; that is, the present prices which must be paid if more goods are purchased. Often these two prices differ, and in determining the total of the inventory the lower of the two

INVENTORY								
Dept. <u>Office Supplies</u>						Sheet No. <u>10</u>		
Section <u>7</u>								
Lot No.	ARTICLE	Quantity	Unit	Cost Price	Cost Extension	Selling Price	Selling Extension	
72	T.C. Yellow Paper	28	reams	1 21	33 88	1 50	42	—
160	Typewriter Paper	67	"	1 10	73 70	1 35	90	45
247	S. Blue Black Ink	5 1/2	doz.	98	5 47	1 80	10	05
253	S. Violet Ink	3 7/2	"	1 30	4 77	2 40	8	80
172	S. Green Ink	1 1/2	"	1 30	1 95	2 40	3	60
18	Blue Black, Tints	1	"	6 25	6 25	10 80	10	80
20	Blue Black, 1/2 Tints	2	"	3 75	7 50	7 20	14	40

INVENTORY AT COST AND SELLING PRICES

prices is taken. The following form shows a type of inventory sheet for recording these prices, and for extending the lower amount:

INVENTORY									
Department <u>Shirts</u>							Page <u>5</u>		
Called by <u>L.S.</u> Ent'd by <u>B.N.K.</u> Ext'd by <u>G.S.L.</u> Exam'd by <u>B.R.T.</u>									
Stock or List No.	DESCRIPTION	Unit	Quantity	Cost		Market		Lower	
				Unit Price	Amount	Unit Price	Amount		
766	Men's Shirts	doz.	8 1/2	24 —	16 —	27 —	18 —	16	—
821	" "	"	1 1/2	33 —	49 50	30 —	45 —	45	—
932	" "	"	7 9/2	36 —	279 —	36 —	279 —	279	—
987	" "	"	1/2	45 —	22 50	42 —	21 —	21	—

INVENTORY AT COST AND MARKET PRICES

For additional problems see page 421.

EXERCISE

Rule and prepare a form similar to the second one on page 252. Let one student call off the following items while another student records them. The first student may then extend the amounts, carry the lower amounts to the proper column, and total the inventory, and the work may be checked by the second, each student entering his initials in the proper spaces:

DEPARTMENT Q

STOCK No.	DESCRIPTION	UNIT	QUANTITY	COST PRICE	MARKET PRICE
804	Shirts	doz.	$1\frac{8}{12}$	\$42	\$36
917	Shirts	"	5	39	36
894	Shirts	"	3	39	39
671	Shirts	"	$1\frac{9}{12}$	27	24
548	Shirts	"	$\frac{9}{12}$	33	30
976	Shirts	"	$\frac{4}{12}$	27	24

BUSINESS STATEMENTS

Profit-and-loss statement. After the inventory has been taken, the merchant is then ready to begin the preparation of the statements which set forth the condition of the business. The first statement which is prepared is the **profit-and-loss, or operating, statement.** This form shows in detail the gross profit (or loss) during the period, the operating expenses, and the resulting net profit (or loss). In addition to the total value of the goods on hand, which is shown by the inventory, other totals must be determined from the merchant's books wherein are recorded the details of all the transactions of the period.

For example, suppose that John Martin, who runs a dry-goods store, finds from his books that his total sales for the year were \$43,424.64, that his inventory of the preceding year showed that he had goods on hand at the beginning of the year to the value of \$8564.26, and that during the year he purchased new merchandise to the amount of \$27,823.82. His inventory just taken on Dec. 31 shows that the goods on hand are now valued at \$7438.82. He further finds that during the year he spent the following amounts for operating expenses: Rent, \$1200; Salaries, \$5508.75;

Advertising, \$1302.68; Insurance, \$285.40; Heat and Light, \$379.20; Depreciation on Store Equipment, \$327.50; Property Taxes, \$318.37; Office Supplies, \$165.72; and Sundry General Expense, \$638.49.

The above items are then arranged in a special order as shown in the following profit-and-loss statement:

John Martin
Profit-and-Loss Statement
For the year ended December 31, 19--

Merchandise Sales		43424 64
Inventory, January 1	8564 26	
Purchases for the year	27823 82	
	<u>36388 08</u>	
Inventory, December 31	7438 82	
Cost of merchandise sold		28949 26
Gross profit on sales		<u>14475 38</u>
Operating Expenses		
Rent Paid	1200 —	
Salaries	5508 75	
Advertising	1302 68	
Insurance	285 40	
Heat and Light	379 20	
Depreciation on Store Equipment	327 50	
Property Taxes	318 37	
Office Supplies	165 72	
Sundry General Expense	638 49	10126 11
Net operating profit for the year		<u>4349 27</u>

PROFIT-AND-LOSS STATEMENT

In determining Martin's gross profit for the year, the cost of the merchandise purchased during the year (\$27,823.82) is first added to the total of the inventory on Jan. 1 (\$8564.26) in order to give the total cost (\$36,388.08) of the merchandise. Then from this sum the total of the inventory on Dec. 31 (\$7438.82) is subtracted to give the true cost (\$28,949.26) of the merchandise sold during the year. The gross profit (\$14,475.38) is then the difference between this last result and the total of the sales (\$43,424.64).

The operating expenses are then added, and their sum (\$10,126.11) is subtracted from the gross profit. The result is Martin's net profit for the year, which is \$4349.27.

This form of operating statement is very convenient, as it offers the merchant a chance to compare the gross profit on

the sales with the total amount of the sales, a relationship which the keen business man is constantly watching. It also shows the important expenses of the business in such a way that a study of the relative amounts expended for the several purposes may be made, and any necessary decreases or increases in the amounts decided upon.

Financial statement. While the profit-and-loss statement summarizes the results of the merchant's transactions for the year, it does not tell much about the financial condition of the business at the end of the period.

Thus, the fact that John Martin in the above case made a net profit of \$4349.27 during the year does not mean that he will have this much more in actual cash at the end of the year.

In considering the financial condition of a business it is necessary to show the relation between certain factors. The first of these factors are the **assets**, or resources, of the business, which include all items of value (property belonging to the business or amounts due the business) that the business owns. The second are the **liabilities**, which are the amounts that the business owes. The third factor is **proprietary interest**, which represents the money, or capital, invested in the business.

Proprietary interest is the excess of assets over liabilities and hence is the net worth of the business at the time under consideration. If the liabilities exceed the assets, the business is insolvent.

In preparing the **financial statement**, or **balance sheet**, which is to set forth the financial condition of the business it is again necessary to turn to the books of the business for the totals of the various asset and liability accounts.

For example, suppose that John Martin, mentioned above, finds from his books that on Dec. 31 the amount of cash on hand in the store and in the bank is \$899.25, that the total amount of Notes Receivable (written promises to pay money due the business) is \$1275.50, and that the various Accounts Receivable (p. 30) stand as follows: L. C. Long owes \$212.40, A. D. Nash owes \$378.25, and M. R. Rich owes \$520.10. His inventory shows him that on Dec. 31 the value of goods on hand is \$7438.82, and the value of his store equipment on the same date is \$3137.50.

He further finds that Notes Payable (written promises which he has made to pay money) total \$380, and that he owes the following Accounts Payable: Land & Co., \$915.45; Orton & Co., \$893.75.

His books also show that on Jan. 1 his investment in the business was \$7323.35, and his profit-and-loss statement (p. 254) shows that during the year he made a net profit of \$4349.27.

All these items are then arranged in a special order as shown in the following financial statement, first the asset accounts, then the liability accounts, and then the proprietary-interest account:

John Martin
Financial Statement
December 31, 19-

<i>Assets</i>					
Cash	899	25			
Notes Receivable	1275	50			
Accounts Receivable					
<i>L. & Long</i>	212	40			
<i>A. O. Nash</i>	378	25			
<i>M. R. Rich</i>	520	10			
Merchandise Inventory	7438	82			
Store Equipment	3137	50			
Total Assets				13861	82
<i>Liabilities</i>					
Notes Payable	380	—			
Accounts Payable					
<i>Land & Co.</i>	915	45			
<i>Orton & Co.</i>	893	75			
Total Liabilities				2189	20
<i>Proprietary Interest</i>					
<i>John Martin, Investment, Jan. 1</i>	7323	35			
<i>John Martin, net profit for year</i>	4349	27			
<i>Proprietary Interest, Dec. 31, 19-</i>				11672	62

FINANCIAL STATEMENT

In determining the proprietary interest, or net worth of Martin's business on Dec. 31, the total of the liabilities (\$2189.20) is subtracted from the total of the assets (\$13,861.82). The result is the net worth at the end of the year, or \$11,672.62.

The statement further shows that this proprietary interest is made up of Martin's investment on Jan. 1 (\$7323.35) and the net profit (\$4349.27) which was made during the year. The sum of these two items is equal to the result of subtracting the liabilities from the assets.

EXERCISES

1. Following the model form on page 254, prepare a profit-and-loss statement from the following facts about the year's business taken from the books of F. H. Bond's dry-goods store:

Sales, \$47,865.87; Inventory, Jan. 1, \$61,730.28; Purchases, \$25,435.52; Inventory, Dec. 31, \$51,480.13.

Operating Expenses: Fuel, \$311.25; Advertising, \$1060.50; Light, \$255.17; Delivery Expense, \$2476.24; Rent, \$1000; Salaries, \$5535.80; Sundry General Expense, \$1119.02.

2. If Bond's investment in Ex. 1 was \$40,000 on Jan. 1 and his assets and liabilities on Dec. 31 of the same year were as follows, prepare a financial statement from these facts:

Assets: Cash, \$2609.82; Notes Receivable, \$856.75; L. D. North, \$460.20; F. M. Fish, \$2946.34; L. F. Spauth, \$2618.56; Merchandise Inventory, see Ex. 1; Store Furniture, \$5297.16.

Liabilities: Notes Payable, \$12,525.39; L. M. Smith, \$8926.78; F. H. Dick, \$2463.43; F. M. Pratt, \$1931.14.

The personal accounts listed above should be grouped under the heading Accounts Receivable or Accounts Payable, as in the model form.

3. Prepare a profit-and-loss statement from the following facts taken from the books of J. D. Brown, a clothing merchant:

Sales, \$53,579.56; Inventory, Jan. 1, \$13,051.82; Purchases, \$35,124.60; Inventory, Dec. 31, \$15,186.04.

Operating Expenses: Insurance, \$1525; Heat and Light, \$2338.29; Rent, \$1920; Wages, \$9216.47; Traveling Expenses, \$1173.98; Advertising, \$1102.16; Sundry Expense, \$4811.07.

4. If Brown's investment in Ex. 3 was \$34,950 on Jan. 1 and his assets and liabilities on Dec. 31 of the same year were as follows, prepare a financial statement from these facts:

Assets: Cash, \$8964.94; Notes Receivable, \$6251.82; L. M. Roberts, \$3103.82; F. H. Smith, \$4026.48; R. E. Ellis, \$1773.35; Merchandise Inventory, see Ex. 3; Furniture and Fixtures, \$750.

Liabilities: Notes Payable, \$2600; F. S. Green, \$3261.97; Stevenson & Co., \$742.27.

For additional problems see page 423.

CHAPTER VIII

INTEREST AND BANKING

SIMPLE INTEREST

Interest. If a merchant borrows money from a bank with which to carry on his business, he must pay for the use of it. The compensation paid for the use of money is called **interest**.

Interest is sometimes paid in other cases where the idea of interest as compensation for the use of borrowed money is not so evident, such as on overdue accounts. But if A fails to settle his account with B at the proper time, he is really withholding B's money for his own use, and may properly be asked to pay interest until the account is settled.

Interest is computed as a per cent of the sum borrowed, which is called the **principal**. The per cent is called the **rate** of interest, and is generally understood to be the rate of borrowing money for 1 yr. unless the contrary is expressly stated.

Thus, if a man borrows \$400 for 1 yr. at 6%, the principal is \$400, and the rate of interest is 6%.

In many states the rate of interest which may be charged is fixed by law. While borrower and lender may agree on a lower rate than this **legal rate**, the charging of a higher rate is forbidden, except under circumstances defined by the law. An agreement for interest greater than the legal rate is called **usury**, and there are various penalties for this offense in the different states.

Computing interest for 1 yr. Since the problem of computing interest involves a sum (the principal) and a per cent (the rate), finding the amount of interest on a given sum for 1 yr. is simply a case of finding a per cent of a number; that is,

$$\text{principal} \times \text{rate} = \text{interest.}$$

Thus, if a man borrows \$1000 for 1 yr. at 7%, the amount of interest which he will have to pay is $\$1000 \times 0.07$, or \$70.

The sum of the principal and the interest is called the **amount**.

Thus, in the above case, the amount of the debt at the end of the year is \$1000 + \$70, or \$1070.

Interest computed in this way is called **simple interest**, to distinguish it from other methods of computing interest which will be discussed later.

ORAL EXERCISES

State the interest for 1 yr. on each of the following :

- | | |
|--------------------------------|----------------------------------|
| 1. \$100 at 6%. | 9. \$200 at $4\frac{1}{4}\%$. |
| 2. \$150 at 6%. | 10. \$150 at 8%. |
| 3. \$200 at 7%. | 11. \$450 at 6%. |
| 4. \$250 at 8%. | 12. \$210 at 7%. |
| 5. \$120 at 5%. | 13. \$300 at $7\frac{1}{2}\%$. |
| 6. \$100 at $7\frac{1}{2}\%$. | 14. \$1750 at 6%. |
| 7. \$400 at $7\frac{1}{2}\%$. | 15. \$2000 at 5%. |
| 8. \$100 at $4\frac{3}{4}\%$. | 16. \$8000 at $4\frac{3}{4}\%$. |

Computing interest for longer periods. In ordinary cases it seldom happens that money is borrowed for a period longer than a year, and generally not for so long a period. Further, when money is borrowed for a period of years some provision is generally made that interest shall be paid at least once a year. It is occasionally necessary, however, to find interest on a principal for a period longer than a year.

In such cases the best method is to find the interest on the principal for 1 yr., and then to multiply the result by the length of the period expressed in years.

EXAMPLE. Find the interest on \$475.25 for $2\frac{1}{2}$ yr. at 5%.

Solution. The interest for 1 yr. is

$$\$475.25 \times 0.05 = \$23.7625.$$

Disregarding the fraction of a cent, the interest for $2\frac{1}{2}$ yr. is

$$\$23.76 \times 2\frac{1}{2} = \$59.40.$$

EXERCISES

Find the interest on each of the following :

- | | |
|--|---|
| 1. \$225 for 2 yr. at 6%. | 8. \$155.75 for $2\frac{1}{4}$ yr. at $5\frac{1}{2}$ %. |
| 2. \$275 for 2 yr. at 7%. | 9. \$925.50 for $2\frac{1}{2}$ yr. at $6\frac{3}{4}$ %. |
| 3. \$450 for 2 yr. at 5%. | 10. \$2125 for $1\frac{1}{2}$ yr. at $5\frac{1}{4}$ %. |
| 4. \$880 for 2 yr. at $4\frac{1}{4}$ %. | 11. \$175 for 1 yr. 6 mo. at 6%. |
| 5. \$400 for 2 yr. at $4\frac{3}{4}$ %. | 12. \$263 for 1 yr. 3 mo. at 5%. |
| 6. \$125.50 for 3 yr. at 6%. | 13. \$425.50 for 1 yr. 6 mo. at $6\frac{1}{2}$ %. |
| 7. \$500 for $1\frac{1}{2}$ yr. at $4\frac{3}{4}$ %. | 14. \$1250 for 1 yr. 6 mo. at $4\frac{1}{4}$ %. |

Computing interest for the commercial year. When computing interest in common business practice the **commercial year** of 360 da., or 12 mo. of 30 da. each, is generally used. Such a method does not give accurate interest, but since it is so convenient, it is in common use. In some states the practice has been made legal, but it is in use in all the states.

Hence, in computing interest for any number of days, the interest for that period is the same fractional part of the interest for 1 yr. that the number of days is of 360 da.

Thus, in finding the interest on \$500 for 60 da. at 5%, the interest for 1 yr. is $\$500 \times 0.05$, or \$25. Then the interest for 60 da. is $\frac{60}{360}$, or $\frac{1}{6}$, of \$25, which is $\$4.16\frac{2}{3}$. Such a result should be considered as \$4.17.

A quicker and easier method of computing interest for periods of less than 1 yr. will be discussed later, but the above example is given to illustrate the general method of computing interest; namely, that *the interest on the principal should be determined for 1 yr. and then the result multiplied by the length of the period expressed in years, either as a fraction, a whole number, or a mixed number.*

ORAL EXERCISES

State the interest on each of the following :

- | | |
|---|--|
| 1. \$100 for 60 da. at 5%. | 5. \$750 for 1 yr. at 6%. |
| 2. \$300 for 90 da. at 6%. | 6. \$250 for 2 yr. at 5%. |
| 3. \$400 for 30 da. at 6%. | 7. \$600 for $2\frac{1}{2}$ yr. at 4%. |
| 4. \$200 for 180 da. at $5\frac{1}{2}$ %. | 8. \$400 for $3\frac{1}{4}$ yr. at 7%. |

Promissory note. When one person borrows money of another the details of the transaction should be put in writing for the protection of all concerned. This is usually accomplished by a written promise on the part of the borrower to pay the sum borrowed to the lender. A written promise to pay a certain sum of money on demand or at a specified time is called a **promissory note**. A typical form of such a note is illustrated below:

\$75.—	CHICAGO, ILL., <u>June 10, 19—</u>
<u>Sixty days</u> after date I promise to pay to the order of <u>Frank R. Smith</u>	
<u>Seventy-five and 00/100</u> ————— Dollars	
at <u>Stockyards Bank</u> for value received with interest at <u>6%</u>	
No. <u>7</u> Due <u>Aug 9, 19—</u>	<u>Richard C. James</u>

PROMISSORY NOTE

For simplicity of explanation promissory notes are considered only as evidencing transactions in which money is borrowed. Sometimes notes are used in place of an immediate settlement of an account or for arranging the details of settlement at some future date. Thus a man may sell goods and agree to take his customer's note instead of a cash payment. Theoretically, the seller then becomes a lender and the purchaser becomes a borrower. Under modern banking conditions such cases are becoming rare, as the purchaser usually borrows the money from his bank and pays his account with the seller.

In the model note above Richard C. James is the borrower, or the **maker** of the note, and Frank R. Smith is the one to whom the money is to be paid, or the **payee**. The sum borrowed, \$75, is called the **face** of the note. The time for which the money is borrowed is 60 da.; that is, 60 da. after June 10, 19—, or on Aug. 9, 19—, the note is due. Hence June 10 is the **date** of the note and Aug. 9 is the **date of maturity**.

The rate of interest is 6%. If the note ran a year, the interest would amount to $\$75 \times 0.06$, or \$4.50. But it runs for only $\frac{60}{360}$, or $\frac{1}{6}$, of a year, and thus the interest due at the date of maturity

is \$0.75. Hence, the **amount** or **value** of the note on the date of maturity is $\$75 + \0.75 , or $\$75.75$.

The words "to the order of" mean that the note is payable to some other person only upon an order signed by the payee. Such an order would be written on the back of the note and is known as an **indorsement**.

For example, Frank R. Smith might desire to transfer the note to J. B. Osgood. In this case he would write across the back the words "Pay to the order of J. B. Osgood" and sign the order. The question of the different types of indorsements will be considered later.

The indorser of a note becomes liable for its payment in case the maker of the note fails to pay the note when due.

The words "for value received" indicate that the note is given in consideration of some value received by the maker. While these words are customarily inserted in notes, they are not now necessary in most states to make a note legal.

Joint-and-several note. It sometimes happens that two or more persons borrow money together for a common purpose. In such a case they might both be makers of a note. When two or more persons sign an agreement in which they jointly and severally promise to pay money, the note is called a **joint-and-several note**. Such a form of note is shown below :

\$ 300.--	MINNEAPOLIS, MINN.,	Aug. 5, 19--
-----Ninety days-----after date we jointly and severally		
promise to pay to the order of-----A. M. Jensen-----		
Three Hundred and 00/100 -----		Dollars
Payable at First National Bank		
Value Received	<i>Arthur Gates</i>	
	<i>Frank R. Simpson</i>	
No. 3 Due Nov. 3, 19--	<i>Charles M. Adams</i>	

JOINT-AND-SEVERAL NOTE

If the words "and severally" are omitted in the above note, it is called a **joint note**. The difference is that in a joint-and-several note, suit for payment may be brought against each of the makers individually or they may

be sued jointly, while in a joint note suit must be made against the makers jointly. In many of the states this distinction has now been abolished by law.

Interest-bearing note. If a note contains a clause specifying the rate of interest, as in the model form on page 261, it is called an **interest-bearing note**. Any note which is not paid when due draws interest from the date of maturity even if no interest clause is included. If no interest is specified, the legal rate of interest may be charged.

The model notes above are called **time notes** since they specify the date on which the money will be paid. If the words "On demand" were substituted for the words "Sixty days after date," the note on page 261 would be called a **demand note**. A demand note in which no interest is specified draws interest at the legal rate after payment is demanded.

Some banks refuse to issue demand notes, and the Federal Reserve Banks will not accept them. The result is that the use of such notes is decreasing.

EXERCISES

Following the model forms write :

1. A 60-day note for \$1525.50, dated today, with interest at 6%. Insert any name for the payee and your name as maker.

It is not necessary to specify the bank at which a note is payable, although this is often done in notes.

2. A 6-month joint-and-several note for \$2376.55, dated today, with interest at $6\frac{1}{2}\%$. The payee is F. J. Dickerson, and the makers are R. T. Babcock and H. E. Weldon.

Such a note is due 6 mo. after date. Thus, if the date of the note is Jan. 7, the note is due July 7, without regard to the actual number of days intervening. The custom of specifying the exact date on which a note is due is growing.

3. A time note for \$165.28 due 87 da. (give date) after today with interest at 7%. The payee is R. G. Reed and the maker is B. F. Graves.

4. A demand note for \$5000, dated today, with interest at 6%. The payee is Jones & Co. and the maker is H. F. Harwood.

5. In Exs. 1-3 find the value (amount) of each note at the date of maturity.

Bankers' 60-day method of computing interest. In years past business men used to borrow money from individuals, and gave their notes in return. Nowadays with the increase in the facilities afforded by modern banks, most business men borrow only from their banks. The majority of such loans are made for short periods — generally for 30 da., 60 da., or 90 da. If the money is needed for a longer period, a new note for another period is made and the old note is paid off from the new loan.

Since 6% is a common rate of interest and the notes run for even periods of 30 da., the computation of interest in such cases is simple and furnishes a short method of computing interest which is known as the **bankers' 60-day method**.

In many banks interest is computed by means of tables, as explained later, but the 60-day method is widely used.

This method is based on the fact that since the commercial year is 360 da., a rate of 6% a year is equivalent to a rate of $\frac{1}{6}\%$ of 6%, or 1%, for 60 da., and similarly for other periods as shown in the following table:

ANNUAL RATE	PERIOD	EQUIVALENT RATE
6%	60 da.	1%
6%	30 da.	$\frac{1}{2}\%$
6%	90 da.	$1\frac{1}{2}\%$
6%	6 da.	0.1%

This table could be extended to include many more equivalent rates, but with practice the student will soon learn to recognize equivalent fractional rates without referring to a table.

Now finding 1% of a number requires multiplying the number by 0.01, and this is easily done by moving the decimal point two places to the left. Hence

1. *To find the interest for 60 da. at 6%, move the decimal point in the principal two places to the left;*

2. To find the interest for 6 da. at 6%, move the decimal point in the principal three places to the left.

The interest for fractional parts of 60 da. or 6 da. is then easily found by taking the corresponding fractional part of the result obtained by the above rules.

EXAMPLES. 1. Find the interest on \$275 for 60 da. at 6%.

Solution. Moving the decimal point two places to the left, the required interest is \$2.75.

2. Find the interest on \$420 for 30 da. at 6%.

Solution. The interest for 60 da. is \$4.20. Hence the interest for 30 da. is $\$4.20 \times \frac{1}{2}$, or \$2.10.

3. Find the interest on \$162 for 1 da. at 6%.

Solution. Moving the decimal point three places to the left, the interest for 6 da. is \$0.162. Hence the interest for 1 da. is $\$0.162 \times \frac{1}{6}$, or \$0.027. Such a result is given to the nearest cent, or as 3¢.

ORAL EXERCISES

State the interest on a note made for each of the following amounts for 60 da. at 6% :

- | | | |
|-----------|-------------|---------------|
| 1. \$750. | 7. \$1764. | 13. \$125.40. |
| 2. \$282. | 8. \$1434. | 14. \$362.40. |
| 3. \$474. | 9. \$2460. | 15. \$470.40. |
| 4. \$138. | 10. \$2832. | 16. \$576.06. |
| 5. \$942. | 11. \$3240. | 17. \$463.92. |
| 6. \$720. | 12. \$4320. | 18. \$975.40. |

19. In Exs. 1-18 state the interest in each case for a 30-day note.

20. In Exs. 1-18 state the interest on each sum for 6 da.; for 1 da.

Interest for any number of days. In finding the interest for any number of days by the 60-day method, it is necessary to separate the time into periods, or parts of periods, of 60 da. or 6 da. before applying the rules just given.

Thus, finding the interest for a period of 67 da. is equivalent to finding the interest for 60 da. + 6 da. + 1 da.; finding the interest for 92 da. is the same as finding it for 60 da. + 30 da. + 2 da.; and so on.

With a little practice the student will soon recognize any number of days as convenient multiples or parts of 60 da. or 6 da.

EXAMPLE. Find the interest on \$175 for 161 da. at 6%.

Solution. Divide 161 da. into convenient periods and proceed as follows:

60 da.	} 120 da.	\$1.75	
60 da.		1.75	
30 da.	($\frac{1}{2}$ of \$1.75)	0.875	
10 da.	($\frac{1}{3}$ of \$0.875)	.2917	(\$0.2916 $\frac{2}{3}$)
1 da.	($\frac{1}{10}$ of \$0.2917)	.0292	(\$0.02917)
<u>161 da.</u>		<u>\$4.6959</u>	or \$4.70

In the above solution it should be noticed that the interest for 120 da. is found by repeating the interest for 60 da. After skill is gained in the method, the interest for 120 da. may be set down at once as \$3.50, the student multiplying \$1.75 by 2 mentally as he thinks "interest for 120 da. on \$175."

The given number of days, 161, can be divided in other ways, as 60 da. + 60 da. + 30 da. + 6 da. + 3 da. + 2 da., but the above method is shorter. In the following exercises the student should first set down the periods into which the required number of days is to be divided and then look them over to be sure that the most convenient division has been chosen. In the student's solution it is unnecessary to set down the explanatory matter given in the parentheses above, as this simply shows how the different items in the model solution were obtained.

EXERCISES

Compute the interest at 6% on each of the following:

- | | |
|---------------------|-------------------------|
| 1. \$240 for 66 da. | 8. \$965 for 24 da. |
| 2. \$300 for 61 da. | 9. \$324 for 90 da. |
| 3. \$360 for 67 da. | 10. \$545.20 for 30 da. |
| 4. \$375 for 63 da. | 11. \$392.40 for 36 da. |
| 5. \$489 for 64 da. | 12. \$437.25 for 31 da. |
| 6. \$273 for 68 da. | 13. \$295.82 for 37 da. |
| 7. \$473 for 19 da. | 14. \$576.93 for 33 da. |

- | | |
|---------------------------|--------------------------|
| 15. \$725.46 for 34 da. | 28. \$1375 for 4 mo. |
| 16. \$1250 for 40 da. | 29. \$2150 for 6 mo. |
| 17. \$2960 for 38 da. | 30. \$1225 for 8 mo. |
| 18. \$7140 for 42 da. | 31. \$496.10 for 3 mo. |
| 19. \$6754 for 45 da. | 32. \$763.25 for 5 mo. |
| 20. \$5963 for 49 da. | 33. \$475.50 for 9 mo. |
| 21. \$4785.40 for 54 da. | 34. \$1652.85 for 2 mo. |
| 22. \$5671.25 for 55 da. | 35. \$1015.30 for 3 mo. |
| 23. \$6954.75 for 126 da. | 36. \$1565.50 for 5 mo. |
| 24. \$2545.10 for 127 da. | 37. \$4923.80 for 4 mo. |
| 25. \$2673.18 for 91 da. | 38. \$3765.70 for 9 mo. |
| 26. \$3152.73 for 96 da. | 39. \$4963.85 for 7 mo. |
| 27. \$1250 for 2 mo. | 40. \$1979.45 for 11 mo. |

41. Find the value (amount) on the date of maturity of a 90-day note for \$225 which draws interest at 6%.

42. A man gave a 60-day note for \$410 with interest at 6%. How much will he have to pay when he takes up the note on the date of maturity?

43. A note for \$375 with interest at 6% is due 3 mo. after date. What will be the amount of the note on the date of maturity?

44. When buying an automobile costing \$1800 a man paid 60% of the purchase price in cash. The unpaid balance was equally divided between three notes due in 30 da., 60 da., and 90 da. respectively, with interest at 6%. What was the total amount paid?

45. In order to take advantage of cash discounts on their purchases a large department store borrows \$20,000 at 6%, knowing that during the month their customers will pay their monthly accounts and that they can then repay the loan. If the loan is not repaid for 36 da., how much interest does the store pay?

Interest for rates other than 6%. The student has now seen the advantages of the 60-day method of computing interest at 6%. Since the interest at 5%, for example, is $\frac{5}{6}$ of the interest at 6%,

this method can be extended to include rates other than 6% by deducting or adding the proper fractional part of the interest found for 6%.

Thus, to find the interest at 3%, deduct $\frac{1}{2}$ of the interest at 6%; to find the interest at 4%, deduct $\frac{1}{3}$ of the interest at 6%; to find the interest at 5%, deduct $\frac{1}{6}$ of the interest at 6%; to find the interest at 7%, add $\frac{1}{6}$ of the interest at 6%; and so on.

Computing interest by the 60-day method at rates other than 6% is illustrated in the following examples.

EXAMPLES. 1. Find the interest on \$124.50 for 93 da. at $5\frac{1}{2}\%$.

Solution. The interest is first computed at 6% as follows:

60 da.	\$1.245
30 da. ($\frac{1}{2}$ of \$1.245)	0.6225
3 da. ($\frac{1}{10}$ of \$0.6225)	.0623 (\$0.06225)
<u>93 da.</u>	<u>\$1.9298</u>
Deduct $\frac{1}{2}$ of the interest at 6%.	.1608 (\$0.1608 $\frac{1}{2}$)
The required interest at $5\frac{1}{2}\%$ is	<u>\$1.7690, or \$1.77</u>

2. Find the interest on \$425 for 42 da. at 7%.

Solution. First find the interest at 6% as follows:

30 da. ($\frac{1}{2}$ of \$4.25)	\$2.125
12 da. ($2 \times$ \$0.425)	0.850
<u>42 da.</u>	<u>\$2.975</u>
Add $\frac{1}{6}$ of the interest at 6%.	.496 (\$0.495 $\frac{5}{6}$)
The required interest at 7% is	<u>\$3.471, or \$3.47</u>

ORAL EXERCISES

State in your own words a method based on the 60-day method for computing interest at each of the following rates:

- | | | | |
|--------|-----------------------|------------------------|------------------------|
| 1. 5%. | 5. 3%. | 9. $3\frac{1}{2}\%$. | 13. $5\frac{1}{4}\%$. |
| 2. 8%. | 6. 2%. | 10. $6\frac{1}{4}\%$. | 14. $5\frac{3}{4}\%$. |
| 3. 4%. | 7. $6\frac{1}{2}\%$. | 11. $7\frac{1}{2}\%$. | 15. $4\frac{1}{4}\%$. |
| 4. 9%. | 8. $6\frac{3}{4}\%$. | 12. $4\frac{1}{2}\%$. | 16. $4\frac{3}{4}\%$. |

EXERCISES

Find the interest in each of the following:

PRINCIPAL	TIME	RATE	PRINCIPAL	TIME	RATE
1. \$165	60 da.	7%	16. \$345.35	18 da.	$7\frac{3}{4}\%$
2. \$127	60 da.	5%	17. \$3225	30 da.	$4\frac{1}{4}\%$
3. \$215	66 da.	8%	18. \$3750	36 da.	$4\frac{3}{4}\%$
4. \$172	67 da.	9%	19. \$4780	31 da.	$4\frac{1}{4}\%$
5. \$233	30 da.	10%	20. \$5125	37 da.	$4\frac{3}{4}\%$
6. \$146	90 da.	$6\frac{1}{2}\%$	21. \$6235	13 da.	7%
7. \$280	96 da.	$5\frac{1}{2}\%$	22. \$1752	15 da.	$6\frac{1}{4}\%$
8. \$325	97 da.	$6\frac{1}{4}\%$	23. \$1855	12 da.	8%
9. \$496	61 da.	$5\frac{3}{4}\%$	24. \$1725	18 da.	7%
10. \$395.25	72 da.	$6\frac{1}{2}\%$	25. \$2350	66 da.	$7\frac{1}{4}\%$
11. \$420.40	15 da.	$6\frac{3}{4}\%$	26. \$4961.50	61 da.	$6\frac{3}{4}\%$
12. \$315.75	75 da.	$5\frac{3}{4}\%$	27. \$4835.75	67 da.	$7\frac{1}{2}\%$
13. \$375.80	70 da.	$5\frac{1}{2}\%$	28. \$3965.10	75 da.	$5\frac{1}{2}\%$
14. \$260.60	68 da.	$7\frac{1}{2}\%$	29. \$2843.20	84 da.	$6\frac{1}{2}\%$
15. \$285.45	12 da.	$7\frac{1}{4}\%$	30. \$4936.75	93 da.	$7\frac{1}{4}\%$

Interest table. In banks and insurance offices where a great many computations of interest have to be made, **simple-interest tables** are frequently used. There are many different types of tables, but the form on page 270 shows a part of such a table for computing interest at 6%.

By following the horizontal lines across to the proper columns it is readily found from the table that interest at 6% on \$10 for 6 da. is \$.01, on \$6 for 20 da. is \$.02, on \$9 for 10 mo. is \$.45, and so on.

The interest at 6% on \$50 for 8 da. is 10 times that for \$5 for 8 da. Hence the interest on \$50 for 8 da. is found by looking in the table for \$5 for 8 da. and setting down the figure there found with the decimal point moved one place to the right; that is, the interest on \$50 for 8 da. is \$0.0667, or \$0.07, and so on.

SIMPLE INTEREST AT 6%										
TIME	\$1	\$2	\$3	\$4	\$5	\$6	\$7	\$8	\$9	\$10
1 da.	.00017	.00033	.0005	.00067	.00083	.001	.00117	.00133	.0015	.00167
2 da.	.00033	.00067	.001	.00133	.00167	.002	.00233	.00267	.003	.00333
3 da.	.0005	.001	.0015	.002	.0025	.003	.0035	.004	.0045	.005
4 da.	.00067	.00133	.002	.00267	.00333	.004	.00467	.00533	.006	.00667
5 da.	.00083	.00167	.0025	.00333	.00417	.005	.00583	.00667	.0075	.00833
6 da.	.001	.002	.003	.004	.005	.006	.007	.008	.009	.01
7 da.	.00117	.00233	.0035	.00467	.00583	.007	.00817	.00933	.0105	.01167
8 da.	.00133	.00267	.004	.00533	.00667	.008	.00933	.01067	.012	.0133
9 da.	.0015	.003	.0045	.00667	.0075	.009	.0105	.012	.0135	.015
10 da.	.00167	.00333	.005	.00667	.00833	.01	.01167	.01333	.015	.01667
20 da.	.00333	.00667	.01	.01333	.01667	.02	.02333	.02667	.03	.03333
1 mo.	.005	.01	.015	.02	.025	.03	.035	.04	.045	.05
2 mo.	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10
3 mo.	.015	.03	.045	.06	.075	.09	.105	.12	.135	.15
4 mo.	.02	.04	.06	.08	.10	.12	.14	.16	.18	.20
5 mo.	.025	.05	.075	.10	.125	.15	.175	.20	.225	.25
6 mo.	.03	.06	.09	.12	.15	.18	.21	.24	.27	.30
7 mo.	.035	.07	.105	.14	.175	.21	.245	.28	.315	.35
8 mo.	.04	.08	.12	.16	.20	.24	.28	.32	.36	.40
9 mo.	.045	.09	.135	.18	.225	.27	.315	.36	.405	.45
10 mo.	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50
11 mo.	.055	.11	.165	.22	.275	.33	.385	.44	.495	.55
1 yr.	.06	.12	.18	.24	.30	.36	.42	.48	.54	.60
2 yr.	.12	.24	.36	.48	.60	.72	.84	.96	1.08	1.20

For periods and amounts not given in the table, the interest may be found by the method shown in the following example.

EXAMPLE. Find the interest on \$750 for 6 mo. 11 da. at 6%.

Solution. Separate \$750 into \$700 + \$50 and the given period into 6 mo. + 10 da. + 1 da., and proceed as follows:

6 mo.	{ \$700	\$21.
	{ \$50	1.50
10 da.	{ \$700	1.167
	{ \$50	0.0833
1 da.	{ \$700	.117
	{ \$50	.0083

The required interest is

\$23.8756, or \$23.88

In using the table in such an example it is convenient to lay the straight edge of a slip of paper along the horizontal line, and then determine the interest on the several amounts and set them down before moving the paper to the next horizontal line.

EXERCISES

Using the interest table on page 270, find the interest at 6% in each of the following:

- | | |
|---------------------------|-----------------------------|
| 1. \$250 for 19 da. | 7. \$4900 for 3 mo. 15 da. |
| 2. \$370 for 1 mo. 20 da. | 8. \$5600 for 1 yr. 28 da. |
| 3. \$460 for 2 mo. 7 da. | 9. \$750 for 5 mo. 17 da. |
| 4. \$120 for 7 mo. 12 da. | 10. \$1250 for 9 mo. 9 da. |
| 5. \$275 for 1 yr. 19 da. | 11. \$3470 for 11 mo. 7 da. |
| 6. \$360 for 10 mo. 8 da. | 12. \$5650 for 1 yr. 25 da. |
13. Find the total interest at 6% on the following items:
- | | |
|------------------------|---------------------------------|
| \$450 for 1 yr. 20 da. | \$212.50 for 2 yr. 1 mo. 10 da. |
| \$495 for 1 yr. 23 da. | \$517.25 for 1 yr. 9 mo. 20 da. |

Exact time between two dates. In computing interest on notes and other obligations it is frequently necessary to determine the exact number of days between two dates. It is the general practice in such cases to exclude the day of the first date but to include the day of the second date. The number of days is then found by counting as in the following example.

EXAMPLE. Find the number of days between Feb. 10 and June 3 of the same year.

Solution. After Feb. 10 there are 28 da. — 10 da., or 18 da., left in February. Then March has 31 da.; April, 30 da.; May, 31 da.; and there are 3 da. in June to be counted. Adding,

$$18 + 31 + 30 + 31 + 3 = 113;$$

that is, the required time is 113 da.

In this case and in the following exercises a leap year should not be considered unless it is specifically mentioned. If the above case had been a leap year, there would have been 19 da. in February, and the time would have been 114 da.

EXERCISES

Find the exact number of days between the following dates in the same year :

1. May 12 and Aug. 14.
2. March 3 and May 10.
3. Sept. 1 and Dec. 1.
4. Oct. 3 and Dec. 2.
5. Jan. 3 and March 5.
6. Feb. 1 and June 2.
7. Feb. 4 and May 1.
8. Jan. 3 and April 7.
9. March 5 and June 10.
10. April 3 and Aug. 5.
11. March 28 and Sept. 3.
12. June 9 and Dec. 1.
13. May 3 and Sept. 14.
14. Feb. 6 and Nov. 11.

15. Solve Exs. 5, 6, 7, 8, and 14 for a leap year.

16. Find the number of days from March 16 to Feb. 7 of the following year; from Dec. 17 to July 7 of the following year; from Sept. 30 to Aug. 19 of the following year.

Table for computing time. In computing the exact number of days between two dates, banks often use tables similar to the one shown below.

FROM ANY DAY OF	TO THE SAME DAY OF											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January . . .	365	31	59	90	120	151	181	212	243	273	304	334
February . . .	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August	153	184	212	243	273	304	334	365	31	61	92	122
September . .	122	153	181	212	243	273	303	334	365	30	61	91
October	92	113	151	182	212	243	273	304	335	365	31	61
November . . .	61	92	120	151	181	212	242	273	304	334	365	30
December . . .	31	62	90	121	151	182	212	243	274	304	332	365

The table shows the exact number of days between any day of one month and the corresponding day of any other month. The following examples illustrate the use of the above table.

EXAMPLE. 1. Find the exact number of days between June 6 and Sept. 6 of the same year.

Solution. In the first column at the left of the table find June. Then following along the horizontal line to the column headed Sept., the figure 92 is found. Thus the number of days between any day of June and the corresponding day of September is 92, or the time from June 6 to Sept. 6 is 92 da.

2. Find the exact number of days between March 1 and June 15.

Solution. By the table, the time from March 1 to June 1 is 92 da. From June 1 to June 15 it is 15 da. — 1 da., or 14 da. Hence the required time is 92 da. + 14 da., or 106 da.

3. Find the exact number of days between Dec. 18 and Feb. 7.

Solution. By the table, the time from Dec. 18 to Feb. 18 is 62 da. From Feb. 18 *back* to Feb. 7 it is 18 da. — 7 da., or 11 da. Hence the required time is 62 da. — 11 da., or 51 da.

ORAL EXERCISES

Using the table on page 272, state the exact number of days between the following dates of the same year :

- | | |
|-------------------------|------------------------|
| 1. March 1 and July 10. | 5. May 28 and Oct. 12. |
| 2. Feb. 2 and Aug. 21. | 6. Aug. 11 and Dec. 1. |
| 3. April 6 and Oct. 3. | 7. Feb. 16 and May 11. |
| 4. July 5 and Dec. 25. | 8. May 19 and Nov. 22. |

Using the same table, state the exact number of days between the following dates where the second date is a day of the following year :

- | | |
|----------------------------|---------------------------|
| 9. Nov. 9 and Feb. 16. | 11. March 10 and Feb. 29. |
| 10. Sept. 25 and April 11. | 12. Oct. 17 and June 1. |

For additional problems see page 425.

Bank discount. As has been stated previously, the borrowing of money in business is usually done through banks, the borrower giving his note to the bank for the amount of the loan. Banks

generally require that interest on such a loan be paid in advance. Interest paid on a note in this way is called **bank discount**.

The student should review the terms used in connection with promissory notes as given on pages 261-263.

The usual method is for the bank to deduct the proper amount for discount (interest) from the face of the note, and to pay the balance, called the **proceeds**, to the borrower. Since the interest is paid in advance, such a note does not contain any interest clause.

The student should see that computing bank discount in such cases is the same as computing simple interest, the only difference being that bank discount represents interest paid in advance.

EXERCISES

1. A borrowed \$1000 at his bank and gave a 60-day note. If the rate of discount (interest) was 6%, what was the bank discount? the proceeds of the note?

2. What is the bank discount on a 90-day note given for a loan of \$650 if the discount rate is $6\frac{1}{2}\%$? What are the proceeds?

In each of the following, find the bank discount and the proceeds :

AMOUNT OF LOAN	TIME	RATE OF DISCOUNT	AMOUNT OF LOAN	TIME	RATE OF DISCOUNT
3. \$865	30 da.	6%	7. \$5675	90 da.	$5\frac{1}{2}\%$
4. \$1125	60 da.	$6\frac{1}{2}\%$	8. \$6240	2 mo.	$6\frac{1}{2}\%$
5. \$980	2 mo.	$5\frac{1}{2}\%$	9. \$845.25	30 da.	7%
6. \$4250	4 mo.	7%	10. \$1047.75	90 da.	$5\frac{1}{2}\%$

Discounting negotiable notes. If a note can be transferred by the payee to any other person by indorsement and delivery of the note, as in the model on page 261, it is called a **negotiable note**.

Sometimes the holder of a negotiable note does not care to wait until the date of maturity to receive the money. He may then offer the note at his bank for discount. If the bank is willing to advance him the money on it, he indorses the note and delivers it to the bank; that is, he sells the note to the

bank. In return for advancing the money, the bank deducts the discount for the number of days between the date of discount and the date of maturity of the note, and pays the proceeds to the holder. This process is called **discounting** a note, and the time for which the discount is charged is known as the **term of discount**. It is the general custom to count the exact number of days in the term of discount, excluding the day of discount but including the day of maturity.

In computing the bank discount on a non-interest-bearing note, the date of maturity must be determined first, then the term of discount, and then the bank discount is computed for this term on the face of the note.

EXAMPLE. On May 31 A. R. Sullivan offers for discount the 60-day note of Henry E. Dunton for \$400, dated May 1. If the bank discounts the note at 6%, find the proceeds.

Solution. The date of maturity of the note is 60 da. after May 1; that is, June 30.

The term of discount is from May 31 to June 30, or 30 da.

The discount for 30 da. is found by the banker's method as follows :

$$\frac{1}{2} \times \$4 = \$2.$$

Then

$$\$400 - \$2 = \$398.$$

Hence the proceeds of the note which the bank pays to A. R. Sullivan are \$398.

ORAL EXERCISES

In the following, state the date of maturity of each note :

DATE OF NOTE	TIME OF NOTE	DATE OF NOTE	TIME OF NOTE
1. Jan. 16	30 da.	6. March 6	3 mo.
2. Jan. 28	1 mo.	7. March 6	90 da.
3. Jan. 20	30 da.	8. Aug. 12	60 da.
4. Jan. 30	1 mo.	9. Feb. 28	2 mo.
5. Jan. 30	60 da.	10. May 6	30 da.

The student should remember that a 30-day note is due exactly 30 da. after date, but that a 1-month note is due on the corresponding day of the following month without regard to the actual number of days in the first month.

In the following, state the term of discount on each note :

DATE OF NOTE	TIME OF NOTE	DATE OF DISCOUNT
11. Feb. 11	60 da.	March 16
12. Feb. 26	2 mo.	April 1
13. March 10	30 da.	March 10
14. March 25	90 da.	May 15
15. April 11	3 mo.	June 1
16. May 14	90 da.	July 1
17. June 15	60 da.	June 21
18. June 30	1 mo.	July 1
19. July 18	2 mo.	July 22
20. Aug. 23	60 da.	Aug. 31

In the following, state the bank discount and the proceeds of each note :

FACE OF NOTE	TERM OF DISCOUNT	RATE OF DISCOUNT	FACE OF NOTE	TERM OF DISCOUNT	RATE OF DISCOUNT
21. \$380	30 da.	6%	27. \$900	75 da.	5%
22. \$275	60 da.	6%	28. \$1000	54 da.	6%
23. \$483	20 da.	6%	29. \$1200	45 da.	6%
24. \$720	70 da.	6%	30. \$1350	30 da.	6%
25. \$500	15 da.	6%	31. \$1150	60 da.	6%
26. \$840	40 da.	7%	32. \$1250	90 da.	6%

EXERCISES

Find the bank discount and the proceeds for each of the following notes :

DATE OF NOTE	FACE OF NOTE	TIME OF NOTE	DATE OF DISCOUNT	RATE OF DISCOUNT
1. Aug. 24	\$2500	4 mo.	Aug. 24	6%
2. Aug. 16	\$406	90 da.	Aug. 16	6%
3. Aug. 22	\$300	2 mo.	Aug. 22	7%

	DATE OF NOTE	FACE OF NOTE	TIME OF NOTE	DATE OF DISCOUNT	RATE OF DISCOUNT
4.	Aug. 4	\$175	30 da.	Aug. 4	6%
5.	Aug. 24	\$7500	90 da.	Aug. 24	6½%
6.	Aug. 24	\$4000	60 da.	Aug. 24	6%
7.	May 3	\$200	60 da.	June 1	6%
8.	May 8	\$325	3 mo.	June 5	5½%
9.	May 10	\$68.50	90 da.	July 1	6%
10.	May 12	\$450	2 mo.	May 25	5%
11.	May 16	\$146.50	90 da.	June 20	6%
12.	May 18	\$270	90 da.	May 31	7%
13.	May 21	\$300	60 da.	May 29	6½%
14.	May 24	\$520	30 da.	June 2	6%
15.	May 26	\$86.90	2 mo.	May 26	6½%
16.	May 28	\$192.50	90 da.	June 27	6%

Discounting interest-bearing notes. Very few interest-bearing notes are offered for discount. When they are, the bank discount is computed on the **value** of the note at maturity; that is, on the face plus the interest due at maturity.

EXAMPLE. On April 2 a 3-month note for \$280, dated March 1 and bearing interest at 6%, is offered for discount. If the discount rate is 6%, what are the proceeds?

Solution. The date of maturity of the note is 3 mo. after March 1; that is, June 1.

The interest due at the date of maturity is found as follows:

$$\$2.80 + \$1.40 = \$4.20.$$

The value of the note is found as follows:

$$\$280 + \$4.20 = \$284.20.$$

The term of discount is from April 2 to June 1, or 60 da.

The bank discount is then $\$284.20 \times 0.01$, or $\$2.84$, to the nearest cent.

Then $\$284.20 - \$2.84 = \$281.36$.

Hence the proceeds of the note are $\$281.36$.

EXERCISES

Find the bank discount and the proceeds for each of the following interest-bearing notes:

DATE OF NOTE	FACE OF NOTE	TIME OF NOTE	RATE OF INTEREST	DATE OF DISCOUNT	RATE OF DISCOUNT
1. March 6	\$500	2 mo.	6%	April 6	6%
2. March 10	\$360	90 da.	6%	May 11	6%
3. March 20	\$960	90 da.	6%	May 6	7%
4. Aug. 20	\$875	60 da.	5%	Sept. 7	6%
5. March 16	\$750	90 da.	6%	March 16	6%

Sometimes a note like the one in Ex. 5 is taken at its face value and no discount is reckoned, as the bank gets the interest for the full time.

When a note is drawn for a loan at a bank, it should be made without interest. Otherwise, the discount may be computed on the value of the note at maturity, and discount would have to be paid on the interest.

Note to yield a required amount. When a man wishes to borrow from a bank to meet a known requirement, he must make the face of his note such an amount that the proceeds will be sufficient for his needs. Since the proceeds of a note for \$1 at the known rate of discount can be readily determined, it is seen that

face of note = required proceeds $\div$ proceeds of \$1.

EXAMPLE. In securing a loan for 60 da. from a bank, a man desires that his note, when discounted at 6%, shall yield \$1188. For what amount should he draw the note?

Solution. The interest on \$1 at 6% for 60 da. is \$0.01.

The proceeds of \$1 for 60 da. at 6% are \$0.99.

Then $\$1188 \div 0.99 = \1200 .

Hence the note should be drawn for \$1200.

EXERCISES

Find the face of a note which will yield each of the proceeds stated when discounted as follows :

	REQUIRED PROCEEDS	TERM OF DISCOUNT	RATE OF DISCOUNT
1.	\$443.25	90 da.	6%
2.	\$640.25	90 da.	7%
3.	\$709.20	3 mo.	6%
4.	\$990.00	2 mo.	6%
5.	\$1089.00	60 da.	5%
6.	\$837.25	90 da.	6%

REVIEW EXERCISES

1. A. B. Comstock wishes to realize \$831.60 as the proceeds of a 60-day note. If the discount rate is 6%, for how much should he draw the note?

2. On Jan. 30 John M. Cox offers for discount a 2-month note for \$1250, dated Jan. 1 and bearing interest at 6%. If the bank discounts the note at 6%, what are the proceeds?

3. Find the total proceeds from discounting the following notes:

FACE OF NOTE	DATE OF NOTE	TIME OF NOTE	DATE OF DISCOUNT	RATE OF DISCOUNT
\$425.25	June 21	2 mo.	June 21	6%
\$301.75	June 17	60 da.	June 23	6%
\$309.04	June 18	60 da.	June 24	6%
\$2000.00	June 20	90 da.	June 20	5%

4. Henry A. Taylor finds on July 6 that his bank balance is \$137.56. He accordingly offers for discount the following notes which he holds: a 2-month note for \$1000, dated July 1; a 60-day note for \$872.40, dated June 29; a 90-day note for \$750, dated June 15; and a 90-day note for \$762.79, dated June 10. If the bank discounts the notes at 6%, what is the amount of Taylor's bank balance after the proceeds are credited to his account?

For additional problems see page 428.

Partial payments. Sometimes the maker of a note desires to pay off part of the note before maturity. Such payments are called **partial payments**, and the date and amount paid are indorsed on the back of the note.

When a partial payment is made on an interest-bearing note, the share of the payment which is applied to the interest due at the date of the payment (accrued interest) is determined before any part is applied to reduce the face of the note.

United States rule. The following method for computing partial payments is known as the **United States rule** :

1. *The payment must be first applied to the discharge of the accrued interest.*
2. *The balance, if any, after paying the interest, may be used to reduce the principal.*
3. *If any payment is less than the accrued interest, the principal remains unchanged until some payment is made, which, added to the omitted payment (or payments), is more than sufficient to discharge the accrued interest.*

Some banks do not observe the third part of this rule.

EXAMPLE. A 1-year note for \$1500, dated Jan. 1 and bearing interest at 6%, has the following partial payments indorsed on it: April 21, \$427.50; Aug. 31, \$300. How much is due at maturity?

Solution. Using the United States rule, proceed as follows :

Face of note	\$1500.00
Interest on \$1500, from Jan. 1 to April 21 (110 da.)	27.50
Value of note on April 21	<u>\$1527.50</u>
Partial payment of April 21	427.50
New principal from April 21	<u>\$1100.00</u>
Interest from April 21 to Aug. 31 (132 da.)	24.20
Value of note on Aug. 31	<u>\$1124.20</u>
Partial payment of Aug. 31	300.00
New principal from Aug. 31	<u>\$824.20</u>
Interest from Aug. 31 to Jan. 1 (123 da.)	16.90
Amount due at date of maturity	<u>\$841.10</u>

The methods of dealing with partial payments are not uniform in all banks. It is suggested that one of the students consult a local bank and find out the methods used.

EXERCISES

Using the United States rule, find the amount due at maturity on each of the following interest-bearing notes:

FACE OF NOTE	DATE OF NOTE	TIME OF NOTE	RATE OF INTEREST	INDORSEMENTS (Payments)
1. \$2500	March 1	4 mo.	6%	{ April 1, \$500 May 18, \$1000
2. \$3500	May 1	6 mo.	6%	{ July 1, \$1000 Aug. 15, \$1200 Oct. 3, \$300
3. \$4350	June 1	5 mo.	6%	{ July 6, \$850 Sept. 1, \$1250
4. \$750.50	Aug. 1	90 da.	7%	{ Sept. 5, \$250 Oct. 7, \$300

Rebate method. When partial payments are made on a note on which interest has been paid in advance, it is evident that a rebate of interest is due the maker of the note.

If the rebate of interest is not adjusted until the date of maturity of the note, the following method is often used:

1. *Compute the interest on each partial payment from the date of the payment to the date of maturity of the note.*

2. *Add these amounts to obtain the total rebate of interest.*

3. *Subtract the sum of the partial payments from the face of the note to find the amount due on the face.*

4. *Subtract the total rebate of interest from the amount due on the face to obtain the amount due at maturity.*

EXAMPLE. On a 6-month note for \$600, dated May 1 and discounted at 6% on that date, the following partial payments were indorsed: July 16, \$200; Sept. 1, \$250. What amount was due on the note at maturity?

Solution. Using the rebate method, proceed as follows:

The date of maturity of the note is 6 mo. after May 1; that is, Nov. 1.

Interest on \$200 from July 16 to Nov. 1 (108 da.)	\$3.60
Interest on \$250 from Sept. 1 to Nov. 1 (61 da.)	2.54
Total rebate of interest	<u>\$6.14</u>
Face of note	\$600.00
Sum of partial payments (\$200 + \$250)	450.00
Amount due on face of note	<u>\$150.00</u>
Total rebate of interest	6.14
Amount due on note at maturity	<u>\$143.86</u>

Banks are not at all uniform in handling such cases. Some banks rebate the interest at the time the payment is made, and other banks cancel the original note when a partial payment is made, adjust the interest, and take another note for the new principal. As suggested before, a student may consult a local bank to find out the method used.

EXERCISES

Using the rebate method, find the amount due at maturity on the following notes which were discounted on the date they were made:

FACE OF NOTE	DATE OF NOTE	TIME OF NOTE	RATE OF DISCOUNT	INDORSEMENTS (Payments)
1. \$6000	June 1	4 mo.	6%	{ July 1, \$1500 Aug. 5, \$650 Aug. 25, \$600 Sept. 1, \$900
2. \$10,000	March 1	90 da.	6%	{ March 31, \$2000 April 15, \$3000 May 10, \$3000
3. \$5000	June 1	90 da.	6%	{ July 16, \$1500 Aug. 6, \$1000
4. \$7500	July 1	90 da.	6%	{ July 16, \$1500 July 31, \$1000 Aug. 15, \$2500 Sept. 10, \$1000

5. The interest on a 90-day note for \$4000, dated May 1 and discounted at 7% on that date, is rebated at 6%. The note bears an indorsement of a partial payment of \$1200 on May 31 and of another payment of \$1800 on June 30. What amount is due on the note at maturity?

6. If interest is rebated at 5%, compute the amount due at maturity on a 60-day note for \$3000, dated April 1 and discounted at 6% on that date, and having the following partial payments indorsed on it: April 16, \$900; May 1, \$600; May 15, \$1000.

7. A 2-year note for \$8000, dated Jan. 1 and discounted on that date at 6%, has the following partial payments indorsed on it: July 1, \$1000; Jan. 1 (2d year), \$2000; June 1 (2d year), \$2500. If the interest is rebated at 6%, compute the amount due at maturity.

In Exs. 7 and 8 the term of the rebate should be figured in months.

Bond and mortgage houses use long-term notes similar to the one given above in some of their transactions.

8. A note for \$9000, dated Jan. 1, to run for 1 yr. 6 mo., is discounted at 6% on the day it is drawn. Compute the amount due at maturity if interest is rebated at the discount rate and the note bears the following indorsements: Nov. 1, \$2000; March 1 (2d year), \$2400; May 1 (2d year), \$3000.

Small loans. When a bank loans money it requires security from its borrowers in some form in order to protect its depositors. Sometimes a person of limited means who owns no real estate or other property of value which he can pledge as security for a loan needs to borrow money in order to meet some unforeseen expenditure.

The needs of this class of borrowers were formerly met by money lenders who charged very high rates of interest, sometimes as high as 200% per month. Generally, the only security which the borrower could give was that he had a weekly or monthly income, part of which he pledged to the money lender. Sometimes household goods or other articles were pledged as security.

Today many states have passed "Small Loan Laws," which license companies to make loans of not more than \$300 at rates varying from 1% to 5% per month, a usual rate being $3\frac{1}{2}\%$. Under these laws no fees or bonuses can be collected for making

the loan, simple interest only is allowed on unpaid balances of the principal, and interest cannot be collected in advance. While the rates seem high, they are necessary in order that the companies may secure money from investors to make these unsecured loans.

The method of computing the interest payments on such loans is illustrated in the following example.

EXAMPLE. A man borrowed \$60 from a licensed small loan agency and agreed to pay the loan in three monthly installments of \$20, with interest at $3\frac{1}{2}\%$ per month. Find the amount paid each month.

Solution. $\$60 \times 0.035 = \2.10 , first month's interest.

$\$2.10 + \$20 = \$22.10$, first monthly payment.

$\$60 - \$20 = \$40$, principal during second month.

$\$40 \times 0.035 = \1.40 , second month's interest.

$\$1.40 + \$20 = \$21.40$, second monthly payment.

$\$20 \times 0.035 = \0.70 , third month's interest.

$\$0.70 + \$20 = \$20.70$, final monthly payment.

The following exercises represent typical transactions with small loan agencies, which have proved more satisfactory in meeting the needs of small borrowers than did the dealings of the money lenders in years past.

EXERCISES

1. A man borrowed \$100 and agreed to repay the principal in four equal monthly installments with interest at $3\frac{1}{2}\%$ per month on the unpaid balances. Compute the amount of each payment during the four months.

2. A loan agency makes a loan of \$300, which is to be repaid in ten equal monthly installments with interest at $3\frac{1}{2}\%$ on the unpaid balances. Find the amount received by the agency at each of the ten payments.

3. A man borrows \$50 and agrees to repay it in weekly installments of \$10. Interest at the rate of 5% per month on unpaid balances is to be paid with each week's payment. Find the amount of each payment.

Count four weeks to a month.

4. A loan of \$250 is repaid in monthly installments of \$50, with interest at 3% per month on unpaid balances. If the interest due is paid each month, find the total amount of each month's payments.

5. Supply the missing items in the following record which shows the details of a loan made by a small loan agency:

NAME, James R. Smith								
ADDRESS, 67 West St.				AMOUNT OF LOAN, \$100				
DATE MADE, June 1				RATE OF INTEREST, $3\frac{1}{2}\%$ per month				
DATE	INTEREST PAID		PAID ON PRINCIPAL	TOTAL PAYMENT		BALANCE OF PRINCIPAL		PAYMENT RECEIVED BY
July 1			\$10					R. S. T.
Aug. 1			20					R. S. T.
Sept. 1			20					M. V. R.
Oct. 1			30					M. V. R.
Nov. 1			20					W. J. M.

ORAL REVIEW EXERCISES

Supply the missing items in the following:

	PRINCIPAL	RATE OF INTEREST	TIME	INTEREST	AMOUNT
1.	\$100	7%	1 yr.	?	?
2.	\$200	$4\frac{1}{2}\%$	1 yr.	?	?
3.	\$350	8%	2 yr.	?	?
4.	\$425	6%	1 yr.	?	?
5.	\$850	4%	?	?	\$884
6.	\$300	4%	?	?	\$312
7.	\$400	?	2 yr.	\$56	?
8.	\$250	8%	$2\frac{1}{2}$ yr.	?	?
9.	\$100	?	$2\frac{1}{2}$ yr.	\$15	?
10.	\$300	?	3 yr.	\$21	?
11.	?	4%	?	\$4	\$104

	PRINCIPAL	RATE OF INTEREST	TIME	INTEREST	AMOUNT
12.	?	6%	2 yr.	\$24	?
13.	?	5%	3 yr.	\$60	?
14.	?	?	1 yr.	\$30	\$780
15.	?	?	2 yr.	\$32	\$232
16.	\$750	4%	?	?	\$810
17.	\$1000	?	2 yr.	?	\$1180
18.	\$2000	5%	?	\$100	?

REVIEW EXERCISES

1. If 60 da. are allowed for payment on an article which sells for \$250 cash, what price should be quoted so as to allow for interest at 6% on the cash price during the credit period?

2. If money is worth 6%, what should be the 90-day credit price on an article which sells for \$3250 cash?

The expression "money is worth 6%" means that the seller figures that he should get 6% interest on his money during the period that he extends credit to the purchaser.

3. If money is worth 8%, find the 90-day credit price on an article which sells for \$1450 cash.

4. If the 60-day credit price of an article is \$5500, and money is worth 6%, find the equivalent cash price.

5. If money is worth 8%, find the equivalent cash price of an article on which a 30-day credit price of \$64.48 is quoted.

6. If money is worth 5%, which is the more economical plan of buying an article, a cash price of \$550 or a 60-day credit price of \$558?

7. On a bill of goods amounting to \$1250 a merchant receives a trade discount of 15% and 10%, and an additional cash discount of 2% if the bill is paid within 10 da. If he borrows the amount needed to pay the bill to the nearest \$50 for 60 da. at 6%, how much does he save by taking advantage of the cash discount?

8. In buying a home a man borrows \$5500 from his bank, and pledges the house as a security for the loan. If the rate of interest is $6\frac{1}{2}\%$, how much must he set aside each month to pay the annual interest charges?

9. A 90-day note for \$1450, dated May 1 and bearing interest at 5%, is discounted on June 5 at 6%. Find the proceeds.

10. Write out rules for computing interest at 4% similar to the rules given for the banker's 60-day method at 6%.

Exact interest. Since $\frac{6}{360}$ is a larger fraction than $\frac{6}{365}$, it is obvious that when interest is computed by counting 360 da. as a year the charge is higher than if the full 365 da. were used. **Exact interest** is simple interest computed by using the exact number of days in the year as a basis.

The United States government uses exact interest, city governments and banks use it when large amounts are involved, and its use is growing among business men. Since the exact time is used, it is the only equitable method of computing interest, but the greater convenience in computing by the 360-day method accounts for the widespread use of the latter method.

In computing exact interest for any fractional part of a year it is necessary to find that fractional part of the year's interest which the given number of days is of 365 da.; that is,

$$\text{exact interest} = \text{principal} \times \text{annual rate} \times \frac{\text{given number of days}}{365 \text{ da.}}$$

A leap year (366 da.) need not be considered in any of the following exercises.

EXAMPLE. Find the exact interest at $7\frac{1}{2}\%$ on a loan of \$432.50 from June 4 to Sept. 10.

Solution. By the table on page 272, the time from June 4 to Sept. 10 is 98 da.

$$\text{Hence} \quad \text{exact interest} = \$432.50 \times 7\frac{1}{2}\% \times \frac{98}{365}$$

The work should be written in the following form, convenient for cancellation, with the rate expressed as hundredths:

$$\text{Then } \$432.50 \times \frac{8.65}{100} \times \frac{3}{365} \times \frac{49}{73} = \frac{\$1271.55}{146} = \$8.709 +.$$

Hence the exact interest from June 4 to Sept. 10 is \$8.71.

EXERCISES

Find the exact interest on:

1. \$565.50 for 95 da. at 6%.
 2. \$496.25 for 75 da. at 7%.
 3. \$573.50 for 45 da. at $6\frac{1}{2}\%$.
 4. \$475.25 for 70 da. at $5\frac{1}{4}\%$.
 5. \$222.50 for 85 da. at $6\frac{1}{4}\%$.
 6. \$525.50 for 96 da. at 7%.
 7. \$254.50 for 58 da. at 5%.
 8. \$422.25 for 64 da. at $5\frac{1}{2}\%$.
 9. \$675.50 for 25 da. at $4\frac{1}{4}\%$.
 10. \$14,964 for 42 da. at $4\frac{3}{4}\%$.
 11. \$17,500 for 39 da. at $5\frac{1}{4}\%$.
 12. \$2150 for 51 da. at $5\frac{3}{4}\%$.
 13. \$3675 for 73 da. at $6\frac{3}{4}\%$.
 14. \$4250 for 82 da. at $6\frac{1}{4}\%$.
 15. \$2755 for 50 da. at $4\frac{1}{2}\%$.
 16. \$1256 for 25 da. at $3\frac{1}{2}\%$.
17. Find the exact interest on \$425 at 6% from May 11 to Sept. 3.
18. Find the exact interest on \$1250 at 5% from Jan. 10 to May 15 of the following year.

Follow the general rule for interest; that is, find the interest for the full year and then for the required fractional part.

19. Find the exact interest on \$2450 at $4\frac{3}{4}\%$ from Dec. 15 to March 8 of the second year following.

20. How much difference is there between the exact interest on \$250 at 7% for 93 da. and the simple interest computed by the banker's 60-day method under the same conditions?

Periodic interest. When a sum is borrowed for a long time it is generally specified that the interest on the principal shall be paid at definite intervals. If, in addition to the interest on the

principal, interest is charged on any unpaid installments of this interest, such an arrangement is called **periodic interest**.

Periodic interest is technically illegal in some states, but it is sometimes actually collected through the use of separate interest-bearing notes for the unpaid installments of the interest on the principal.

EXAMPLE. On a loan for \$1500, made for 3 yr. at 6%, the interest was to be paid semiannually. If no payments of interest were made, and interest at 6% on the unpaid installments was collected through the use of separate notes, find the periodic interest due at maturity.

Solution. The installment of interest due each 6 mo. is

$$^{\circ} \$15 \times 3 = \$45.$$

The first unpaid installment of interest draws interest at 6% for 2 yr. 6 mo., the second for 2 yr., and so on.

Interest on \$1500 (principal) for 3 yr. at 6%	\$270.00
Interest on \$45 (1st payment) for 2½ yr. at 6%	6.75
Interest on \$45 (2d payment) for 2 yr. at 6%	5.40
Interest on \$45 (3d payment) for 1½ yr. at 6%	4.05
Interest on \$45 (4th payment) for 1 yr. at 6%	2.70
Interest on \$45 (5th payment) for 6 mo. at 6%	1.35
Periodic interest due at maturity	<u>\$290.25</u>

To save computing the interest separately for each payment, it is easier to begin with the last payment in such a case, compute the interest for 6 mo., double it for 1 yr., and so on. The items are then totaled as above.

EXERCISES

On the following loans the interest was to be paid as specified, but no payments were made and interest at the same rate was collected by separate notes on the unpaid installments. Find the periodic interest due at maturity:

1. \$450 for 3 yr. at 6%, interest due annually.
2. \$1250 for 3 yr. at 7%, interest due annually.
3. \$1725 for 2 yr. at 5%, interest due semiannually.

4. \$2250 for 1 yr. 6 mo. at $5\frac{1}{2}\%$, interest due quarterly.
5. \$3160 for 2 yr. at $4\frac{3}{4}\%$, interest due quarterly.
6. \$1725 for $2\frac{1}{2}$ yr. at $7\frac{1}{4}\%$, interest due semiannually.

COMPOUND INTEREST

Compound interest. When the unpaid interest is added to the principal at definite intervals, and interest is paid on this new principal, the interest is said to be **compounded**. Interest charged in this way is called **compound interest**.

Interest may be compounded annually, semiannually, quarterly, or even monthly. Savings banks commonly pay compound interest on sums deposited with them, and it is used in computations relating to sinking funds and life insurance.

In most states the charging of compound interest is not sanctioned by law, but if mutually agreed upon, it does not constitute usury.

Compound interest is not essentially unfair, since the same result can be obtained by taking the interest payments when due and reinvesting them at the same rate.

The method of computing compound interest is shown in the following example.

EXAMPLE. If the interest is compounded semiannually at $4\frac{1}{2}\%$, find the compound interest on \$4500 for 2 yr.

Solution. A semiannual rate of $2\frac{1}{4}\%$ is equivalent to an annual rate of $4\frac{1}{2}\%$.

Principal for 1st half year	\$4500.00
Interest on 1st principal ($\$4500 \times 0.02\frac{1}{4}$)	101.25
Principal for 2d half year	<u>\$4601.25</u>
Interest on 2d principal ($\$4601.25 \times 0.02\frac{1}{4}$)	103.53
Principal for 3d half year	<u>\$4704.78</u>
Interest on 3d principal ($\$4704.78 \times 0.02\frac{1}{4}$)	105.86
Principal for 4th half year	<u>\$4810.64</u>
Interest on 4th principal ($\$4810.64 \times 0.02\frac{1}{4}$)	108.24
Compound amount at end of 2d year	<u>\$4918.88</u>
Original principal	4500.00
Compound interest for 2 yr.	<u>\$418.88</u>

EXERCISES

Find the compound interest on :

1. \$250 for 2 yr. at 6%, compounded annually.
2. \$425 for 2 yr. at 6%, compounded semiannually.
3. \$385 for 2 yr. 6 mo. at 7%, compounded semiannually.
4. \$750 for 1 yr. at 4%, compounded quarterly.

Find the amount of :

5. \$1250 for 1 yr. 3 mo. at 5%, compounded quarterly.
6. \$1500 for 1 yr. 8 mo. at $6\frac{1}{2}\%$, compounded semiannually.
7. \$1650 for 1 yr. 9 mo. at 7%, compounded semiannually.

Compound-interest table. Since the computation of compound interest is an involved operation, it is usually determined by the use of tables which show the amount of \$1 compounded at different rates. Part of such a table is given on page 292.

The following examples illustrate the use of the table.

EXAMPLES. 1. Find the amount of \$1 at 6%, compounded annually for 6 yr.

Solution. Look for 6 in the column headed "Year," and follow along the horizontal line to the column headed "6%." The number thus found is 1.418519. Hence the amount of \$1 at 6%, compounded annually for 6 yr., is \$1.42, to the nearest cent.

2. Find the amount of \$10 at 8%, compounded semiannually for 8 yr.

Solution. In the 8 yr. there will be 16 interest periods. Hence 8% compounded semiannually for 8 yr. is the same as 4% compounded annually for 16 yr. Using the table as in Ex. 1, the amount of \$1 for this time is found to be \$1.87298. Hence the amount of \$10 at 8%, compounded semiannually for 8 yr., is \$18.73, to the nearest cent.

The amount of any principal at compound interest is thus found by multiplying the amount of \$1 for the given rate and period by the number of dollars in the principal. If the compound interest alone is desired, the principal should be subtracted from the compound amount.

The table given below is not complete, but is sufficient for the exercises which follow. Complete compound-interest tables make it possible to compute the interest at any rate for any period, and compounded for periods of different lengths.

AMOUNT OF \$1 COMPOUNDED ANNUALLY								
YEAR	2%	4%	4½%	5%	6%	7%	8%	YEAR
1	1.02000	1.04000	1.04500	1.05000	1.060000	1.070000	1.080000	1
2	1.04040	1.08160	1.09203	1.10250	1.123600	1.144900	1.166400	2
3	1.06121	1.12486	1.14117	1.15763	1.191016	1.225043	1.259712	3
4	1.08243	1.16986	1.19252	1.21551	1.262477	1.310796	1.360489	4
5	1.10408	1.21665	1.24618	1.27628	1.338225	1.402551	1.469328	5
6	1.12616	1.26532	1.30226	1.34010	1.418519	1.500730	1.586874	6
7	1.14869	1.31593	1.36086	1.40710	1.503630	1.605781	1.713824	7
8	1.17166	1.36857	1.42210	1.47746	1.593848	1.718186	1.850930	8
9	1.19509	1.42331	1.48610	1.55133	1.689479	1.838459	1.999004	9
10	1.21899	1.48024	1.55297	1.62889	1.790847	1.967151	2.158925	10
11	1.24337	1.53945	1.62285	1.71034	1.898298	2.104852	2.331639	11
12	1.26824	1.60103	1.69588	1.79586	2.012196	2.252191	2.518170	12
13	1.29361	1.66507	1.77220	1.88565	2.132928	2.409845	2.719623	13
14	1.31948	1.73168	1.85194	1.97993	2.260904	2.578534	2.937193	14
15	1.34587	1.80094	1.93528	2.07893	2.396558	2.759031	3.172169	15
16	1.37279	1.87298	2.02237	2.18287	2.540351	2.952163	3.425942	16
17	1.40024	1.94790	2.11338	2.29202	2.692772	3.158815	3.700018	17
18	1.42825	2.02582	2.20848	2.40662	2.854339	3.379932	3.996019	18
19	1.45681	2.10685	2.30786	2.52695	3.025599	3.616527	4.315701	19
20	1.48595	2.19112	2.41171	2.65330	3.207135	3.869684	4.660957	20

ORAL EXERCISES

Use the compound-interest table in the following exercises:

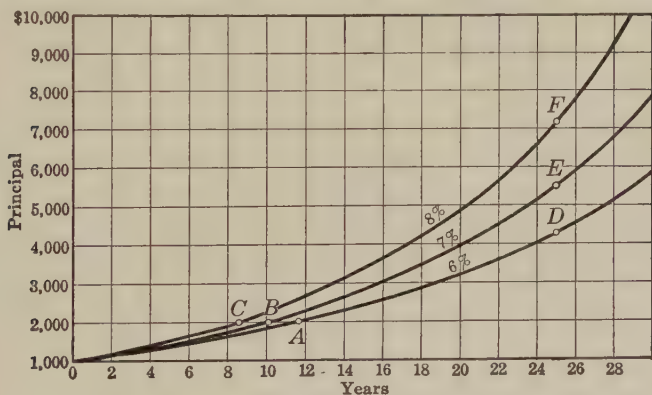
1. What is the amount of \$1 for 7 yr., compounded annually at 5%? at 4%? at 4½%? at 7%?
2. What is the interest on \$10 for 10 yr., compounded annually at 2%? at 4%? at 6%? at 8%?
3. In how many years will \$1 practically double itself if compounded annually at 4%? at 4½%? at 6%? at 7%? at 8%?
4. What is the amount of \$100 for 14 yr., compounded annually at 4½%? at 5%? at 6%? at 8%?

EXERCISES

Using the table on page 292, find the compound interest on:

1. \$150 at 6%, compounded annually for 16 yr.
2. \$455 at $4\frac{1}{2}\%$, compounded annually for 11 yr.
3. \$1275 at 8%, compounded semiannually for 5 yr.
4. \$1675 at 9%, compounded semiannually for 3 yr.
5. \$1250 at 8%, compounded quarterly for 2 yr.; for 4 yr.
6. \$1625 at 8%, compounded semiannually for 3 yr. 6 mo.
7. For how long will \$1000 have to be placed at interest at 6%, compounded annually, to amount to approximately \$3000?
8. A man invests \$500 at $4\frac{1}{2}\%$, compounded annually, on the day that his son is 5 yr. old. How much will the investment amount to on the son's twenty-first birthday?
9. At what rate of interest, compounded semiannually, must \$1000 be placed so that it shall amount to \$2191.12 in 10 yr.?

Growth of money. While the compound-interest table on page 292 shows how money increases when invested at compound



GROWTH OF MONEY AT INTEREST COMPOUNDED SEMIANNUALLY

interest, the graph above shows more plainly the growth of money at rates of 6%, 7%, and 8%, compounded semiannually. The student should compare the curves in the graph with the amounts given in the table.

By referring to the graph it is seen that \$1000 invested at 6%, compounded semiannually, doubles itself in a little less than 12 yr.; to be exact, in 11 yr. 264 da. (see point *A* of graph). The same amount invested at 7%, compounded semiannually, doubles itself in 10 yr. 27 da. (point *B*), and at 8%, compounded semiannually, in 8 yr. 304 da. (point *C*).

During a period of 25 yr., \$1000 compounded semiannually at 6% amounts to \$4383.91 (point *D*); at 7%, to \$5585.03 (point *E*); and at 8%, to \$7106.82 (point *F*).

ORAL EXERCISES

Referring to the graph on page 293, answer the following:

1. In how many years (approximately) will \$1000 become \$3000 if compounded semiannually at 6%? at 7%? at 8%?
2. In how many years (approximately) will \$2000 become \$10,000 if compounded semiannually at 6%? at 7%? at 8%?
3. How many times (approximately) has the original investment been increased at each of the three rates in 20 yr.?
4. Consider Ex. 3 at the end of 28 yr.

Sinking fund. When a large sum of money is borrowed to carry out some extensive public improvement, such as the laying out of a municipal park system, it is necessary to adopt a plan whereby the debt shall be met when it is due. If at regular intervals a definite sum is invested at compound interest, the accumulation of the amounts invested and the interest will eventually meet the debt. Such an accumulated sum which is used to meet some future obligation is known as a **sinking fund**.

The general principle of a sinking fund is that of compound interest, but it differs from the ordinary application of compound interest in that at each period the invested principal is increased not only by the interest due but also by the addition of the new installment to be invested according to the plan of the fund.

Sinking funds are used in making provision for the payment of both public and corporation debts at future dates, or for the replacing of property when it is depreciated. The computations in connection with sinking funds are too involved for consideration in schools, but the following table and exercises will help the students to appreciate how a fund grows when regularly increased and invested at compound interest.

The following table shows how \$1, invested on the first day of each year at interest compounded annually, will accumulate:

SINKING-FUND TABLE					
YEAR	2%	4%	4½%	5%	6%
1	1.02	1.04	1.045	1.05	1.06
2	2.0604	2.1216	2.137025	2.1525	2.1836
3	3.121608	3.246464	3.278191	3.310125	3.374616
4	4.204040	4.416323	4.470710	4.525631	4.637093
5	5.308121	5.632975	5.716892	5.801913	5.975319
6	6.434283	6.898294	7.019152	7.142008	7.393838
7	7.582969	8.214226	8.380014	8.549109	8.897468
8	8.754628	9.582795	9.802114	10.026564	10.491316
9	9.949721	11.006107	11.288209	11.577893	12.180795
10	11.168715	12.486351	12.841879	13.206787	13.971643
11	12.412090	14.025805	14.464032	14.917127	15.869941
12	13.680332	15.626838	16.159913	16.712983	17.882138
13	14.973938	17.291911	17.932109	18.498632	20.015066
14	16.293417	19.023588	19.784054	20.578564	22.275970
15	17.639285	20.824531	21.719337	22.657492	24.672528
16	19.012071	22.697512	23.741707	24.840366	27.212880
17	20.412312	24.645413	25.855084	27.132385	29.905653
18	21.840559	26.671229	28.063562	29.539004	32.759992
19	23.297370	28.778078	30.371423	32.065954	35.785591
20	24.783317	30.969201	32.783137	34.719252	38.992727

By referring to the table in the same way as for the compound-interest table on page 292, it is seen that an annual deposit of \$1 at 5%, compounded annually, will accumulate to \$22.657492,

or \$22.66, to the nearest cent, in 15 yr. Hence an annual deposit of \$500 under the same conditions would accumulate to $\$22.657492 \times 500$, or \$11,328.75, to the nearest cent, and similarly for any other sum.

ORAL EXERCISES

State the accumulated value of the following:

	ANNUAL DEPOSIT	INTEREST COMPOUNDED ANNUALLY AT	TIME
1.	\$1	2%	5 yr.
2.	\$10	$4\frac{1}{2}\%$	15 yr.
3.	\$100	4%	14 yr.
4.	\$100	6%	18 yr.
5.	\$1000	5%	17 yr.

6. If money can be invested at 5%, compounded annually, how large an annual payment must be made so that a sinking fund shall amount to \$347,192.52 at the end of 20 yr.?

7. A sinking fund is established to pay off the money borrowed to erect a school building which cost approximately \$125,000. If the annual payments are invested at 4%, compounded annually, how much must be set aside each year so that the whole indebtedness can be canceled at the end of 10 yr.?

8. To take care of the indebtedness caused by extending their plant a corporation sets aside \$10,000 annually towards a sinking fund. If the money is invested at 5%, compounded annually, how large a fund will they have at the end of 14 yr.?

SAVINGS-BANK ACCOUNTS

Nature of savings banks. Banks chartered by a state for the purpose of receiving savings as deposits are called **savings banks**. An account at such a bank does not carry with it the privilege of drawing checks (see page 26) against the amount on deposit, but

the deposits (or any part of them) are payable to the depositor, or to his order, upon demand.

Sometimes savings banks require that notice of intention to withdraw money be given some days in advance.

The amount that may be deposited by any one person in such a bank is generally limited to less than \$5000, and the regulations governing such banks are carefully prescribed by the states so as to insure safety to the depositors.

Dividends. Savings banks invest the money deposited with them and thus secure an income from which they pay interest to the depositor according to the amount of his account. Such payments of interest are called **dividends**.

Dividends are either paid in cash or added to the amount on deposit at stated intervals—generally semiannually or quarterly, the former being the more common. In most banks only such money as is on deposit at the beginning of a dividend period draws interest.

Thus, if a bank pays dividends semiannually, on Jan. 1 and July 1, money shall be on deposit on these dates in order to draw interest. Since the interest is added to the principal, savings banks pay compound interest.

The days on which money begins to draw interest under the rules of the bank are called **interest days**. Sometimes banks allow interest to begin on a specified day of the month following the deposit, but credit interest only semiannually or quarterly.

Thus a bank which pays dividends quarterly may permit interest to begin on Aug. 5 on a deposit made on July 10, but credits interest to the depositor's account only on Jan. 1, April 1, July 1, and Oct. 1.

A common practice is to pay interest only on the smallest amount on deposit during any dividend period. In computing interest, savings banks disregard any fraction of a dollar; that is, they pay interest only on even dollars.

Such a method is illustrated in the form on page 298, which shows the record of James M. Smith's account at the First Savings Bank. This bank pays dividends semiannually, on Jan. 1 and

July 1, at the rate of 4% per year, and requires that money shall be on deposit on these dates to draw interest. Interest is computed on the smallest balance on deposit during the dividend period.

FIRST SAVINGS BANK									
In Account with <i>James M. Smith</i>									
Date		Withdrawals		Interest		Deposits		Balance	
<i>19--</i>									
<i>Aug.</i>	<i>20</i>					<i>300</i>	<i>—</i>	<i>300</i>	<i>—</i>
<i>Sept.</i>	<i>13</i>	<i>30</i>	<i>—</i>					<i>270</i>	<i>—</i>
<i>Nov.</i>	<i>22</i>	<i>25</i>	<i>—</i>					<i>245</i>	<i>—</i>
<i>Dec.</i>	<i>17</i>	<i>80</i>	<i>—</i>					<i>165</i>	<i>—</i>
<i>19--</i>									
<i>Feb.</i>	<i>10</i>	<i>24</i>	<i>50</i>					<i>140</i>	<i>50</i>
<i>"</i>	<i>18</i>					<i>150</i>	<i>—</i>	<i>290</i>	<i>50</i>
<i>April</i>	<i>2</i>	<i>69</i>	<i>55</i>					<i>220</i>	<i>95</i>
<i>"</i>	<i>20</i>	<i>10</i>	<i>—</i>					<i>210</i>	<i>95</i>
<i>May</i>	<i>10</i>	<i>32</i>	<i>—</i>					<i>178</i>	<i>95</i>
<i>July</i>	<i>1</i>	<i>Div. *58</i>		<i>2</i>	<i>80</i>			<i>181</i>	<i>75</i>

SAVINGS-BANK ACCOUNT

Smith opened his account on Aug. 20 by depositing \$300, which was credited in the column headed "Deposits" and entered in the "Balance" column. On Sept. 13, Nov. 22, and Dec. 17 he made withdrawals of money as shown in that column, and the "Balance" column was correspondingly adjusted.

A dividend of interest for 6 mo. would be payable on Jan. 1, but Smith had no money on deposit on the preceding July 1, so that no interest was due him at that time.

On Feb. 10, April 2, April 20, and May 10 he made other withdrawals and on Feb. 18 he made a deposit of \$150. The "Balance" column shows the proper deduction or addition for each of these transactions.

A second dividend of interest for 6 mo. was due on July 1, and between Jan. 1 and July 1 the smallest balance on deposit was \$140.50, that of Feb. 10. Hence, disregarding the fractional part of a dollar, the interest due is $\$140 \times 0.02$, or \$2.80. The bank credited this as Dividend #58 to Smith's account, as shown by the last entry. Hence Smith's balance on July 1 was \$181.75.

EXERCISES

1. The dividend rate of the People's Savings Bank is 4% per year, payable on Jan. 1 and July 1 on deposits on record of these dates, and interest is computed on the smallest balance during the dividend period. Copy and complete the following account:

PEOPLE'S SAVINGS BANK							
IN ACCOUNT WITH				George W. Stone			
DATE		WITHDRAWALS		INTEREST		DEPOSITS	
July	1					1000	1000
"	8					200	1200
Sept.	12					200	?
Oct.	24					100	?
Jan.	1	Div. # 46		?			?

2. The account of Fred R. Roe at the People's Savings Bank in Ex. 1 shows the following deposits: Feb. 16, \$300; Feb. 21, \$50; March 24, \$50; April 21, \$50; April 23, \$200; July 20, \$50; Aug. 11, \$50; Aug. 25, \$50; Sept. 6, \$50; Sept. 21, \$50; Oct. 21, \$45. The account also shows the following withdrawals: May 18, \$50; July 1, \$100; Oct. 11, \$20. Find his balance on Jan. 1 following the last deposit.

Rule a form similar to the account on page 298, record the different items, and compute the dividends due according to the conditions given in Ex. 1.

3. On Jan. 1, April 1, July 1, and Oct. 1 the Central Savings Bank credits interest at 4% per year on the smallest balance on deposit during an interest period, provided the money has been on deposit since the first day of the period. Copy and complete the following account:

CENTRAL SAVINGS BANK									
IN ACCOUNT WITH Dora W. Stevenson									
DATE		WITHDRAWALS		INTEREST		DEPOSITS		BALANCE	
19—									
Oct.	7					700		700	00
"	29					300		?	
Dec.	31					58	06	?	
19—									
Jan.	1	Div. # 69		?				?	
"	24	58	06					?	
"	26					120		?	
March	1	50						?	
"	3					243	96	?	
April	1	Div. # 70		?				?	
"	10	1000						?	
May	2	88	90					?	
"	10	50						?	
July	1	Div. # 71		?				?	

4. The dividend rate of the Union Savings Bank is 4% per year. The bank credits interest quarterly on Jan. 1, April 1, July 1, and Oct. 1, computing the interest monthly on the smallest balance on deposit during the preceding month. The account of Frank R. Black at this bank shows the following deposits: June 12, \$500; Aug. 8, \$75; Nov. 7, \$60; Dec. 20, \$45. The account shows the following withdrawals: July 5, \$25; Sept. 10, \$60; Nov. 17, \$100. Find his balance on Jan. 1 following the last deposit.

The student should rule a form and begin as follows:

DATE		WITHDRAWALS		INTEREST		DEPOSITS		BALANCE	
June	12					500		500	
July	5	25						475	
Aug.	8					75		550	
Sept.	10	60						490	
Oct.	1	Div. # 64		4	79			494	79

The deposit on June 12 draws no interest for June since it was not on deposit on June 1. Hence no interest is credited on July 1. Due to the withdrawal on July 5, the smallest balance for July is \$475, on which the interest rate is $\frac{1}{3}\%$ per month. Hence the July interest is $\$475 \times 0.00\frac{1}{3}$, or \$1.58. The smallest balance for August is also \$475, so that the August interest is likewise \$1.58. The deposit on Aug. 8 draws no interest until Sept. 1. Due to the withdrawal on Sept. 10, the smallest September balance is \$490, on which the interest is \$1.63. The sum of these three interest items is \$4.79, which is credited on Oct. 1 and added to the amount on deposit.

The student should now complete the solution.

5. The account of Myra J. Peterson with the Union Savings Bank in Ex. 4 shows the following deposits: July 1, \$125; Oct. 5, \$100; Oct. 15, \$100. The account also shows the following withdrawals: Sept. 6, \$50; Nov. 2, \$50. Find her balance on April 1 following the last withdrawal.

Postal savings bank. In connection with the postal service the United States government conducts a savings bank known as the **postal savings bank**. Any person ten years of age or over may open one account in his own name and may make deposits on this account at any time until the deposits amount to \$2500 exclusive of accumulated interest. Fractional parts of a dollar are not accepted for deposit.

Postal savings certificates. The post office at which the deposit is made acknowledges its receipt by issuing a **postal savings certificate** for the amount of the deposit. These certificates are issued in the following denominations: \$1, \$2, \$5, \$10, \$20, and \$50. Each certificate bears the name of the depositor and the number of his account, and is not transferable.

Since fractional parts of a dollar are not accepted for deposit, a person may save amounts less than \$1 by purchasing **postal savings stamps** at 10¢ each. A postal savings card is furnished to which ten of these stamps may be affixed. When the card is filled it may be turned in at the post office as a deposit of \$1, or may be redeemed in cash.

Withdrawals may be made from an account by surrendering to the post office where the deposits were made certificates of sufficient value to equal the amount to be withdrawn.

Interest. The government pays interest on postal savings deposits at 2% per year. Interest begins on the first day of the month following that in which the postal savings certificate is issued, and is due and payable at the end of each full year thereafter. No interest is paid for fractional parts of a year, and unless interest is withdrawn and redeposited it does not draw interest.

Postal savings bonds. A depositor may exchange all or part of his deposits for postal savings bonds which are issued in denominations of \$20, \$100, and \$500, and bear interest at $2\frac{1}{2}\%$ per year. Such exchange must be made as of Jan. 1 or July 1 of any year, and the interest on the bonds is payable on these dates.

ORAL EXERCISES

1. Frank Smith deposits \$100 in a postal savings bank on April 19. When does interest commence on this deposit? When is interest first payable on this deposit? How much interest will he receive on that date?

2. Marie Smith is a depositor in a postal savings bank. If she desires to withdraw \$500, how many 20-dollar certificates must she surrender to the post office?

3. A man deposits \$67 in a postal savings bank. Using as large denominations as possible, state the number and denomination of the certificates which he receives.

4. On Jan. 1 a man exchanges his deposit of \$1000 in a postal savings bank for a postal savings bond. How much interest is due him on July 1 of the same year?

5. How much interest will the man in Ex. 4 receive each year from his bond?

EXERCISES

1. Clara Jones made the following deposits in a postal savings bank: March 2, 1924, \$250; March 19, 1924, \$100; July 16, 1924, \$150; Sept. 12, 1924, \$100. How much interest was due her on May 1, 1925?

2. If no withdrawals of interest are made in Ex. 1, how much can she receive on Oct. 1, 1925, by surrendering all her certificates?

3. On Jan. 1, 1924, a man has \$780, which has been on deposit in a postal savings bank for a full year. How much interest is due him on that date?

4. If the man in Ex. 3 exchanges his deposit for postal savings bonds, how much more interest will he receive during the next year than he did on Jan. 1, 1924?

5. Beginning Jan. 31, 1924, a man deposits \$50 in a postal savings bank on the last day of each month for a full year. If no withdrawals of interest or principal are made, how much interest is due him on Jan. 1, 1926? on Feb. 1, 1926?

DOMESTIC EXCHANGE

Domestic exchange. If James A. Snow, a New York merchant, purchases goods from Dunbar & Co., a Chicago wholesale house, it would be both inconvenient and risky if Snow had to send money to Chicago to settle his account every time that he made a purchase. Through the use of checks, bank drafts, and other forms which will be discussed later, modern business now has a safe and convenient method of paying bills at a distance. Such a process of settling debts or transmitting credits without sending cash is known as **exchange**, and when effected between two points in this country is known as **domestic exchange**.

NEW YORK, <u>March 6, 19</u> —	No. <u>41</u>
UNION NATIONAL BANK	
Pay to the order of <u>Dunbar & Co.</u>	<u>\$906.50</u>
<u>Nine hundred six and 50/100</u>	Dollars
<u>James A. Snow</u>	

CHECK

Check. A **check** is a written order upon a bank by a depositor for the payment of his money to himself or to a specified person or firm. The form above illustrates the check that James A. Snow,

mentioned above, might make out to Dunbar & Co. in payment of a purchase amounting to \$906.50.

In this check, Dunbar & Co. is the **payee**, the one to whom the money is to be paid. Snow mails the check to them, and when they receive it they may indorse the check across the back as follows:

Dunbar & Co.

Such an indorsement is known as a **blank indorsement**, since it no longer specifies any person to whom money is to be paid. Any person holding this check may present it for cashing. Business houses often use a stamp for indorsing their checks, which in this case might read as follows:

FOR DEPOSIT ONLY
TO THE CREDIT OF
DUNBAR & CO.

Such an indorsement is known as a **limited indorsement**, since the check cannot be cashed but must be placed to the credit of Dunbar & Co. as a deposit.

After it is indorsed Dunbar & Co. deposit the check at the bank with which they do business, the Webster National Bank of Chicago, who credit the amount to Dunbar & Co.'s account.

The Webster National Bank of Chicago handles all their New York business through the Scott National Bank of New York. They accordingly send the check to this bank, making the following additional indorsement:

PAY TO THE ORDER OF THE
SCOTT NATIONAL BANK
NEW YORK CITY.
INDORSEMENT GUARANTEED
MARCH 9, 19—
WEBSTER NATIONAL BANK
CHICAGO, ILL.
H. A. DOE, CASHIER

The Scott National Bank sends the check to the clearing house in New York, making the following additional indorsement:

PAY TO THE ORDER OF
ANY BANK, BANKER, OR
TRUST CO. ALL PRIOR IN-
DORSEMENTS GUARANTEED.
SCOTT NATIONAL BANK,
MARCH 12, 19—
A. G. ROE, CASHIER

The details of clearing-house procedure will be explained later.

The check now returns to the Union National Bank, and the amount of the check is charged by them against the account of James A. Snow, the **maker** of the check.

Bank draft. Personal checks like the one above are not always accepted, as the maker of the check may not have sufficient funds at his bank to pay the check when it is presented, or he may be unknown to the person to whom the check is sent.

ATLANTA, GA., <u>March 1, 19—</u> No. <u>83</u>	
CITY NATIONAL BANK	
Pay to the order of <u>A. M. Taylor</u>	<u>\$160.⁰⁰</u>
EXACTLY ONE HUNDRED SIXTY DOLLARS NO CENTS	
Dollars	
To Franklin National Bank New York	<u>D. McKingley</u> Cashier

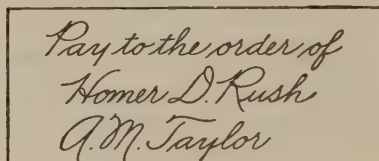
BANK DRAFT

Banks often keep deposits in other banks located in different cities, and issue checks against these accounts. A check of one bank upon another is known as a **bank draft**. This form of making remittances is often used by business men, since it is certain that the check will be paid when presented.

The form above shows a bank draft which A. M. Taylor, of Atlanta, Ga., bought of the City National Bank, in order to remit \$160 to Homer D. Rush of New York City.

If Taylor had an account at the City National Bank, the amount of the draft was charged against it; if not, he paid cash for the draft. The draft is drawn in favor of A. M. Taylor on the Franklin National Bank of New York, where the City National Bank has a deposit.

Taylor then mails the draft to Rush in New York City, indorsing it as follows:



Pay to the order of
Homer D. Rush
A. M. Taylor

Such an indorsement is known as a **full indorsement**, and no one without the indorsement of Rush can cash the draft.

The draft on page 305 might have been made payable to Rush. If, however, the draft became detached from the letter accompanying it, there might be difficulty in finding out from whom it was sent. This difficulty is avoided by making the draft payable to Taylor and by his indorsing it to Rush.

EXERCISES

1. J. E. Wood, of New York, draws a check on the Second National Bank to the order of H. R. May, dated today, for \$150. Write the check and indorse it in blank.

2. H. R. May, of St. Louis, draws a check on the Unity National Bank to the order of F. E. Jones for \$112.25, dated today. Write the check and indorse it to the order of J. B. Merrill.

3. W. B. Elliott, of Boston, draws a check on the Granite National Bank, to the order of W. E. Sherrill for \$649.79, dated today. Write the check and indorse it for deposit only.

4. E. L. Watters, of Janesville, Wis., buys a draft on San Francisco from the First National Bank, payable to himself. The draft is drawn on the West Coast National Bank and is dated today. Write the bank draft and indorse it to the order of Howard A. Milton.

In Exs. 4, 5, and 6 use your name as cashier.

5. J. E. Toothaker, of Tampa, Fla., buys a draft on New York for \$168.75 from the Traders' National Bank, payable to himself. The draft is drawn on the Scott National Bank and is dated today. Write the draft and indorse it to the order of Kirby & Son.

6. E. D. Winslow, of Austin, Tex., buys a draft on Boston from the Ranchers' National Bank for \$132.25, payable to himself. The draft is drawn on the Granite National Bank and is dated today. Write the bank draft and indorse it to D. L. Cox.

7. On checks drawn on distant places, banks sometimes charge a small collection fee on account of the expense of collecting the checks. A man deposits the following checks at his bank, and on each starred amount a collection charge of $\frac{1}{10}\%$ is made: \$165.25; *\$234.15; \$155.67; *\$215.67; \$68.45; *\$456.70; \$106.50; *\$202.96; \$128.30; \$145.12; *\$101.24; \$54.66. Deducting the collection charges, find the amount placed to the man's credit at the bank.

8. William H. Stewart deposits the following checks at his bank: *\$212.56; \$156.23; \$36.75; *\$86.75; \$198.76; \$335.42; *\$413.22; \$9.49; *\$79.42; \$566.43; *\$654.32; *\$228.55; \$12.27; *\$47.56; *\$765.90; *\$203.44; *\$198.54; \$38.64; *\$63.25; \$199.01. On each starred amount a collection charge of $\frac{1}{10}\%$ is made, with a minimum charge of 10¢ in case the collection fee is less than this amount. Find the amount placed to Stewart's credit.

Sight draft. Sometimes when a business man wishes to collect an amount due him, he "draws" on the person who owes him the money.

Suppose, for example, that A. M. Drake, of Scranton, Pa., owes Burke & Co., of Trenton, N.J., \$250.75 for goods sold with the understanding that the seller is to draw on the purchaser for the amount of the invoice. Burke & Co. make out the draft, known as a **sight draft**, which is shown on page 308.

Burke & Co. indorse the draft and send it to their bank in Trenton, and the bank forwards it to a bank in Scranton. This bank presents the draft to Drake, and if he honors and pays it,

the bank remits the amount to Burke's bank in Trenton. The Trenton bank then notifies Burke & Co. that the draft is collected and credits the proceeds to the account of Burke & Co.

<u>\$ 250.75</u>	<u>TRENTON, N. J.,</u>	<u>May 1, 19--</u>				
-----At sight-----						
-----pay to the order of-----						
-----Ourselves-----						
-----Two hundred fifty and 75/100-----						
-----Dollars						
<i>Value received and charge to account of</i>						
<table style="width: 100%; border: none;"> <tr> <td style="width: 50%; border-bottom: 1px solid black;">To <u>A. M. Drake</u></td> <td rowspan="2" style="width: 5%; border: none; vertical-align: middle;">}</td> <td rowspan="2" style="width: 45%; border-bottom: 1px solid black; vertical-align: middle;"><i>Burke & Co</i></td> </tr> <tr> <td style="border-bottom: 1px solid black;">No. <u>17</u> <u>Scranton, Pa.</u></td> </tr> </table>			To <u>A. M. Drake</u>	}	<i>Burke & Co</i>	No. <u>17</u> <u>Scranton, Pa.</u>
To <u>A. M. Drake</u>	}	<i>Burke & Co</i>				
No. <u>17</u> <u>Scranton, Pa.</u>						

SIGHT DRAFT

In case Drake refuses to honor the draft, it is returned to the Trenton bank, which notifies Burke & Co. that Drake has refused to pay the draft.

Drafts are not generally used in the collection of invoices unless the terms of the invoice call for it. Thus an invoice may bear the terms, "Subject to draft in 30 da." Sight drafts are also used in the collection of overdue accounts.

While a man is not obliged to honor a draft drawn upon him, he generally does so unless he has some valid reason for refusing, in order to maintain his credit reputation.

A collection charge of $\frac{1}{10}\%$ is generally made by a bank for collecting such a draft.

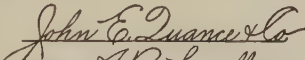
The use of drafts in collecting an invoice is now generally replaced by the trade acceptance, which will be discussed later.

ORAL EXERCISES

If $\frac{1}{10}\%$ is charged for collection, state the proceeds of collected sight drafts for the following amounts:

- | | | | |
|-----------|-----------|-----------|---------------|
| 1. \$450. | 4. \$670. | 7. \$382. | 10. \$425.25. |
| 2. \$280. | 5. \$720. | 8. \$464. | 11. \$617.75. |
| 3. \$360. | 6. \$830. | 9. \$816. | 12. \$325.50. |

Trade acceptance. A trade acceptance is a form of draft drawn by a seller on a purchaser. For example, suppose that John E. Quance & Co., of Portland, Ore., sell goods amounting to \$1400 to Nelson R. Conner, of Denver, Colo., under terms which provide that the sale is subject to a 30-day trade acceptance. The following form shows the trade acceptance that Quance & Co. fill out and attach to the invoice which is sent to Conner:

TRADE ACCEPTANCE			
No. <u>710</u>		Portland, Ore., May 18, 19--	
--Thirty--days after date pay		to the order of <u>Ourselves</u>	
Fourteen hundred 00/100-----		-----Dollars \$ <u>1400.00</u>	
The obligation of the acceptor of goods from		hereof arises out of the purchase the drawer	
To <u>Nelson R. Conner,</u>	ACCEPTED	 By <u>A. B. Lowell</u>	
Denver, Colo.			
Due <u>June 15,</u> 19--			
DATE <u>May 19, 19--</u>	PAYABLE AT <u>Nelson National Bank</u>	LOCATION OF BANK <u>Portland, Ore.</u>	
	SIGNATURE <u>Nelson R. Conner</u>	PER <u>S. E. Loring</u>	

TRADE ACCEPTANCE

Upon its receipt Conner accepts the draft by filling in the acceptance across the face as shown, and returns it to Quance & Co. The trade acceptance is now, in effect, a promissory note, since it has the purchaser's indorsement promising to pay the money under the conditions stated. Quance & Co. treat this as any promissory note, and can offer it for discount at their bank, thus obtaining cash which they may need to carry on their business.

The use of trade acceptances avoids open book accounts, since in their place the merchant holds trade acceptances receivable, which can be discounted when necessary.

EXERCISES

1. If John E. Quance & Co. discount the above trade acceptance on May 22 at 6%, what amount will the bank place to their credit?

2. On June 3 S. E. Loring, of New York City, ships goods amounting to \$1565.25 to D. V. Drake, of Scranton, Pa., subject

5. If the Gem Specialty Co. in Ex. 4 discounts the trade acceptance on Feb. 28 at 6%, what are the proceeds?

61596	Westfield, Sta. 1, Mass.	3746
OFFICE NUMBER		ORIGINAL NUMBER

United States Postal Money Order <u>Oct. 16th</u>	10 DOLLARS XX 10 CENTS AMOUNT FOR WHICH ISSUED	Westfield, Sta. 1, Mass. 61596 OFFICE NUMBER
--	---	---

COUPON FOR PAYING OFFICE
NOT TO BE DETACHED BY SOLDER

THE POSTMASTER AT	10 DOLLARS 10 CENTS	PAID TO: WHITE WOODS FOR DOLLARS DOLLARS 10 CENTS 10
-------------------	------------------------------	--

Arvey, J. Lee
 WILL PAY AMOUNT STATED ABOVE TO ORDER OF PAYEE NAMED IN ATTACHED COUPON OF SAME NUMBER. IF ISSUED WITHIN THE CONTINENTAL UNITED STATES, ALASKA EXCEPTED, THE POSTMASTER AT ANY MONEY ORDER OFFICE IN THE CONTINENTAL UNITED STATES, ALASKA EXCEPTED, WILL PAY IF PRESENTED WITHIN THIRTY DAYS FROM DATE OF ISSUE.

PAYING OFFICE	<i>Wm. H. Foote</i> F.R.B. POSTMASTER	RECEIVED PAYMENT: FACSIMILE. OF NO VALUE
---------------	--	--

STAMP HERE

Postal money order. A person who wishes to make a remittance to another at a distant point can go to a post office and purchase, for a small fee, a **postal money order** for the desired amount, payable to the order of the person to whom the money is to be sent. The purchaser then mails the money order to the payee, who collects the money at his own post office (or through his bank if he so desires). The above form shows a postal money order.

The maximum amount for which a single money order is issued is \$100; if a larger amount is required, additional orders must be purchased. If a

postal money order is lost, a duplicate may be obtained from the Post Office Department at Washington.

When an order is issued, the money is not transmitted from one post office to another; it is simply received by the government at one office and paid out at another.

The fees charged in the United States for domestic postal money orders are as follows:

Not exceeding \$2.50	3¢	\$30.01 to \$40.00	15¢
\$2.51 to \$5.00	5¢	\$40.01 to \$50.00	18¢
\$5.01 to \$10.00	8¢	\$50.01 to \$60.00	20¢
\$10.01 to \$20.00	10¢	\$60.01 to \$75.00	25¢
\$20.01 to \$30.00	12¢	\$75.01 to \$100.00	30¢

ORAL EXERCISES

State the amount (face plus fee) charged for postal money orders for each of the following sums:

1.	2.	3.	4.	5.
34¢	\$1.12	\$3.16	\$25.20	\$18.25
99¢	\$43.14	\$7.32	\$15.65	\$13.10
\$1.87	\$58.57	\$64.23	\$110.75	\$43.12
\$2.56	\$74.28	\$85.47	\$123.45	\$38.25

6. The 30-cent fee for a postal money order for \$100 is equivalent to what per cent of the face of the order?

EXPRESS MONEY ORDER WHEN COUNTERSIGNED BY AGENT AT POINT OF ISSUE D-6954840		D-6954840 American Express Company MONEY ORDER REMITTERS RECEIPT KEEP IT
ADDRESS TO TRANSMIT AND Pay on presentation to <i>G. P. Beck</i> or order \$50 00 The Sum of <i>Fifty and no</i> Dollars Counter <i>20</i> CENTS NOT CASHABLE AT THE HIGHEST PAYABLE NATIONAL BANK - IN NO CASE TO EXCEED FIFTY DOLLARS		AMOUNT OF ORDER DOLLARS 50 CENTS 00 DATE <i>17th</i> - SENT TO <i>G. P. Beck</i> BY <i>L. L. Hagg</i>
SIGNED BY <i>L. L. Hagg</i> Agent BOSTON, MASS. State of <i>MASS.</i> Date <i>December 22</i> 19 <i>18</i> ANY EXCESSIVE DELAY IN THE DELIVERY OF THIS ORDER REMAINS AT THE ORDERER'S RISK		IF THE ABOVE DEMONSTRATION ORDER IS USED OR A STUDY OF THE COMPANY WILL BE MADE TO DETERMINE THE FACE VALUE THEREOF FROM PRESENTATION OF THIS RECEIPT AND EXECUTION OF THIS RECEIPT AGREEMENT FOR RETURN

EXPRESS MONEY ORDER

Express money order. Express companies also issue money orders which are payable at any of their agencies. The above form shows an **express money order**.

The fees charged by the American Express Company for money orders for amounts up to and including \$100 are as follows:

Not exceeding \$2.50	5¢
\$2.51 to \$5.00	7¢
\$5.01 to \$10.00	10¢
\$10.01 to \$25.00	12¢
\$25.01 to \$50.00	15¢
\$50.01 to \$75.00	18¢
\$75.01 to \$100.00	20¢

ORAL EXERCISES

State the amount (face plus fee) charged for express money orders for each of the following sums:

1.	2.	3.	4.	5.
\$8.75	\$11.53	\$21.45	\$54.25	\$33.70
\$9.14	\$23.55	\$35.67	\$72.59	\$7.65
\$11.63	\$55.73	\$81.54	\$45.60	\$28.32
\$5.40	\$31.10	\$19.56	\$17.45	\$63.27

Telegraphic money order. When it is necessary to transmit money quickly, it may be done by means of a **telegraphic money order**. The person who desires to transmit the money goes to a telegraph office and orders the "money telegraphed" to the person to whom he is making the remittance. The telegraph company then sends a telegram to its agent at that place directing him to pay the specified amount to the person designated in the message.

Such messages are sent in code so as to guard against an unauthorized person's receiving the money and to lessen the expense. Banks and trust companies frequently transfer credits in this way.

The cost of telegraphing money varies according to the amount and the distance which it is sent. The transfer charges between Boston, Mass., and Cleveland, Ohio, are as follows:

\$0.01 to \$25.00	25¢	\$50.01 to \$75.00	60¢
\$25.01 to \$50.00	35¢	\$75.01 to \$100.00	85¢

For each additional \$100, or fraction thereof, up to and including
 \$3000 25¢

For each additional \$100, or fraction thereof, over
 \$3000 20¢

The transfer charges on \$160 between Boston, Mass., and Cleveland, Ohio, are computed as follows :

Charges on first \$100	= \$0.85
Charges on \$60	= .25
Total charges	<u>\$1.10</u>

The \$60 after the first \$100 takes the rate given for an additional \$100 below \$3000.

ORAL EXERCISES

State the amount (face plus transfer charge) charged for transmitting each of the following amounts between Boston and Cleveland by telegraph :

1.	2.	3.	4.
\$150.00	\$250.00	\$325.00	\$175.00
\$26.50	\$85.10	\$75.60	\$87.45
\$300.00	\$225.00	\$500.00	\$275.00
\$400.00	\$101.01	\$69.50	\$48.95

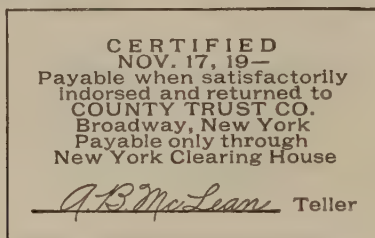
EXERCISES

Find the amount (face plus transfer charge) charged for transmitting each of the following amounts between Boston and Cleveland by telegraph :

1.	2.	3.	4.
\$2000	\$3800	\$3765.52	\$2165.90
\$1850	\$4200	\$6978.63	\$3874.33
\$1550	\$5000	\$2347.98	\$5468.65
\$3500	\$6500	\$6115.60	\$7842.73

Certified check. Although checks pass freely as money, checks drawn or presented by unknown persons are not accepted as money without caution on account of the danger of fraud.

If the maker of a check desires to assure the one to whom it is sent that the check will be paid when presented at his bank, he may have the check **certified**. He may do this by taking it to his bank, where a statement similar to the following is stamped or written across the face of the check:



When a check is certified the bank assumes responsibility for its payment. The bank takes the amount of the check from the depositor's account and holds it in a separate account until the check is presented for payment. Such a check is good as long as the bank is solvent.

A certified check is handled in exactly the same way as an ordinary check, as described on pages 303-305.

Clearing house. During the course of a business day a city bank receives and pays many checks drawn on other banks, some in distant places and others in the same city. Accordingly, each bank has a daily balance to settle with each of the other banks in the same city; to some it must make payments and from others it receives payments. The different banks often form a central agency, called a **clearing house**, through which they effect the settlement of these balances and exchange the checks which they have paid. Messengers from the different banks meet daily at the clearing house, each bringing with him the checks that his bank has paid for each of the other banks during the preceding day.

For example, suppose that the Union National Bank, the Second National Bank, and the Merchants' National Bank settle their balances through a clearing house. Other banks may be represented in this clearing

house, but, for purposes of explanation, only the operations of the three banks will be considered.

On a certain day the statements submitted at the clearing house by the messenger from each bank show that on the preceding day the total of the checks paid by the Union National Bank was as follows:

For the Second National Bank	\$6246.73
For the Merchants' National Bank	7808.67
	<u>\$14,055.40</u>

The total of the checks paid by the Second National Bank was as follows:

For the Union National Bank	\$5869.41
For the Merchants' National Bank	2718.47
	<u>\$8587.88</u>

The total of the checks paid by the Merchants' National Bank was as follows:

For the Union National Bank	\$4551.73
For the Second National Bank	4054.67
	<u>\$8606.40</u>

The manager of the clearing house now prepares a statement, which is called the **clearing-house manager's sheet**. On this sheet the total of the checks paid for each bank by the other banks is entered in the "Debits" column, and the total of the checks paid by each bank for the other banks is entered in the "Credits" column, as shown below:

NAME OF BANK	DEBITS		CREDITS		DEBIT BALANCE		CREDIT BALANCE	
Union National	10421	14	14055	40			3634	26
Second National	10301	40	8587	88	1713	52		
Merchants' National	10527	14	8606	40	1920	74		
TOTALS	31249	68	31249	68	3634	26	3634	26

When balanced, this sheet shows that the Union National has paid out \$3634.26 more than was paid out for its account, that the Second National paid out \$1713.52 less than was paid out for its account, and that the Merchants' National paid \$1920.74 less than was paid out for its account. Hence these two banks turn over to the clearing house the amount of their debit balances, and the Union National receives a check for \$3634.26 from the clearing house. In this way the balances are settled with the exchange of the least amount of cash.

Of course, all the checks that a bank receives are not payable at other banks in the same city. The procedure in the case of a check on an out-of-town bank is illustrated in the following case :

Suppose that A. W. Hoyt, of Chicago, sends a check drawn on the Webster National Bank, of Chicago, to J. B. West, of Springfield, Mass., in payment of a debt of \$500. West deposits the check at the Traders' Bank, of Springfield, and it is credited to his account. The Springfield bank has no account with any Chicago bank, but it has one with the Grant National Bank, of Boston, and hence the check is sent there and credited to their account at the Boston bank. The Grant National Bank may wish to increase their balance at the Franklin National Bank, of New York, and accordingly the check is sent along with others to that bank for credit. The Franklin National Bank handles all its Chicago business through the Sumner National Bank, of Chicago, and hence they send the check to them, where it is credited to the account of the Franklin National Bank. The Sumner National Bank sends the check to the Chicago clearing house with others paid for the account of the Webster National Bank, and the balance is adjusted as described above. The Webster National Bank will receive the check with the rest of the day's clearings, and will charge the amount to Hoyt's account. The check will finally be returned to Hoyt at the end of the month with his other checks paid during the month.

If banks do not operate through a clearing house, messengers are sent to each of the other banks with the checks, the balance due or payable is determined, and the necessary adjustments are made by sending cash or checks. Obviously, it is more work to settle balances in this way, and it means the carrying of checks or cash to several banks instead of to a central agency.

EXERCISE

Below are given the amounts which each of five banks operating through a clearing house paid for each of the other banks during a business day. Prepare a clearing-house manager's sheet, similar to the one given on page 315, from the following data :

The Union National Bank paid checks as follows :

For the Second National Bank	\$12,375.25
For the Merchants' National Bank	8,417.17
For the Commercial Trust Co.	10,511.12
For the Second State Bank	<u>4,308.26</u>

The Second National Bank paid checks as follows:

For the Union National Bank	\$11,546.20
For the Merchants' National Bank	6,313.35
For the Commercial Trust Co.	9,432.60
For the Second State Bank	<u>2,854.22</u>

The Merchants' National Bank paid checks as follows:

For the Union National Bank	\$8,567.25
For the Second National Bank	10,371.20
For the Commercial Trust Co.	4,575.50
For the Second State Bank	<u>3,218.75</u>

The Commercial Trust Co. paid checks as follows:

For the Union National Bank	\$9,238.70
For the Second National Bank	7,456.23
For the Merchants' National Bank	5,285.90
For the Second State Bank	<u>3,475.30</u>

The Second State Bank paid checks as follows:

For the Union National Bank	\$6,795.23
For the Second National Bank	5,432.75
For the Merchants' National Bank	7,386.30
For the Commercial Trust Co.	<u>2,731.90</u>

FOREIGN EXCHANGE

Foreign exchange. The process of settling obligations between creditors in one country and debtors in another is called **foreign exchange**.

The methods of transfer and collection are much the same as for similar cases of domestic exchange.

Bill of exchange. Drafts drawn on foreign money centers are usually called **bills of exchange** and are divided into three classes: (a) **bankers' bills**, which are drawn by one banker upon another; (b) **commercial bills**, which are drawn by one merchant upon another; and (c) **documentary bills**, which are drawn by one

merchant upon another and are secured by the assignment and transfer of the bill of lading and the insurance policy covering merchandise which is being shipped.

The following form illustrates a typical banker's bill of exchange:

£ 5000.0.0.	<i>New York</i> NOV. 12, 19--	No. 16899
NORTHERN TRUST COMPANY		
<i>On Demand (DUPLICATE UNPAID) please</i>		
<i>pay to the order of</i> JOHN BORDAN		
-----FIVE THOUSAND POUNDS STERLING-----		
<i>To</i> <u>MACKELLAR'S BANK, LTD.</u> <u>LONDON, ENGLAND</u>	NORTHERN TRUST COMPANY <i>R. T. Stevens</i> MANAGER FOREIGN DEPARTMENT <i>Edw. L. Webster</i> ASST. MANAGER FOREIGN DEPARTMENT	

BANKER'S BILL OF EXCHANGE

Bills of exchange are generally issued in duplicate, as in the above form, which is the original banker's bill. Sometimes the two copies are sent in different mails, but more frequently the original is sent and the duplicate is filed where it is available in case of necessity. The original and duplicate are so worded, as in the above form, that the payment of one of them cancels the other.

Mint par of exchange. The standard unit of exchange in any country is called the **monetary unit**.

Thus the monetary unit in the United States is the dollar; in England, the pound sterling; in France, the franc; and in Germany, the mark.

The value of the monetary unit of one country expressed in terms of the monetary unit of another country, both units being considered on the gold basis, is called the **mint par of exchange**.

Thus, the United States gold dollar contains 23.22 troy grains of pure gold, and the English pound sterling contains 113.0016 troy grains. Hence the pound sterling contains $113.0016 \div 23.22$, or 4.8665 times as much pure gold as the gold dollar, and is worth 4.8665 times as much as \$1, or \$4.8665. Hence the mint par of exchange with respect to England is \$4.8665.

Foreign money. The following tables show the systems of money in use in some of the foreign countries, the monetary unit being shown in heavy type:

ENGLAND

4 farthings = 1 penny (d.)
 12 pence = 1 shilling (s.)
 20 shillings = 1 **pound** (£)
 Mint par of exchange, \$4.8665

FRANCE

100 centimes (c.) = 1 **franc** (fr.)
 Mint par of exchange, 19.3¢

GERMANY

100 pfennigs (pf.) = 1 **mark** (M.)
 Mint par of exchange, 23.8¢

The Swiss, Belgian, and Italian systems are the same as the French, and the mint par of exchange is the same. In Italy, however, the unit is called the **lira** (plural, **lire**) instead of the franc.

In Japan the unit is the **yen**, and in Mexico the unit is the **peso**. The mint par value of these units is \$0.498. In Sweden and Norway the unit is the **crown**, which has a mint par value of \$0.268.

Where accurate results are not essential, it is convenient to use the following approximate equivalents of foreign coins: pound, \$5; shilling, 25¢; penny, 2¢; mark, 25¢; franc or lira, 20¢; peso or yen, 50¢; and crown, 27¢.

ORAL EXERCISES

Using the mint par value, state the value in United States money of the following amounts:

- | | | | |
|-----------|------------|-------------|----------------|
| 1. £10. | 5. 10 fr. | 9. 1000 fr. | 13. 500 M. |
| 2. £50. | 6. 100 fr. | 10. 10 M. | 14. 1000 M. |
| 3. £100. | 7. 200 fr. | 11. 100 M. | 15. 100 pesos. |
| 4. £1000. | 8. 500 fr. | 12. 300 M. | 16. 200 lire. |

Using the approximate equivalents given on page 319, state the value in United States money of the following amounts :

- | | | | |
|------------------|-------------|--------------|----------------|
| 17. £8 6s. | 21. £35 6s. | 25. 1200 fr. | 29. 250 lire. |
| 18. £11 10s. | 22. 80 fr. | 26. 820 fr. | 30. 300 pesos. |
| 19. £15 10s. | 23. 120 fr. | 27. 1400 M. | 31. 250 yen. |
| 20. £12 10s. 6d. | 24. 650 fr. | 28. 65 lire. | 32. 50 crowns. |

EXERCISES

In the following exercises use the mint par of exchange.

1. Express in United States money: £35 10s.; £80 5s.; £95 15s.; £12 15s.; £56 5s.; £75 16s.; £82 12s.; £45 18s.

First express £35 10s. as £35.5. Then multiply this result by the mint par value.

2. A business man in New York received a draft from London for £256 15s. What was the value of the draft in United States money?

3. A business man in Chicago received a draft from Paris for 2250 fr. What was the value of the draft in United States money?

4. A tourist who visited several cities in Europe found that his hotel expenses in each city were as follows:

In London: £1 15s. per day for 12 da.

In Paris: 30 fr. per day for 15 da.

In Geneva: 35 fr. per day for 5 da.

In Rome: 32 lire per day for 10 da.

In Berlin: 30 M. per day for 6 da.

What was the total amount of his hotel expenses, expressed in United States money?

5. A Philadelphia business man received the following drafts:

From Brussels: a draft for 5000 fr.

From Christiania: a draft for 150 crowns.

From Paris: a draft for 4500 fr.

What was the total value of these drafts in United States money?

Rate of exchange. The rate of exchange is the market value in one country of the bills of exchange on another country. The rate of exchange between any two countries is always fluctuating.

If, for example, business men in the United States owe business men in England exactly the same amount that those in England owe those in the United States, debts between them could be settled without the actual transmission of money, and exchange would be **at par**; that is, the rate would be the same as the mint par of exchange. But if English business men owe American business men more than the Americans owe the English, exchange in the United States would be **at a discount** and in England **at a premium**. That is, there would not be enough American bills of exchange in England to meet the demand for them, and there would be an oversupply of English bills in America. Hence an Englishman would have to pay more than par for a bill of exchange on America, while an American could buy bills on England at less than par.

The rates of premium and discount are limited by the cost of shipping gold bullion from one country to another. Thus, if it is cheaper to ship the gold than to buy bills of exchange, the gold will usually be shipped to settle the balances.

Exchange on Great Britain is generally quoted as the number of dollars to the pound sterling; exchange on France, Belgium, and Switzerland as the number of cents to the franc; exchange on Germany, as the number of cents to the mark; and so on for other countries.

The cost of shipping gold used to be about $\frac{1}{2}\%$ of its value. Hence, if A in New York owed B in London and if A could not buy a bill of exchange on London for less than \$4.89, it would be cheaper for him to send gold to London in payment of the debt. On the other hand, if D in London owed C in New York and C could not sell a draft on D for more than \$4.84, it would be cheaper for C to import gold.

Owing to conditions resulting from the World War, the placing of gold embargoes by many countries, the large amounts owed to the United States on account of loans and excessive imports,

and the tremendous depreciation of many foreign currencies, the normal conditions of foreign exchange have been seriously affected. The mint par of exchange is now of but little significance, except in bullion transactions, and it will be a long time before conditions again return to normal. The above paragraphs set forth the basic principles of foreign exchange, but it cannot be expected that any textbook can take into consideration the almost daily changes in the rates and conditions of foreign exchange.

Under stable financial conditions paper currency is secured by an adequate supply of gold. Owing to the shortages of gold, many European countries have issued vast quantities of paper money without such security. The result is an inflation of currency, which is attended by a rise in prices and a consequent depreciation in the value of the currency.

The following table illustrates the influence of the World War on rates of exchange:

MONETARY UNIT	BEFORE THE WORLD WAR	AFTER THE WORLD WAR
English pound	\$4.86	\$3.95
German mark	23.6¢	0.65¢
French franc	19.2¢	7.32¢
Belgian franc	19.2¢	7.18¢
Swiss franc	19.3¢	18.12¢

While the quotations given since the World War are from recent transactions, on account of rapid fluctuations they are of but little value as a guide to conditions from day to day. At the discretion of the instructor, current quotations from the daily newspapers may be used for this comparison and for the following exercises.

ORAL EXERCISES

Using the above quotations for before the World War, state the value in United States money of drafts for each of the following amounts:

- | | | |
|----------|-----------|-----------------------|
| 1. £200. | 5. 400 M. | 9. 90 fr., French. |
| 2. £500. | 6. 700 M. | 10. 300 fr., French. |
| 3. £800. | 7. 900 M. | 11. 600 fr., Swiss. |
| 4. £150. | 8. 300 M. | 12. 150 fr., Belgian. |

EXERCISES

Using the quotations for after the World War given on page 322, find the value in United States money of drafts for each of the following amounts :

- | | | |
|----------------------|--------------|------------------------|
| 1. 600 fr., French. | 6. £750. | 11. 900 fr., Belgian. |
| 2. 1150 fr., French. | 7. £280 10s. | 12. 1240 fr., Belgian. |
| 3. 750 fr., French. | 8. £550 15s. | 13. 825 fr., Belgian. |
| 4. 920 fr., Swiss. | 9. 1050 M. | 14. 350 fr., Swiss. |
| 5. 800 fr., Swiss. | 10. 235 M. | 15. 425 fr., Swiss. |

16. If exchange is quoted at \$4.756, find the cost of a draft on London for £108 10s.

17. If exchange is quoted at 18.5¢, find the cost of a draft on Paris for 675 fr.

18. A business man in Baltimore offered for deposit a draft on Brussels for 1650 fr. If exchange was quoted at 17.5¢, how much was placed to his credit?

In Exs. 18 and 19 disregard any commission for collecting or issuing the drafts.

19. A. D. Roswell, of St. Louis, bought a draft on London for £175 15s. If exchange was quoted at \$4.865, what was the amount of the check which he gave in payment for the draft?

Foreign money order. In sending small sums from one country to another, the **international postal money order** and the **foreign express money order** are extensively used.

The foreign postal money order is similar to the domestic form. In transmitting money by express money order, the express company simply issues a receipt to the person sending the money and then orders their agent in the country to which the money is sent to pay the amount to the person designated.

Express companies also issue foreign drafts, which are purchased in much the same way as domestic bank drafts.

The fee for an international postal money order is 10¢ for an order equivalent to \$10 in United States money, and 10¢ for each additional \$10 up to the maximum amount of a single order, \$100.

The rates for transmitting money by foreign express money orders are determined daily in accordance with the current rates of exchange.

These rates are slightly higher than the market quotations of exchange in order to cover the cost of handling the order. On small orders the rates are relatively higher than when large amounts are to be transmitted. There is, however, no fixed fee to be paid. It is suggested that a student consult the American Express Company's local office for current rates.

ORAL EXERCISES

1. A man in Denver wished to place the sum of 1100 fr. at the disposal of his daughter in Paris. If francs were quoted at 19¢, what was the cost of transmitting the money by express money order?

2. A firm in Milwaukee wished to send a sum equivalent to \$1800 to a firm in Berlin. If the money was transmitted by express money order, and the mark was quoted at 18¢, what amount in marks was received by the Berlin firm?

3. State the cost, including the fee as stated on page 323, for international postal money orders, payable in English funds, equivalent to the following amounts: \$20; \$30; \$60; \$100; \$150.

In the last case two orders, one for \$100 and one for \$50, must be purchased.

4. A business man desired to transmit to Manchester, England, a sum equivalent to \$175. If he bought international postal money orders, what was the total cost of the orders?

EXERCISES

1. If on the day that you wish to send an express money order to Wales, the rate is quoted at \$4.85 to the pound, how much will you have to pay for an order for £85?

2. A business man in Buffalo, N. Y., sent an international money order to Berne, Switzerland, for 308.5 fr. If the franc was quoted at 18.5¢, what did the order cost in dollars, including the fee?

4. An importer in Philadelphia had to remit 640 M. to a firm in Cologne, Germany. What was the cost, including fees, for sending the money by international postal money orders if the mark was quoted at 15.6¢?

Traveler's check. It is always dangerous when traveling to carry large amounts of cash, but it is also difficult for a traveler to cash personal checks in localities where he is unknown. A convenient and safe method of carrying funds is afforded by the use of **traveler's checks**, which require no identification to cash them. The following form is an illustration of a typical traveler's check:



BANKER'S TRAVELER'S CHECK

A person intending to travel can go to a bank or express office and there purchase a sufficient number of these checks to cover his estimated needs for cash during the trip. The checks are sold at their face value plus a small commission to cover the cost of issuing them. The name of the payee is left blank, and the checks are printed in amounts from \$10 up to larger amounts, so that the traveler can obtain cash in convenient amounts. The purchaser signs his name on each check before

it is issued to him. Later, when he wishes to cash a check, he fills in the name of the payee and again signs his name on a different part of the check in the payee's presence, the similarity of the two signatures thus identifying him.

Traveler's checks are good anywhere in this country or abroad, but when cashed in foreign countries the face of the check is payable at the current rates of exchange.

ORAL EXERCISES

If $\frac{1}{2}\%$ of the face is charged for issuing traveler's checks, state the cost of purchasing:

1. Ten checks for \$10 each.
2. Six checks for \$50 each.
3. Twelve checks for \$20 each.
4. Six checks for \$10 each, and six checks for \$20 each.
5. Ten checks for \$100 each.
6. Four checks for \$200 each.
7. Eight checks for \$50 each, and four checks for \$100 each.
8. Four checks for \$50 each, and two checks for \$100 each.
9. Fifteen checks for \$50 each.
10. Twenty checks for \$20 each.
11. Ten checks for \$100 each.
12. Twelve checks for \$50 each.
13. Five checks for \$20 each, and three checks for \$50 each.
14. Ten checks for \$10 each, and two checks for \$100 each.
15. Five checks for \$10 each, and five checks for \$20 each.

Letter of credit. If a person intending to travel goes to a bank and deposits cash or securities, the bank will give him a **letter of credit**, which is a form of letter addressed to other banks in different places with which the issuing bank does business. This letter authorizes the other banks, which are known as **correspondents**, to cash any draft drawn by the holder of the letter upon the issuing bank up to the amount specified therein.

The following form shows a letter of credit and the one on page 328 shows typical indorsements on the back of this letter.

SECURITY TRUST COMPANY	
CIRCULAR LETTER OF CREDIT	
No <u>D56789</u>	New York, <u>NOVEMBER 10, 19--</u>
<i>To Messieurs,</i> <i>The Bankers mentioned in</i> <i>our List of Correspondents</i> <i>Gentlemen:</i>	
<i>We beg to introduce to you</i>	
MR. ROBERT THOMPSON-----whom we hereby authorize to draw on us at sight up to the aggregate amount of <u>FIVE THOUSAND-----00/100 Dollars (U. S. \$ 5,000.00)</u>	
<i>Kindly negotiate his drafts at your buying rate for demand drafts on</i> <i>New York, indorsing the amount of each draft on the back of this letter.</i> <i>Each draft should be marked "Drawn under T.L.C. No <u>D56789</u>," and</i> <i>the signature affixed thereto in your presence should be identical with the</i> <i>one below.</i>	
<i>We engage that such drafts shall meet with due honor if issued on or</i> <i>before--May 15,-----19--</i>	
<i>Recommending-----MR. THOMPSON-----to your usual courtesies,</i> <i>we remain, gentlemen,</i>	
<i>Yours faithfully,</i>	
<i>Holder's signature</i> <u>Robert Thompson</u> HW.DL	<i>Security Trust Company</i> <u>F. K. Jamieson</u> Vice President <u>C. S. Allen</u> Asst. Secretary

LETTER OF CREDIT

The holder signs his name to the letter when it is issued, and this signature is compared with the signature on the drafts

which he signs when he desires to obtain cash. The bank cashing the draft indorses the amount paid on the back of the letter. These indorsements thus show how much has been drawn against the letter, and when the specified amount has been used up, the last bank takes up the letter and forwards it to the bank which issued it. Cash can be obtained on a letter of credit only at a bank specified in the list of correspondents attached to it.

THE AMOUNT DRAWN AGAINST THIS LETTER OF CREDIT MUST NOT EXCEED <u>Five thousand</u> U. S. DOLLARS. INDORSEMENTS OF PAYMENTS ONCE MADE HEREON MUST BE ALLOWED TO REMAIN WITHOUT ERASURE OR ALTERATION. THIS LETTER OF CREDIT MUST BE CANCELLED WHEN EXHAUSTED AND ATTACHED TO FINAL DRAFT DRAWN HEREUNDER.				
DATE WHEN PAID	BY WHOM PAID	NAME OF CITY WHERE PAID	AMOUNT IN DOLLARS	
			IN WORDS	IN FIGURES
<i>Nov. 15, 19—</i>	<i>River Trust Co.</i>	<i>Boston</i>	<i>Three hundred</i>	<i>\$300.—</i>
<i>Dec. 24, 19—</i>	<i>Midland Bank</i>	<i>London, Eng.</i>	<i>Five hundred</i>	<i>\$500.—</i>

REVERSE OF LETTER OF CREDIT

Letters of credit issued by banks in the United States are usually made payable in dollars, but when presented in foreign countries drafts on them are cashed at the current rates of exchange. Banks generally charge a commission of from $\frac{1}{2}\%$ to $\frac{3}{4}\%$ for issuing letters of credit in dollars, and 1% for issuing letters payable in pounds sterling.

REVIEW EXERCISES

1. A traveler buys a letter of credit in Boston for \$4000. What is the cost of the letter, including the commission at $\frac{1}{2}\%$?
2. While in London, the traveler in Ex. 1 draws £50 against his letter of credit. If exchange that day is quoted at \$4.76 to the pound, find the amount in dollars which is indorsed on the letter.
3. Basset & Co., of Philadelphia, drew a sight draft on Longman & Co., of Manchester, England, for £150. If the pound was quoted at \$4.855, find the value of the draft in dollars.
4. An exporter in New York drew a sight draft on a firm in Brussels for \$16,285. If exchange was quoted at $16\frac{1}{2}\phi$, find the amount in francs paid by the Brussels firm to meet the draft.

5. A Boston broker bought the following bills of exchange: £1275 12 s. at \$4.855; 1450 fr. at 18.5¢; 11,360 M. at 21.6¢. If a commission of $\frac{1}{8}\%$ was charged for issuing the drafts, find the total cost in dollars.

6. A cotton exporter in New York drew a draft on a firm in Manchester, England, for a sum equivalent to \$5665.55. If exchange was quoted at \$4.86, what was the face of the draft in pounds?

7. If a commission of $\frac{1}{8}\%$ was charged on the draft in Ex. 6, what was the amount of the commission (in dollars)?

8. A Chicago importer bought the following drafts with which to pay for merchandise:

On London: £385 5s. at \$4.865

On Paris: 7500 fr. at 18¢

On Hamburg: 4250 M. at 21¢

If a commission of $\frac{1}{8}\%$ was charged, what was the total cost of the drafts?

CHAPTER IX

BUSINESS ORGANIZATION

SOLE PROPRIETORSHIP

Sole proprietorship. A business is usually owned and operated by one of three types of organization : individual ownership, partnership, or corporation.

A business owned and operated by an individual is classed as a **sole proprietorship**, since the owner does not necessarily have to consult with others; and as he alone is responsible for the administration of the business, he receives all the profits and suffers all the losses.

PARTNERSHIP

Nature of a partnership. A **partnership** is a contract relation between persons, known as **partners**, who combine their property, labor, or skill in a business and who agree to share the profits and losses jointly. In a partnership each of the partners has certain rights and obligations. It is important, therefore, that the fundamental points of such an organization be understood by those who plan to enter business.

The amount of money or other property which a partner contributes is called his **investment**. The total of the partners' investments constitutes the **capital** of the partnership.

The agreement covering a partnership may be verbal or written, but preferably the latter. Usually, the clauses in the written articles of partnership cover the following points :

- | | |
|--|--|
| 1. Names of the partners and name of the firm. | 5. Capital to be invested and provisions for its withdrawal. |
| 2. Nature of the business. | 6. Salaries to be paid to the partners. |
| 3. Place of business. | 7. Interest to be paid on invested capital. |
| 4. Date of commencement and duration of partnership. | 8. Division of profits and losses. |
| | 9. Provisions for dissolution. |

Partners who actually take part in the conduct of the business and who are known publicly as partners are called **real partners**. Ordinarily real partners are liable for all the debts of the partnership. Partners who are known publicly as partners but who have no investment and no share in the profits are known as **nominal partners**. Such partners share the liability of real partners for the debts of the business. Partners who are not known to the public as partners but who receive a share of the profits are called **silent partners**. Such a partner is liable for the debts of the business as soon as his connection with the partnership is known publicly. Partners whose liability for the debts of the business are limited are called **limited partners**. In such cases the liability is usually limited to the amount invested.

The method of forming a limited partnership is prescribed by laws, which vary in the different states. Usually there must be one partner who is the manager of the business and whose liability is not limited.

Division of profits and losses. There are three general methods of distributing the profits and losses of a partnership:

1. In proportion to a partner's investment.
2. In some arbitrary division.
3. In proportion to the average investment.

Proportional division of profits. If the investments of the partners are equal and the profits (or losses) are divided in proportion to the investment, it is obvious that each partner receives an equal share of the profit.

EXAMPLE. In a partnership formed by A and B, profits are divided in proportion to the investment. If each invests \$4000 and the net profit for a year is \$2000, how much does each partner receive?

Solution. In this case each partner receives $\frac{1}{2}$ of the profits. Hence A and B each receive $\$2000 \times \frac{1}{2}$, or \$1000.

If the investments are unequal and the profits are divided in proportion to the investment, the general method of determining the share of each partner is shown in the following example.

EXAMPLE. In a partnership formed by C and D, profits are divided in proportion to the investment. If C invests \$3250 and D invests \$2450, how much does each partner receive in a year in which the net profit is \$1596?

Solution. $\$3250 + \$2450 = \$5700$, capital.

$\$1596 \div \$5700 = 0.28$ (or 28%), rate of profit.

$\$3250 \times 0.28 = \910 , C's profit.

$\$2450 \times 0.28 = \686 , D's profit.

Check. $\$910 + \$686 = \$1596$, total profit.

This last method might have been applied in the first example on page 331, but where the investments are equal, the amount of each partner's profit is always the amount of the net profit divided by the number of partners. Hence the method there used is easier.

When simple ratios are involved, an example like the second one above can frequently be solved by inspection. Thus, if C had invested \$3000 and D \$2000, the capital of the firm would have been \$5000. Hence C would have received $\frac{3}{5}$ of the net profit and D would have received $\frac{2}{5}$.

Arbitrary division of profits. Often the business experience and skill of one partner or the time which one partner devotes to the business entitles him to receive a larger share of the profits than the other partners do, even though their investments are equal. The arbitrary method in which the profits are to be divided is generally prescribed by the articles of partnership.

If no division of profits is indicated in the articles of partnership, profits are divided equally, even though the investments are unequal.

EXAMPLE. A partnership is formed by E and F, and each invests \$8000. On account of his greater business experience E is to receive 60% of the profits and F is to receive 40%. How much does each partner receive in a year in which the net profit is \$2000?

Solution. $\$2000 \times 0.60 = \1200 , E's profit.

$\$2000 \times 0.40 = \800 , F's profit.

Check. $\$1200 + \$800 = \$2000$, total profit.

ORAL EXERCISES

State each partner's share of profit in the following cases:

	INVESTMENT		BASIS OF DIVISION	PROFIT
	A	B		
1.	\$1000	\$2000	Investment	\$150
2.	\$1500	\$4500	A, $\frac{2}{3}$; B, $\frac{1}{3}$	\$1200
3.	\$1750	\$3500	Investment	\$600

	INVESTMENT		BASIS OF DIVISION	PROFIT
	A	B		
4.	\$3000	\$2000	A, $\frac{2}{5}$; B, $\frac{3}{5}$	\$500
5.	\$3000	\$3000	Equal	\$800
6.	\$1000	\$5000	A, $\frac{1}{3}$; B, $\frac{2}{3}$	\$750
7.	\$1500	\$6000	Investment	\$600
8.	\$2000	\$4000	A, $\frac{3}{8}$; B, $\frac{5}{8}$	\$960
9.	\$1500	\$2000	A, $\frac{1}{3}$; B, $\frac{2}{3}$	\$690
10.	\$1500	\$7500	Investment	\$1200

EXERCISES

Find each partner's share of the profit in the following cases:

	INVESTMENT			BASIS OF DIVISION	PROFIT
	A	B	C		
1.	\$9000	\$9000	\$6000	Investment	\$2850
2.	\$9000	\$9000	\$6000	A, $\frac{1}{3}$; B, $\frac{1}{2}$; C, $\frac{1}{6}$	\$2850
3.	\$5000	\$5000	\$5000	A, $\frac{2}{5}$; B, $\frac{2}{5}$; C, $\frac{1}{5}$	\$3000
4.	\$6000	\$8000	\$2000	Investment	\$2400
5.	\$7500	\$2500	\$5000	A, $\frac{1}{2}$; B, $\frac{1}{3}$; C, $\frac{1}{6}$	\$2280
6.	\$3500	\$3500	\$5000	Equal	\$1500
7.	\$5000	\$3000	\$4000	A, $\frac{1}{2}$; B, $\frac{1}{4}$; C, $\frac{1}{4}$	\$1020
8.	\$6500	\$4500	\$5500	A, $\frac{3}{8}$; B, $\frac{1}{3}$; C, $\frac{7}{24}$	\$1800
9.	\$6600	\$5600	\$4600	Investment	\$1950
10.	\$3750	\$5250	\$4500	Investment	\$1350

11. Three men form a partnership, A investing \$5000; B, \$4000; and C, \$7000. Profits and losses are to be shared as follows: A, $\frac{2}{7}$; B, $\frac{2}{7}$; and C, $\frac{3}{7}$. In a year in which the net profit is \$4228.35, find each partner's share and the per cent which is returned upon the investment of each partner.

12. If the profits in Ex. 11 had been shared in proportion to the investments of the partners, what would then have been each partner's share?

13. In a partnership formed by A, B, and C, the net profit for the first 6 mo. was \$2847.90 and the net loss for the next 6 mo. was \$762.24. Each partner invested \$7500, but the profits and losses are shared as follows: A, 30%; B, 30%; and C, 40%. Prepare a table which shows each partner's share of the profit or loss for each half year and his net income for the year.

14. In a partnership formed by A and B, A invests \$10,000, B invests \$12,500, and profits and losses are shared equally. The net loss for a year was \$2746.70. How much better would it have been for A if losses had been shared in proportion to the investment?

15. In a partnership, A invests \$8000 and B invests \$12,000. The articles of partnership provide that profits in excess of \$2400 shall be divided equally, but that profits less than \$2400 (and losses) shall be divided in proportion to the investment. If the profits for three years are as follows: 1st year \$1789.50, 2d year \$3487.20, 3d year \$2510.80, find each partner's share of the profits for each year and their totals for the three years.

Division of profits by average investment. Sometimes, owing to changes during the year in the amount invested by each partner, one partner's share of the capital has been invested in the business longer than another's. A division of profits and losses which takes into account not only the amount invested but also the time in which the investment has actually been used in the business is known as a division according to the **average investment**. Such a method is illustrated in the following example.

EXAMPLE. On Jan. 1, A and B form a partnership, each investing \$5000. On July 1, A invests \$1000 more and on Oct. 1 he withdraws \$1000. On March 1, B withdraws \$1000 and on Dec. 1 he invests \$1000. If profits and losses are shared in proportion to the average investment, find each partner's share of a net profit of \$1425.

Although the investment of each partner on Jan. 1 and on Dec. 31 is \$5000 in each case, it is apparent that A has had more money invested for a longer time than B. Hence, to divide the profits equally would be unfair to A.

Solution. Considering A's investment first, he had \$5000 invested from Jan. 1 to June 30 (6 mo.), \$6000 from July 1 to Sept. 30 (3 mo.), and \$5000 from Oct. 1 to Dec. 31 (3 mo.).

\$5000 for 6 mo. is equivalent to	\$30,000 for 1 mo.
\$6000 for 3 mo. is equivalent to	18,000 for 1 mo.
\$5000 for 3 mo. is equivalent to	15,000 for 1 mo.
	12 $\overline{) \$63,000}$ for 1 mo.
A's average investment for the year is	\$5,250

Similarly, B had \$5000 invested from Jan. 1 to Feb. 28 (2 mo.), \$4000 from March 1 to Nov. 30 (9 mo.), and \$5000 from Dec. 1 to Dec. 31 (1 mo.).

\$5000 for 2 mo. is equivalent to	\$10,000 for 1 mo.
\$4000 for 9 mo. is equivalent to	36,000 for 1 mo.
	5,000 for 1 mo.
	12 $\overline{) \$51,000}$ for 1 mo.
B's average investment for the year is	\$4,250

$\$5250 + \$4250 = \$9500$, total average investment of firm.
 $\$1425$ (net profit) $\div$ $\$9500 = 0.15$ (or 15%), rate of profit.
 $\$5250 \times 0.15 = \787.50 , A's share of profit.
 $\$4250 \times 0.15 = \637.50 , B's share of profit.

If the profit in the above case had been divided only in accordance with the amount invested, each partner would have received \$712.50.

EXERCISES

1. On Jan. 1, C invests \$10,000 and D invests \$7500 in a partnership in which they agree to share profits and losses in proportion to the average investment. On July 1, C withdraws \$2000, and on Sept. 1, D withdraws \$1000. If the net profit for the year is \$2200, find the share of each partner.

2. On July 1, E and F form a partnership in which E invests \$25,000, F invests \$15,000, and profits and losses are to be shared in proportion to the average investment. On Sept. 1, E withdraws \$1000 and F withdraws \$500; on Dec. 1, each withdraws \$1000; and on March 1, F invests \$1000. If the net profit for the year is \$6000, what is the share of each partner?

3. On Jan. 1, G invests \$4500 in a partnership with H who invests \$6000, and profits and losses are to be shared in proportion to the average investment. On April 1, G invests \$1500 and H withdraws \$1000. If the net loss for the year is \$1350, how much of this loss should be borne by each partner?

Brown & Hart
Financial Statement
December 31, 19-

<i>Assets</i>			
Cash	1243	50	
Notes Receivable	2833	25	
Accounts Receivable	3387	25	
Merchandise Inventory	4365	60	
Real Estate	1500	—	
Buildings and Fixtures	6500	—	
Store Equipment	1800	—	
Prepaid Expenses	366	20	
Total Assets			21995 80
<i>Liabilities</i>			
Notes Payable	3436	30	
Accounts Payable	1050	—	
Total Liabilities			4486 30
<i>Proprietary Interest</i>			
C. H. Brown; Investment	8000	—	
C. H. Brown; Drawing, Cr.	2142.50		
	10142	50	
J. S. Hart; Investment	8000	—	
J. S. Hart; Drawing, Dr.	633	—	
Total Proprietary Interest			17509 50

FINANCIAL STATEMENT OF A PARTNERSHIP

Financial statement of a partnership. On page 256 the proper form for the financial statement of a business owned and operated by an individual was shown. The illustration above shows the form for the financial statement of a partnership.

In partnerships, the investment of each partner is kept separate from transactions of the business with the members of the firm *as individuals*. Since such transactions do not affect a partner's investment, they are recorded in a separate account called a partner's **Personal**, or **Drawing, account**. The balance of a Drawing account represents either an amount due the partner by the business or an amount due the business by the partner.

EXERCISES

Following the model on page 336, make a financial statement from the following data:

1. On Dec. 31, 19—, the books of the partnership of Sommer & Conway show the following balances:

Assets: Cash, \$1150; Notes Receivable, \$2180.60; Accounts Receivable, \$2408.45; Merchandise Inventory, \$11,210.50; Real Estate, \$2000; Store Building, \$7350; Store Equipment, \$2150.

Liabilities: Notes Payable, \$2475; Accounts Payable, \$3782.75.

Proprietary Interest: A. L. Sommer, Investment, \$12,000; A. L. Sommer, Drawing, Dr., \$183.40; M. H. Conway, Investment, \$10,000; M. H. Conway, Drawing, Cr., \$375.20.

2. On Dec. 31, 19—, the books of the partnership of Green & Linden show the following balances:

Assets: Cash, \$1255.20; Notes Receivable, \$2050; Accounts Receivable, \$3170; Merchandise Inventory, \$13,284.85; Store Equipment, \$1450.50.

Liabilities: Notes Payable, \$1158.25; Accounts Payable, \$2827.80.

Proprietary Interest: W. B. Green, Investment, \$8500; W. B. Green, Drawing, Cr., \$100; M. H. Linden, Investment, \$8500; M. H. Linden, Drawing, Cr., \$124.50.

3. On Dec. 31, 19—, the books of the partnership of Benton & Wood show the following balances:

Assets: Cash, \$830.60; Notes Receivable, \$1527.50; Accounts Receivable, \$3786.25; Securities on hand, \$3480; Merchandise Inventory, \$12,418.19; Prepaid Expenses, \$287.50.

Liabilities: Accounts Payable, \$2640.78.

Proprietary Interest: C. M. Benton, Investment, \$12,500; C. M. Benton, Drawing, Dr., \$112.63; W. L. Wood, Investment, \$7500; W. L. Wood, Drawing, Dr., \$198.11.

REVIEW EXERCISES

1. In a partnership formed by A, B, and C each partner invests \$8750, and profits and losses are shared as follows: A, 35%; B, 35%, C, 30%. The net profits or losses of the firm for 5 yr. were as follows: 1st year, profit, \$2784.28; 2d year, loss, \$642.96; 3d year, profit, \$3973.39; 4th year, profit, \$4747.42; 5th year, loss, \$846.80. What is each partner's share of the annual profit or loss, and what is his net income for the whole period?

2. In a partnership, A, B, and C share profits as follows: A, 50%; B, 30%; C, 20%. They decide to add to their operations a new business in which profits and losses are to be divided equally. The first year the old business showed a profit of \$12,582.40 and the new business showed a loss of \$3822.90. The second year the old business showed a loss of \$1875.50 and the new business showed a profit of \$8752.20. What is the annual net return to each partner and the net return for the 2-year period?

3. In a partnership in which profits and losses are divided in proportion to the average investment, A's investment account shows the following transactions: Jan. 1, invested \$7500; March 15, withdrew \$2500; June 1, invested \$3500; Sept. 1, invested \$1500; Dec. 1, withdrew \$2500. B's investment account shows the following transactions: Jan. 1, invested \$6000; May 1, invested \$2000; July 15, withdrew \$1750; Oct. 15, invested \$1000; Dec. 15, withdrew \$1250. If the net profit for the year was \$3761.90, find the share of each partner.

4. Three men form a partnership, A investing \$20,000, B investing \$10,000, and C investing \$4000. Profits and losses are shared as follows: A, $\frac{3}{5}$; B, $\frac{1}{4}$; and C, $\frac{3}{20}$. If the profits for the year were \$4276.90, find the share of each partner.

5. In Ex. 4 find the per cent of return which each partner received upon his investment.

6. Prepare the financial statement of the firm of Roe & Law at the close of business on Dec. 31, 19—, from the following data:

Assets: Cash, \$275.84; Accounts Receivable, \$971.37; Inventories, \$11,382.11; Real Estate, \$1000; Buildings, \$3945.48; Machinery, \$8573.66; Small Tools, \$868.

Liabilities: Notes Payable, \$2400; Accounts Payable, \$1842.79.

Proprietary Interest: F. R. Roe, Investment, \$10,000; F. R. Roe, Drawing, Cr., \$149.25; J. W. Law, Investment, \$12,500; J. W. Law, Drawing, Cr., \$124.42.

* CORPORATIONS

STOCKS

Nature of a corporation. Under the provisions of the laws of the separate states, individuals may form an association, known as a **corporation**, for the purpose of carrying on a business. The individuals forming the association are known as the **incorporators**, and the association is said to be **incorporated**.

The investment of the corporation for the purpose of carrying on the business is called the **capital stock**, and the business is said to be **capitalized** for the amount of the capital stock.

The capital stock corresponds to the proprietor's investment in a sole proprietorship or to the total of the partner's investment in a partnership.

The capital stock is divided into convenient units known as **shares**, and the owners of the shares are called the **stockholders** of the corporation.

Advantages of corporate organization. The advantages of incorporating a business are as follows:

1. The existence of the corporation is not dependent upon the life and continued activity of any of its stockholders.

2. In a legal sense, the corporation acts as an individual in any cases that arise, and it is not necessary to deal with the stockholders as individuals.

3. A corporation may buy and sell real estate the same as an individual.

4. The business usually benefits from the unification and continuance of business policies which are possible under this type of organization.

5. The liabilities of the stockholders for corporate indebtedness is generally limited to the amount of stock which they own.

The advantages under 1 and 5, above, give corporations a decided advantage over partnerships in securing capital.

Forming a corporation. When the incorporators of a business have satisfied the requirements of the laws of the state under which the incorporation is made, a **charter**, or **certificate of incorporation**, is issued by the state.

The charter may be a formal document issued by a state official, or it may be an authorization from the state in some other form. If it is a formal certificate, it generally specifies the name, the capitalization, the location, the purposes, and the duration of the corporation.

The maximum amount of capital stock which may be issued, as defined in the charter, is known as the **authorized capital stock**. Most states require that a certain part of the authorized capital stock shall be subscribed and paid for before the corporation has legal existence. Such stock is known as **paid-up capital stock**.

Capital stock which persons have agreed to purchase is known as **subscribed capital stock**. Any part of the authorized capital stock which is not subscribed is called **unsubscribed capital stock**.

The first issue of capital stock is generally disposed of by subscription. The persons agreeing to invest in the corporation sign a subscription list, similar to the one shown on page 341.

Stock may be paid for in full at the time of subscription or at some specified date, or a portion may be paid down and the balance in installments as agreed upon. Stock may sometimes be paid for in property or in services.

Thus the owner of an established business may incorporate it in connection with other individuals, and he may be allotted a certain amount of the stock for the property of his business and another amount for his services in connection with forming the corporation.

Capital stock which has been purchased from the stockholders by the corporation, or which is donated to it by the stockholders, is called **treasury stock**.

Thus, a corporation capitalized for \$100,000 has an authorized capital stock of \$100,000. If half of it is subscribed, the subscribed capital stock

SUBSCRIPTION LIST

We, the undersigned, do hereby subscribe for and agree to take the number of shares of the Ohio Manufacturing Company's stock set opposite our respective names:

Date	Name of Stockholder	Address	Number of Shares	Amount
Sept. 1	J. L. Clark	Janesville, Wis.	50	5000 —
"	F. M. Barr	Janesville, Wis.	40	4000 —
"	E. M. Read	Janesville, Wis.	35	3500 —
"	F. W. Brown	Janesville, Wis.	50	5000 —
"	W. R. Parker	Janesville, Wis.	50	5000 —
"	A. B. Cork	Janesville, Wis.	25	2500 —

SUBSCRIPTION LIST

is \$50,000, and if half of the stock subscribed is paid for, the paid-up capital stock is \$25,000. In this case the unsubscribed capital stock is \$50,000. In case stock to the value of \$2000 is purchased by the corporation from stockholders, the treasury stock is \$2000.

ORAL EXERCISES

1. Define a corporation, and mention one.
2. Under what laws are corporations formed?
3. What is the charter of a corporation?
4. Define in your own words the following terms: incorporators, subscribers, stockholders, capital stock, authorized capital stock, unsubscribed capital stock, paid-up capital stock, treasury stock.

5. Discuss the advantages of corporate organization of a business by comparing them with the corresponding conditions prevailing in sole proprietorships and partnerships.

6. The authorized capital stock of the Ohio Manufacturing Company, whose subscription list is shown on page 341, is \$50,000. After the subscriptions shown in the list are made, how much of the company's stock remains unsubscribed?

7. If it is required in Ex. 6 that a payment of 20% be made on each subscription on Sept. 10, how much is due on that date?

8. If the stock in Ex. 7 must be paid for in full on Jan. 1, how much is due on that date? What is then the amount of the Ohio Manufacturing Company's paid-up capital stock?

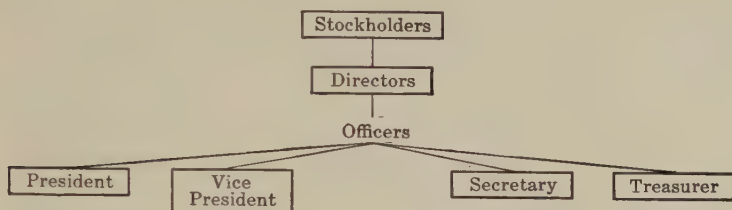
Corporate administration. The original stockholders of the corporation, those who have purchased the original issue of the capital stock, meet and elect a **board of directors** which is to supervise the conduct of the business and to pass judgment upon questions of general management. The stockholders also adopt rules, known as **by-laws**, which cover the details of management and organization.

Each stockholder is entitled to a vote for each share of stock which he owns, and if he is unable to attend the meeting may authorize someone else to vote for him. Such a procedure is called **voting by proxy**. After the first meeting at which the organization is effected, annual meetings of the stockholders are held, and any holder of stock in the corporation is entitled to attend and vote at such a meeting.

At such meetings directors are reelected or new ones chosen, changes in the by-laws are considered, and so on. Thus the stockholders, through the directors, are the real managers of the business.

The board of directors elect the **officers** of the corporation who conduct its actual management. The customary officers of a corporation are a president, vice-president, secretary, and treasurer.

The following chart shows graphically how the business of a corporation is administered :



CORPORATE ORGANIZATION

Stock certificate. Ownership of a share of stock is evidenced by a **stock certificate**, although generally one certificate is issued for the total number of shares purchased. The following form illustrates a stock certificate which shows that the holder owns 100 shares of stock in the Citizens Trust Company of New York :



STOCK CERTIFICATE

The holder of this certificate is entitled by his rights as a stockholder to vote at the stockholders' meetings, to share in the profits earned by the corporation, or to participate in a distribution of the assets if the corporation is dissolved.

A stockholder cannot claim ownership of any particular property of the corporation, since he has a right only to that part of all the corporation property that the number of shares which he owns is of the total number of shares issued.

Values of stock. The **par value** of stock is the value assigned to a share when it is issued. A common par value for shares is \$100. Generally the par value is decided upon first, and then the total capital desired is divided by the par value to determine the number of shares to issue.

Thus, if the par value of a share is \$100, a corporation which is to be capitalized for \$100,000 would issue 1000 shares of stock.

Sometimes stock is issued without a stated par value.

The **book value** of a share is the result obtained by dividing the excess of assets over liabilities of the corporation by the number of shares of stock issued.

The **market value** of a share depends upon whether or not the company is making a profit in its business, upon the financial condition of the business, and upon present and future (estimated) conditions of supply and demand for the product of the company.

Depending upon the conditions mentioned above, the market value may be above or below the book value. Thus, if the assets of a corporation are \$125,000 in excess of its liabilities, and there are 1000 shares of stock outstanding, the book value of a share is $\$125,000 \div 1000$, or \$125. The market value might, however, be more or less than this amount.

Common stock. The different conditions under which certain issues of stock are made give rise in some states to different classes of stock. **Common stock**, the most usual type, carries the same rights and obligations to all its owners. When capital stock is referred to without qualifications, it is understood that the entire stock of the corporation is common stock.

Dividends. The net profit of a corporation is distributed among the stockholders in the form of **dividends**. Each share of the same kind of stock participates equally in such a distribution, so that the amount received by a stockholder depends upon the number of shares which he holds.

The method of determining and paying dividends is discussed later.

Preferred stock. Stock which grants its owners preference over the common stockholders in various points is called **preferred stock**. Generally, preferred stock shares in the distribution of dividends before any dividends are paid on the common stock. The dividend rate on preferred stock is generally fixed at the time of issue.

If dividends are not paid on preferred stock for one or more years, it is sometimes provided that when payments of dividends are resumed these unpaid dividends must be paid before dividends are paid on any other type of stock. Such stock is known as **cumulative preferred stock**. If unpaid dividends are not so payable, the stock is known as **noncumulative preferred stock**.

The laws of the different states in regard to the meaning and issuance of preferred stock vary so widely that in a detailed study the special conditions governing the case should be investigated.

When the dividend rate on preferred stock is fixed and a company has a successful year, it frequently happens that the dividend on the common stock is larger than that on the preferred stock, and hence the common stock is more valuable, since it offers greater returns to investors.

ORAL EXERCISES

1. Referring to the stock certificate on page 343, state the name of the corporation.

2. Define in your own words the following terms: board of directors, stock certificate, par value, book value, market value, common stock, preferred stock.

3. The subscription list of a corporation to be capitalized at \$75,000 shows that \$60,000 has been subscribed. Of this amount \$15,000 has been paid in cash, and the balance is to be paid in five equal monthly installments. What is the amount of each installment?

4. In Ex. 3, what is the authorized capital stock of the corporation? What is the subscribed capital stock? What is the amount of the unsubscribed stock?

5. A corporation is to be formed with a capital stock of 1500 shares of par value \$100. The subscriptions total 1200 shares, of which 600 shares are to be paid for in property and 600 in cash. What is the total amount subscribed? What is the value of the property received for the 600 shares? What is the amount of the unsubscribed stock?

6. The excess of assets of a corporation over liabilities is \$10,750,000. If there are 10,000 shares of stock outstanding, what is the book value of a share?

Payment of dividends. After the net profit of a corporation for the year is determined, the amount is transferred to the Surplus (undivided profits) account. The directors then decide what amount they can declare as dividends on the various classes of stock. The whole amount transferred to Surplus is not always distributed in dividends, as part of it may be allowed to remain for emergencies.

Dividends may be declared annually, semiannually, or quarterly, and generally as a per cent on the par value of a share. On stock which does not have a stated par value, dividends are declared as a specified amount per share.

Thus, if the directors declare an annual dividend of 8% on the common stock, each common stockholder will receive 8% of the par value of the stock which he owns. Thus, if the par value is \$100, he will receive \$8 per share; if the par value is \$50, he will receive \$4 per share, and so on.

The directors might also have declared a quarterly dividend of 2% in the above case, or a semiannual dividend of 4%, with the same annual return to the stockholders.

Dividends may be paid in cash (check), or in many of the states by issuing more stock of the company to the stockholders.

In case it is necessary to increase the authorized capital stock to take care of the payment of a stock dividend, permission must be obtained from the proper state officials.

The stockholders have no legal claim to any portion of the undivided profits until the directors declare the dividend.

Payments of the dividends are made to those stockholders whose names appear upon the stock records of the corporation.

In large corporations whose stocks are constantly changing hands, dividends are usually declared payable to those stockholders whose names are recorded on the stock books on a certain date.

Thus, the following notice is often found in the financial pages of newspapers: "The annual dividend of the — Company is payable to stockholders of record at the close of business on Dec. 31, 19—."

ORAL EXERCISES

State the amount of the dividend in each of the following cases:

	PAR VALUE PER SHARE	ANNUAL DIVIDEND RATE	DIVIDEND PAYABLE
1.	\$100	7%	Annually
2.	\$50	8%	Semiannually
3.	\$100	6%	Quarterly
4.	\$100	7%	Quarterly
5.	\$50	12%	Semiannually
6.	\$75	8%	Annually
7.	\$100	9%	Semiannually
8.	\$100	10%	Quarterly
9.	\$50	7%	Semiannually
10.	\$75	6%	Quarterly

EXERCISES

1. The net profit of a corporation for a year is \$21,975.50. If the capital stock outstanding amounts to \$150,000 and the directors declare a cash dividend of 12%, what is the amount of the dividend? What is the balance in Surplus after the dividend is paid?

2. If a corporation which has capital stock of \$500,000 outstanding declares quarterly dividends of $1\frac{3}{4}\%$, what is the total of the annual dividends? What amount is received annually by the owner of 47 shares?

In this text, unless otherwise stated, the par value of a share is to be taken as \$100.

3. Semiannual dividends of $2\frac{1}{4}\%$ are declared by a corporation which has capital stock to the amount of \$250,000 outstanding. What is the annual income for the holder of 65 shares? How much is paid to the stockholders annually?

4. In a year in which the gross profits of a corporation are \$389,516.34, the expenses are \$176,237.69. If the directors declare a dividend of 8% on the outstanding capital stock which amounts to \$1,250,000 and reserve the balance for emergencies, what is the amount thus reserved?

5. A manufacturing company which has a capital stock of \$450,000 makes a net profit for the year of \$77,828.49. If the directors decide to reserve \$23,828.49 for contingencies and to pay dividends from the remainder, what annual rate of dividend should they declare?

6. The par value of a share of the Pennsylvania Railroad is \$50. If a semiannual dividend of 3% is paid on its capital stock of \$499,296,400, what amount is paid to the stockholders?

7. During a half year a manufacturing company which has a capital stock of \$2,500,000 made a net profit of \$136,472 and declared a semiannual dividend of $3\frac{1}{2}\%$. If 15% of the net profit was reserved for plant extensions, what balance was left in the Surplus account after the dividends were paid?

8. The dividend rate on the preferred stock of a company is fixed at 7%, and the directors declare a dividend of 12% on the common stock. If the amount of preferred stock issued is \$200,000 and the amount of the common stock is \$300,000, what amount is paid in dividends to the stockholders?

9. A corporation pays dividends of 8% on its preferred stock, which amounts to \$100,000. The following table shows the net profits and rate of dividend on its common stock, which amounts to \$150,000, for a period of three years:

YEAR	NET PROFIT	DIVIDEND RATE
1st	\$28,285.70	8%
2d	19,189.20	6%
3d	23,571.92	8%

a. What was the amount of dividends paid each year on the common stock?

b. What was the total amount distributed as dividends each year?

c. What balance remained in the Surplus account after the dividends for the three years were paid?

10. If the Ohio Manufacturing Co., whose subscription list is shown on page 341, declares a dividend of 8% on its capital stock, what amount is received by each stockholder whose name appears on the list?

11. The preferred stock of a corporation pays $7\frac{1}{2}\%$ and amounts to \$200,000, while the common stock amounts to \$250,000. From a net profit of \$66,514.75 the directors pay the dividends on the preferred stock and one of 12% on the common stock. What is the total amount paid in dividends, and what is the balance of the Surplus account after the dividends are paid?

12. The directors of a corporation declare quarterly dividends of $1\frac{3}{4}\%$ on the preferred stock and $2\frac{1}{2}\%$ on the common stock. If the number of shares outstanding is 3250 for the preferred stock and 3750 for the common stock, what amount is paid out during the year in dividends?

13. If T. M. Nixon owns 15 shares of the preferred and 27 shares of the common stock of the corporation in Ex. 12, what is the income from his holdings for the year?

14. A manufacturing corporation capitalized at \$750,000 earned a net profit for each quarter as follows:

1st quarter . .	\$23,718.48	3d quarter . .	\$18,916.83
2d quarter . .	17,432.57	4th quarter . .	19,127.54

If a quarterly dividend of $2\frac{1}{2}\%$ was paid, what was the balance of the Surplus account at the end of the year?

Stock exchange. Certain stocks of large corporations are bought and sold almost daily. Organizations known as **stock exchanges** have been formed in many of the larger cities to provide facilities for trading in stocks.

The New York Stock Exchange is the largest in America. A corporation which desires to have its stock traded in on the floor of this exchange must satisfy all the requirements set by the officials of the exchange. The stock is then "listed on the exchange."

Stock broker. A person who buys and sells stocks for others on commission is called a **stock broker**. Stocks are usually bought and sold through stock brokers.

The rates of commission prescribed by the New York Stock Exchange which may be charged to nonmembers by brokers for buying and selling stocks are as follows:

COMMISSION RATES		
SELLING PRICE PER SHARE	RATE PER 100 SHARES	
Under \$10	7	50
\$10 and over, but under \$125	15	00
\$125 and over	20	00

Proportional parts of the above rates are charged for lots less than 100 shares, but a minimum charge of \$1 is made where the commission at the above rates would be less than \$1.

Stock quotations. The prices at which the different shares may be bought or sold are posted in the stock exchanges and are known as **stock quotations**.

The market value of a share, as shown by the quotations, may be more than the par value of the share, in which case the stock is said to sell **above par**, or **at a premium**.

If the stock of a corporation continues to pay large dividends, people will be willing to pay more than par for it. If stock of par value \$100 is selling at \$106.50 per share, the purchaser must pay a premium of \$6.50 per share for it in addition to his broker's commission.

If a share sells for less than its par value, it is said to sell **below par**, or **at a discount**.

If a corporation has suffered extensive losses and its future prospects are uncertain, its stock may decline in value and sell below par. If a person sells stock of par value \$100 at \$87.50 per share, he sells at a discount of \$12.50 per share, and in addition must pay his broker's commission.

Stock quotations are generally given without the dollar sign. Thus a quotation of 85 means that shares of par value \$100 may be bought at \$85; one of $114\frac{1}{2}$ means that the price is \$114.50 per share, and so on. On the New York Stock Exchange quotations are expressed in dollars and eighths of a dollar.

EXAMPLES. 1. Find the cost of purchasing through a broker 100 shares of stock which is quoted at $98\frac{1}{4}$.

Solution. $\$98.25 \times 100 = \9825 , cost of stock.

By referring to the table on page 350, it is seen that the broker's commission on stock selling under \$125 per share is \$15 per 100 shares.

Hence $\$9825 + \$15 = \$9840$, total cost.

2. Find the amount received from selling through a broker 25 shares of stock quoted at 114.

Solution. $\$114 \times 25 = \2850 , selling price.

The broker's commission is \$15 per 100 shares, or 15¢ per share.

Then $\$0.15 \times 25 = \3.75 , commission.

Hence $\$2850 - \$3.75 = \$2846.25$, amount received from sale.

3. Find the cost of purchasing through a broker 4 shares of stock quoted at $131\frac{3}{8}$.

Solution. $\$131.375 \times 4 = \525.50 , cost of stock.

The broker's commission is \$20 per 100 shares, or 20¢ per share. But in this case the commission would be only 80¢, and hence the minimum charge of \$1 is made.

Then $\$525.50 + \$1 = \$526.50$, total cost.

Since stock must generally be bought or sold through a broker, it is understood that in the following pages whenever stock transactions are involved the broker's commission is always to be figured as in the above cases.

The table on page 352 shows the quotations for certain stocks as listed on the New York Stock Exchange at a recent date.

In the first and second columns is shown the highest and the lowest price respectively at which the stock sold during the preceding year. The third column shows the number of shares sold on the day for which the

STOCK-EXCHANGE QUOTATIONS

PRECEDING YEAR		SALES	NAME OF STOCK AND DIVIDEND RATE	OPENING	HIGH	LOW	CLOSING	NET CHANGE
High	Low							
147 $\frac{3}{8}$	111	3000	Am. Car & Fy. (12) . . .	125	127 $\frac{1}{4}$	124	126 $\frac{1}{2}$	+ 1 $\frac{1}{4}$
116 $\frac{1}{4}$	105 $\frac{3}{4}$	300	Am. Car & Fy, pf. (7) . .	112	112	112	112	+ 2
109 $\frac{1}{4}$	74	2800	Am. Locomo. (6)	84	86	83	85	+ 1 $\frac{1}{2}$
107	95 $\frac{1}{2}$	200	Am. Locomo., pf. (7) . . .	102 $\frac{1}{2}$	102 $\frac{1}{2}$	102	102	+ 2 $\frac{3}{8}$
165 $\frac{1}{2}$	55 $\frac{1}{2}$	21,000	Am. Woolen (7)	67	70 $\frac{7}{8}$	65 $\frac{3}{4}$	70 $\frac{3}{4}$	+ 2
105 $\frac{1}{2}$	88 $\frac{1}{2}$	200	Am. Woolen, pf. (7)	95 $\frac{1}{2}$	95 $\frac{1}{2}$	95 $\frac{1}{2}$	95 $\frac{1}{2}$	+ 2 $\frac{3}{8}$
90 $\frac{3}{8}$	76	1300	Atch. T. & S. F. (6)	83 $\frac{1}{2}$	84 $\frac{1}{2}$	83 $\frac{1}{2}$	84 $\frac{1}{2}$	+ 1
82	72	1070	Atch. T. & S. F., pf. (5) . .	79 $\frac{1}{2}$	79 $\frac{1}{2}$	79	79	+ 1
148 $\frac{1}{2}$	78	24,180	Baldwin Loco. (7)	88 $\frac{1}{2}$	92 $\frac{3}{8}$	87 $\frac{1}{2}$	91 $\frac{1}{2}$	+ 2 $\frac{1}{2}$
102 $\frac{1}{2}$	92	750	Baldwin Loco. pf. (7) . . .	99 $\frac{1}{2}$	100	99 $\frac{1}{2}$	100	+ 1
44 $\frac{3}{8}$	21	2800	Chi. Mil. & St. P.	29 $\frac{1}{2}$	30 $\frac{1}{4}$	29 $\frac{1}{2}$	30	+ 1 $\frac{1}{2}$
65	36 $\frac{3}{4}$	4400	Chi. Mil. & St. P., pf. . . .	44 $\frac{1}{4}$	46	44	45 $\frac{3}{8}$	+ 1
278 $\frac{1}{2}$	70	43,600	Crucible Steel (8)	87	97 $\frac{1}{2}$	85 $\frac{1}{2}$	96 $\frac{1}{2}$	+ 9 $\frac{3}{8}$
172	116 $\frac{1}{2}$	3700	General Electric (8)	121 $\frac{1}{2}$	122 $\frac{1}{2}$	120 $\frac{1}{2}$	122	+ 3 $\frac{1}{2}$
97 $\frac{1}{2}$	80 $\frac{7}{8}$	200	Illinois Central (7)	88 $\frac{1}{2}$	88 $\frac{1}{2}$	88 $\frac{1}{2}$	88 $\frac{1}{2}$	+ 1 $\frac{1}{2}$
155	120	200	Kresge (S. S.) (6)	54 $\frac{1}{2}$	56	53 $\frac{1}{2}$	56	+ 1 $\frac{1}{2}$
112 $\frac{1}{2}$	94	200	Louis. & Nash. (7)	102 $\frac{3}{8}$	103	102 $\frac{7}{8}$	103	+ 1
125	96	100	Nat. Biscuit (7)	103	103	103	103	+ 1
84 $\frac{1}{2}$	64 $\frac{1}{2}$	2800	N.Y. Central (5)	73	74	72 $\frac{1}{2}$	73	+ 1 $\frac{1}{2}$
124	95 $\frac{3}{4}$	400	Pullman (8)	106	107	106	107	+ 1
229	180	100	Tidewater Oil (16)	168	168	168	168	+ 2
143 $\frac{3}{4}$	54 $\frac{1}{2}$	21,500	U. S. Rubber (8)	67 $\frac{3}{8}$	71	65	70 $\frac{1}{2}$	+ 2 $\frac{3}{8}$
115 $\frac{1}{2}$	95 $\frac{1}{2}$	300	U. S. Rubber, 1st pf. (8) . .	103 $\frac{1}{4}$	103 $\frac{3}{4}$	103 $\frac{3}{4}$	103 $\frac{3}{4}$	+ 2
109	76 $\frac{1}{2}$	33,000	U. S. Steel (5)	82 $\frac{3}{8}$	83 $\frac{3}{4}$	82 $\frac{1}{2}$	83 $\frac{1}{2}$	+ 1
115 $\frac{3}{4}$	104 $\frac{1}{4}$	500	U. S. Steel, pf. (7)	109	109	109	109	+ 1

report was made. The fourth column shows the name of the stock in abbreviated form and the rate of dividend last declared. The fifth, sixth, seventh, and eighth columns show the opening, highest, lowest, and closing prices respectively for the stock on the day considered. The ninth column shows the net change (advance indicated by + and decline by -) between the closing prices of the preceding day and this day.

The abbreviation "pf." in the fourth column stands for "preferred." If the profits of a corporation are such that the rate of dividends on the common stock is greater than the fixed rate on the preferred stock, the quotation for the common stock is often higher.

Thus, the highest price at which the common stock of the American Car and Foundry Co. sold in the preceding year was $147\frac{3}{8}$; the lowest, 111. On the day of the report 3000 shares were sold, and during the day the stock opened at 125, advanced to $127\frac{1}{4}$ for some sales, other sales were made at the lowest price of 124, and the stock closed at $126\frac{1}{2}$. The net change over the closing price of the preceding day was $+1\frac{1}{4}$, spoken of as an advance of $1\frac{1}{4}$ points. .

ORAL EXERCISES

The exercises on this page and on the following pages which involve stock transactions refer to the table on page 352, and all sales and purchases are made through a broker. Unless preferred stock (pf.) is specifically mentioned, common stock is to be understood.

1. State the cost of 100 shares of Illinois Central bought at the lowest price for the day.

2. The first eight quotations show the prices of the common stock and the preferred stock of four companies. Using the highest prices for the day, which stock in each case is worth more and how much more per share?

As previously stated, the par value of a share is to be considered as \$100 unless otherwise stated.

3. If a broker sells 100 shares of United States Steel at the highest price of the day, how much does his principal receive from the sale?

4. State the cost of 100 shares of stock of the National Biscuit Co. if bought at the opening price.

5. Which two prices for the common stock of the United States Rubber Co. showed the greatest change? How great a difference was there between these two prices?

6. State the cost of 100 shares of each of the following stocks if bought at the opening price:

- | | |
|-----------------------------------|-------------------------------------|
| <i>a.</i> Baldwin Locomotive. | <i>d.</i> Louisville & Nashville. |
| <i>b.</i> Baldwin Locomotive, pf. | <i>e.</i> United States Rubber, pf. |
| <i>c.</i> Kresge. | <i>f.</i> General Electric. |

7. State the amount received from the sale of 100 shares of each of the following stocks if the sales were made at the highest price of the day:

- | | |
|--------------------------------|--|
| <i>a.</i> United States Steel. | <i>e.</i> Chicago, Milwaukee & St. Paul. |
| <i>b.</i> Pullman. | <i>d.</i> American Woolen. |
| | <i>e.</i> Atchison, Topeka & Santa Fe, pf. |
| | <i>f.</i> American Locomotive, pf. |

EXERCISES

Find the cost of 500 shares of each of the following stocks if purchased at the opening price:

1. United States Rubber.
2. American Locomotive.
3. Chicago, Milwaukee & St. Paul, pf.
4. New York Central.
5. Atchison, Topeka & Santa Fe.
6. Crucible Steel.
7. Baldwin Locomotive, pf.
8. Louisville & Nashville.
9. General Electric.
10. American Woolen.

Find the amount received from the sale of 300 shares of each of the following stocks if sold at the highest price for the day:

11. American Car & Foundry, pf.
12. American Locomotive, pf.
13. Chicago, Milwaukee & St. Paul.
14. Atchison, Topeka & Santa Fe, pf.
15. Illinois Central.
16. Kresge.
17. Pullman.
18. National Biscuit.
19. Tidewater Oil.
20. United States Rubber, pf.

Profit and loss on stock transactions. The method of finding the profit or loss which is incurred by buying and selling stocks at different prices is shown in the following example.

EXAMPLE. A man purchased 500 shares of United States Steel at $82\frac{3}{8}$ and later sold the stock at $94\frac{7}{8}$. Find the profit on the transaction.

Solution Brokerage must be considered on both the purchase and the sale at \$15 per 100 shares, or 15¢ per share.

$$\$82.375 + \$0.15 = \$82.525, \text{ cost per share.}$$

$$\$94.875 - \$0.15 = \$94.725, \text{ receipts per share.}$$

$$\$94.725 - \$82.525 = \$12.20, \text{ profit per share.}$$

$$\$12.20 \times 500 = \$6100, \text{ total profit.}$$

EXERCISES

Find the profit or loss on the sale of 300 shares of each of the following stocks if they were purchased at the lowest price for the day (p. 352) and sold at the price indicated below:

1. Crucible Steel, 97.
2. American Locomotive, pf., $99\frac{5}{8}$.
3. Baldwin Locomotive, $93\frac{7}{8}$.
4. National Biscuit, $107\frac{1}{2}$.
5. American Woolen, $61\frac{3}{8}$.
6. Louisville & Nashville, $105\frac{1}{8}$.
7. Atchison, Topeka & Santa Fe, $80\frac{1}{4}$.
8. Chicago, Milwaukee & St. Paul, $28\frac{5}{8}$.
9. American Car & Foundry, pf., 118.
10. American Car & Foundry, $132\frac{1}{8}$.

Find the profit on 200 shares of each of the following stocks, if bought at the lowest price for the day and sold at the highest price:

11. American Locomotive.
12. United States Steel.
13. American Woolen.
14. Baldwin Locomotive.
15. General Electric.
16. Kresge.
17. United States Rubber.
18. Chicago, Milwaukee & St. Paul, pf.
19. American Car & Foundry.
20. New York Central.

21. Last year B bought 500 shares of United States Steel preferred stock at the lowest price of the year and now sells it at the opening price for the day. How much is his profit?

22. Last year C bought 400 shares of American Car and Foundry at the highest price of the year and now sells it at the highest price for the day. How much is his loss?

23. D gave his broker an order to buy 2000 shares of Atchison, Topeka & Santa Fe preferred stock and to sell 2500 shares of United States Steel. If the broker bought at the lowest price for the day and sold at the highest price, what balance does he place to D's credit?

24. E owns 1500 shares of Pullman which he bought at $98\frac{1}{2}$ and 500 shares of Chicago, Milwaukee & St. Paul which he bought at $37\frac{7}{8}$. If he instructs his broker to sell both lots at the highest prices for the day, how much is his net profit or loss?

25. F bought Crucible Steel at the lowest price of the day and later sells it at $88\frac{5}{8}$. If his gain on the transaction is \$1537.50, how many shares did he buy?

26. G bought National Biscuit at 96 and instructs his broker to sell his stock at the opening price for the day. If his profit is \$1340, how many shares did he buy?

27. If H bought 600 shares of American Car & Foundry at $121\frac{3}{8}$ and sold enough at the closing price for the day to give him a profit of \$955, how many shares did he sell?

28. If K lost \$1220 when he bought stock at $84\frac{3}{8}$ and later sold it at $81\frac{5}{8}$, how many shares did he buy and sell?

Income from stock investments. Since dividends are usually declared and paid upon the par value of a share of stock, the per cent of income returned upon the amount of an investment in stocks is larger than the dividend rate if the stock is bought below par, and smaller if it is bought above par. The following examples illustrate the method of determining the per cent of income received upon the investment.

EXAMPLES. 1. If stock is bought when quoted at 122 and the dividend rate is 8%, what is the per cent of income on the investment?

Solution. $\$100 \times 0.08 = \8 , annual income per share.

$\$122 + \$0.15 = \$122.15$, cost of purchasing 1 share.

$\$8 \div \$122.15 = 0.0654 +$, return on investment.

Hence the rate of return on the investment is approximately 6.5%.

2. If stock which pays dividends of 8% is bought at 70 $\frac{1}{4}$, what is the per cent of income on the investment?

Solution. $\$100 \times 0.08 = \8 , annual income per share.

$\$70.25 + \$0.15 = \$70.40$, cost of purchasing 1 share.

$\$8 \div \$70.40 = 0.1136 +$, return on investment.

Hence the rate of return upon the investment is approximately 11.4%.

EXERCISES

Find the per cent of income upon an amount invested in each of the following stocks if they are bought at the closing prices for the day (p. 352), and if the stocks continue to pay dividends at the rates shown in the fourth column of the table:

1. Tidewater Oil.

6. Louisville & Nashville.

2. American Woolen.

7. United States Steel.

3. American Locomotive, pf.

8. United States Steel, pf.

4. New York Central.

9. American Car & Foundry.

5. Pullman.

10. Baldwin Locomotive.

11. Which of the two investments will yield the greater per cent of income, a stock paying 5% if bought at 73, or a stock paying 8% if bought at 122? How much greater?

12. If A buys at 102 United States Rubber preferred stock which pays 8%, what is the per cent of income on the investment?

13. If A receives annually \$800 in dividends upon the stock in Ex. 12, what was the amount of his investment?

14. If B invests \$19,030 in American Sugar at 95, how many shares did he buy? If he receives \$1400 in dividends during the year, what rate of dividend was paid?

15. In Ex. 14 what was the rate of return upon the investment?

The student should notice that there is a short way to solve Ex. 15.

16. The par value of Pennsylvania Railroad stock is \$50 per share. If a dividend of 75¢ per share is payable quarterly upon this stock, what is the annual dividend rate? What is the per cent of income upon an investment in this stock if it is purchased at 45?

The quotation in Ex. 16 means \$45 per share, and similarly for Ex. 17.

17. The par value of the stock of the Kelley-Springfield Tire Co. is \$25 per share. If a dividend of \$1 per share is paid quarterly on this stock, what is the per cent of income on the investment if the stock can be purchased at 47 $\frac{3}{4}$?

From the data given find the per cent of income on investments in each of the following stocks:

STOCK	PAR VALUE	DIVIDEND RATE	DIVIDENDS PAYABLE	PURCHASE PRICE
18. Delaware, Lacka- wanna & West Virginia	\$50	5%	Quarterly	220 $\frac{1}{2}$
19. American Locomo- tive	\$100	1 $\frac{1}{2}$ %	Quarterly	85 $\frac{3}{8}$
20. Atchison, Topeka & Santa Fe, pf.	\$100	2 $\frac{1}{2}$ %	Semiannually	79 $\frac{1}{8}$
21. General Electric	\$100	2%	Quarterly	120 $\frac{1}{2}$
22. International Nickel	\$25	1 $\frac{1}{2}$ %	Quarterly	15 $\frac{3}{8}$
23. American Car & Foundry	\$100	3%	Quarterly	126 $\frac{1}{2}$

Investing to yield a certain income. Often an investor desires that his investment shall yield him a certain amount of income per year and needs to know how much money he must invest in a certain stock in order to secure the desired amount of income.

EXAMPLE. If a man can purchase at $74\frac{3}{4}$ a desirable stock which pays 6%, how much will he have to invest to secure an income of \$1200 per year?

Solution. Since the stock pays 6%, the annual income per share is \$6.

Then $\$1200 \div \$6 = 200$, number of shares,

and $\$74.75 + \$0.15 = \$74.90$, cost per share.

Hence $\$74.90 \times 200 = \$14,980$, amount of investment.

EXERCISES

1. What per cent of income does the man in the above example receive upon his investment?

2. If a man desires an income of \$2100 a year from his investment, how much must he invest in a stock paying 7% if the market is $95\frac{1}{2}$?

3. In order to secure an income of \$1600, how much must be invested in General Electric stock which pays 8% and is quoted at $120\frac{1}{2}$?

4. Find the per cent of income received upon each investment in Exs. 2 and 3.

5. What amount should be invested in Atchison, Topeka & Santa Fe stock which pays 6% and is quoted at $84\frac{1}{8}$ in order to secure an annual income of \$1800?

6. What amount should be invested in American Car & Foundry stock which pays 12% and is quoted at 124 in order to secure an annual income of \$1200?

7. Find the per cent of income received upon each investment in Exs. 5 and 6.

8. If a man buys 100 shares of stock paying 5% and quoted at $122\frac{5}{8}$, and 100 shares of stock paying 5% and quoted at $71\frac{3}{8}$, what is the amount of his total investment? What is his annual income from the stocks? What per cent of return does he receive on the investment in each stock? Which stock pays him the larger per cent of return, and what per cent more does it pay?

BONDS

Types of bonds. When a corporation desires to borrow money for a long period of time for the purpose of new construction or for the improvement and extension of existing property, it generally does so by issuing bonds. A **bond** is a written promise to pay money with interest, and occupies the same position with relation to long-term financial transactions of a corporation that the promissory note does in evidencing the short-term obligations of ordinary business borrowing. Bonds are also issued by the governments of a country, state, or smaller political unit.

While a stock certificate is evidence of part ownership of the assets of a corporation, a bond of the corporation is only evidence that money has been loaned to it.

Bonds of a national government are called **government bonds**; those of a state, city, or other municipal organization are called **municipal bonds**; and those of a corporation are called **corporation bonds**. Corporation bonds may be classified as **railroad bonds** and **industrial bonds**, the latter class including all bonds of corporations other than railroads.

The face value of a bond is usually \$1000, \$500, or \$100, although bonds with a face value of \$50 are occasionally issued. Bonds are generally issued for periods which vary from 10 yr. to 50 yr.

The statement of all the conditions which relate to the issuing and payment appears upon the face of the bond and is known as the **bond recital**.

Bonds are further classified according to the security provided for their payment. When the security takes the form of a claim on property the bond is called a **mortgage bond**, and the holders may make claim upon the security if the bonds are not paid when due.

The different types of mortgage bonds are called first, second, general, or blanket mortgage bonds, to indicate the type of security given or the order in which claims may be satisfied if the bonds are not paid when due.

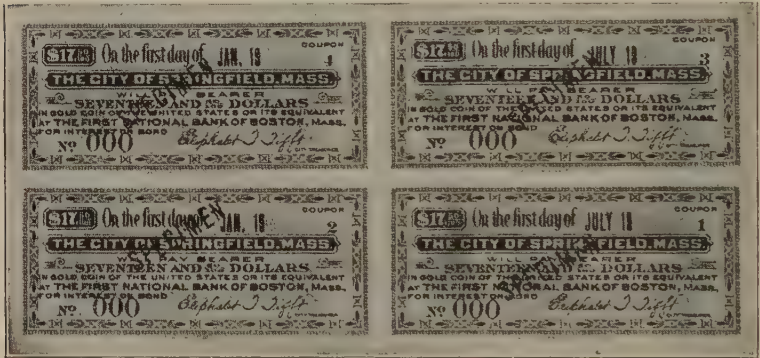
Thus, the Chicago & Northwestern general mortgage 4's, 1987, are bonds of the Chicago & Northwestern Railway Co. which are secured by a general mortgage on the company's property, which bear interest at 4%, and which are due in 1987.



COUPON BOND

Bonds in which the payment depends upon the unsecured credit of the corporation which issues them are known as **debenture bonds**. If bonds are secured by the deposit of securities of value with a trustee, they are known as **collateral trust bonds**. If the payment of interest upon a bond is made only if earned, such a bond is called an **income bond**.

Bond interest. Bond interest is usually payable semiannually, and this interest must be paid when due or the bondholders may satisfy their claims through legal procedure. Consequently, unlike stock dividends which depend upon earnings, bond interest is a fixed charge for the corporation which issues the bonds.



BOND COUPONS

For convenience in collecting the interest payments when due, bonds are often issued with interest notes, or coupons, attached to the bond. Such a bond is known as a **coupon bond**.

The illustration on page 361 shows the first sheet of a coupon bond and the illustration above shows four of the coupons which are printed on sheets attached to the bond.

The holder of the bond simply detaches the proper coupon when the interest is due and presents it at the office of the corporation (or through his bank or broker) and receives the amount of interest due. Since these coupons, like the bond itself, are payable to the bearer, they should be handled with as much care as cash.

The name of the bondholder is sometimes recorded in the office of the corporation which issues the bond, and in such

cases the interest when due and the face of the bond at maturity are paid only to him. Such a bond is called a **registered bond**. Sometimes bond issues provide for the registration of the face of the bond, but the interest is paid by coupons which are payable to the bearer.

In stating the particulars of a bond issue the months in which the interest is payable are generally stated in abbreviated form. Thus, "Jan. & J." means that the interest is payable on Jan. 1 and July 1, "Apr. & O." means "April 1 and Oct. 1," and so on.

ORAL EXERCISES

1. Explain the difference between a bond and a stock certificate of a corporation.
2. Explain the difference between a coupon bond and a registered bond.
3. What is the face of the bond illustrated on page 361? What rate of interest does it bear? What amount is payable on each interest date?
4. What is the difference between a government bond and a municipal bond?

At the discretion of the instructor the class may secure information, through one or two representatives, about the bonded indebtedness of the local municipality. Such information may cover the types of bonds issued and the rates of interest which they bear.

Buying and selling bonds. Bonds are generally bought and sold through bond houses, investment bankers, private bankers, or brokers.

In all stock-exchange transactions in bonds a commission is charged for buying and selling, but when bonds are bought from a regular bond house the prices quoted include the commission. The commission for buying and selling bonds on the New York Stock Exchange is \$15 for each \$10,000 par (face) value, which is equivalent to \$1.50 on each bond for \$1000.

The market prices of bonds vary according to the rate of interest, date of maturity, the security of the bonds, and market

conditions. The following table shows recent prices for certain issues of bonds on the New York Stock Exchange:

BOND QUOTATIONS	
Baltimore & Ohio, 6's	91½
Central Leather, 5's	90¾
Chesapeake & Ohio, general, 4½'s	76½
Chicago, Burlington & Quincy, general, 4's	80
Chicago, Milwaukee & St. Paul, general, 4½'s	77½
Chicago, Milwaukee & St. Paul, general, 4's, 1934	62½
Chicago & Northwestern, general, 5's	93
International Mercantile Marine, sinking fund, 6's	83⅝
New York Central, debenture, 6's	91⅞
Pennsylvania R. R., guaranteed, 4½'s	98¾
Pennsylvania R. R., convertible, 5's	99¾
Southern Pacific, convertible, 5's	99¾
Union Pacific, 6's	99¾
United States Rubber, 7's	96¾
United States Steel, sinking fund, 5's	94¼

These quotations are based on a par value of \$100; that is, the quotation for Baltimore & Ohio, 6's, means that a bond of par value \$1000 costs \$915 on the exchange.

Cost of bonds, including brokerage. The following example illustrates the method of finding the cost of bonds when brokerage is added.

EXAMPLE. At the price quoted in the above table, what is the cost of purchasing through a broker bonds of par value \$10,000 of Chicago & Northwestern, general, 5's?

Solution. Since the quotation of 93 is based on a par value of \$100, a bond of par value \$1000 will cost \$930. The brokerage (p. 363) for each \$1000 of par value is \$1.50.

Then $\$930 + \$1.50 = \$931.50$, cost per \$1000.

Hence $\$931.50 \times 10 = \9315 , total cost.

The purchaser also pays the interest accrued since the last interest date, as explained later, but in the following exercises the question of accrued interest is not to be considered.

EXERCISES

The exercises refer to the table of quotations on page 364, and commission is to be included in each case.

1. What is the cost of purchasing bonds of par value \$50,000 of New York Central, debenture, 6's?

2. A instructs his broker to sell bonds of par value \$15,000 of Chicago, Milwaukee & St. Paul, general, $4\frac{1}{2}$'s, and to buy bonds of par value \$15,000 of Chesapeake & Ohio, general, $4\frac{1}{2}$'s. What balance should the broker pay to A?

The commission is deducted from the receipts in the case of a sale.

3. If B instructs his broker to buy the following bonds, each of par value \$1000, find the total cost:

NO. PURCHASED	ISSUE
5	Pennsylvania R. R., guaranteed, $4\frac{1}{2}$'s
5	United States Rubber, 7's
5	United States Steel, sinking fund, 5's
5	Baltimore & Ohio, 6's

4. If C instructs his broker to sell the following bonds, each of par value \$1000, find the proceeds from the sale:

NO. SOLD	ISSUE
4	Central Leather, 5's
10	Chicago, Milwaukee & St. Paul, general, 4's, 1934
5	Chicago & Northwestern, general, 5's
6	Chicago, Burlington & Quincy, general, 4's

5. Find the total cost of the following purchases of bonds:

ISSUE	PAR VALUE
Pennsylvania R. R., convertible, 5's	\$12,000
Southern Pacific, convertible, 5's	\$10,000
Union Pacific, 6's	\$10,000
International Mercantile Marine, sinking fund, 6's	\$5,000

6. Find the total proceeds from the sale of bonds of par value \$5000 of each of the four issues which have the highest quotations in the table.

7. Find the total cost of purchasing five bonds of par value \$1000 of each of the four issues which have the lowest quotations in the table.

8. A city borrowed \$250,000 in order to build a new high school and issued 10-year coupon bonds which bear interest at $5\frac{1}{2}\%$. If these bonds were sold through a broker at $100\frac{3}{8}$, how much was received by the city?

9. If a man purchased five bonds of par value \$1000 each of the issue mentioned in Ex. 8, how much did they cost him? What amount of interest does he receive annually from his bonds?

Cost of bonds, including brokerage and interest. When a bond is bought between two interest dates, the purchaser must pay the interest which has accrued since the last interest day in addition to the market price of the bond. This is only fair to the last holder of the bond, as otherwise he would receive no return for the period which he has held the bond before disposing of it. The interest accrued is computed at the rate specified in the bond on the basis of 360 da. to the year.

For this reason bonds are frequently spoken of as being sold at a certain price "and interest."

The following example shows the method of determining the cost of a bond which is bought through a broker and on which accrued interest is paid.

EXAMPLE. On July 1, A purchases through his broker 10 bonds of par value \$1000 each of Illinois Central, 5's. If interest on the bonds is payable June & D. and the market price is $93\frac{3}{4}$, what is the total cost of the bonds?

Solution. Interest on these bonds is payable on June 1 and Dec. 1, and hence accrued interest must be paid for 1 mo.

$$\$5 \times \frac{1}{6} = \$4.17, \text{ interest on each bond for 1 mo. (p. 267)}$$

$$1.50, \text{ brokerage on each bond (p. 363)}$$

$$937.50, \text{ market price of each bond}$$

$$\$943.17, \text{ total cost of each bond}$$

$$\$943.17 \times 10 = \$9431.70, \text{ total cost of the bonds}$$

EXERCISES

In each of the following cases find the total cost of purchasing bonds of par value \$1000 each through a broker:

NO. BOUGHT	DATE BOUGHT	MARKET QUOTATION	RATE OF INTEREST	INTEREST DATES
1. 10	Feb. 15	103 $\frac{5}{8}$	6%	Jan. & J.
2. 5	June 1	89 $\frac{1}{8}$	4 $\frac{1}{4}$ %	Apr. & O.
3. 8	April 30	97 $\frac{3}{4}$	4 $\frac{1}{2}$ %	Feb. & A.
4. 10	July 10	102 $\frac{1}{4}$	7%	May & N.
5. 6	May 16	101 $\frac{3}{8}$	5 $\frac{1}{2}$ %	Mar. & S.
6. 5	Jan. 31	104 $\frac{7}{8}$	6 $\frac{1}{2}$ %	June & D.

Income from bond investments. Determining the per cent of income which is received upon an investment in bonds involves the consideration of the rate of interest which the bond bears, the total price paid for the bond, and the date of maturity of the bond.

Thus a bond of par value \$1000 and paying 6% might cost \$1125.50. At maturity only \$1000 will be received for the bond, and hence the purchaser must regard the annual income of \$60 as consisting of two parts: one a return of income at the prevailing rate, say 5%, on his investment, and the other an annual return of a portion of the premium paid for the bond.

Thus when a bond is bought above par, the investor realizes a per cent upon his investment which is lower than the rate of interest which the bond pays. The exact determination of such a per cent can only be made by the methods of higher algebra, and hence is unsuited to a textbook in arithmetic.

This fact is mentioned only as a caution to those who buy bonds, as it is very easy for one to deceive himself upon this question.

The per cent of income which is received from bonds when bought at certain prices has been worked out in bond tables, used by bond houses, bankers, and brokers. The table on page 368 shows a part of such a set of tables, which are too large for inclusion in a textbook, by which the per cent of income derived from purchasing a 20-year bond on which interest is payable semiannually may be determined.

20-YEAR BOND TABLE

(Interest payable semiannually)

PER CENT OF ANNUAL INCOME	RATE OF INTEREST ON BOND						
	3%	3½%	4%	4½%	5%	6%	7%
3.70	90.17	97.19	104.21	111.24	118.26	132.30	146.35
3¾	89.51	96.50	103.50	110.49	117.48	131.46	145.44
3.80	88.86	95.82	102.78	109.74	116.70	130.63	144.55
3⅞	87.90	94.81	101.73	108.64	115.56	129.39	143.22
3.90	87.58	94.48	101.38	108.28	115.18	128.98	142.78
4.	86.32	93.16	100.00	106.84	113.68	127.36	141.03
4.10	85.09	91.86	98.64	105.42	112.20	125.76	139.32
4½	84.78	91.54	98.31	105.07	111.84	125.37	138.90
4.20	83.87	90.59	97.31	104.03	110.75	124.19	137.63
4¼	83.27	89.96	96.65	103.35	110.04	123.42	136.80
4.30	82.68	89.34	96.00	102.66	109.33	122.65	135.98
4⅝	81.80	88.42	95.04	101.65	108.27	121.51	134.75
4.40	81.51	88.11	94.72	101.32	107.93	121.14	134.35
4½	80.35	86.90	93.45	100.00	106.55	119.65	132.74
4.60	79.22	85.72	92.21	98.70	105.19	118.18	131.16
4⅝	78.94	85.42	91.90	98.38	104.86	117.82	130.77
4.70	78.11	84.55	90.99	97.43	103.86	116.74	129.61
4¾	77.57	83.98	90.39	96.80	103.20	116.02	128.84
4.80	77.02	83.40	89.79	96.17	102.55	115.32	128.08
4⅞	76.22	82.56	88.90	95.24	101.59	114.27	126.95
4.90	75.95	82.28	88.61	94.94	101.27	113.92	126.58
5.	74.90	81.17	87.45	93.72	100.00	112.55	125.10
5.10	73.86	80.09	86.31	92.53	98.76	111.20	123.65
5½	73.61	79.82	86.03	92.24	98.45	110.87	123.29
5.20	72.85	79.02	85.19	91.36	97.53	109.87	122.22
5¼	72.34	78.49	84.64	90.78	96.93	109.22	121.51
5.30	71.85	77.97	84.09	90.21	96.33	108.57	120.81
5⅝	71.11	77.19	83.27	89.36	95.44	107.60	119.77
5.40	70.87	76.94	83.01	89.07	95.14	107.28	119.42
5½	69.90	75.92	81.94	87.96	93.98	106.02	118.06
5⅞	68.72	74.68	80.64	86.59	92.55	104.47	116.38
5¾	67.57	73.46	79.36	85.26	91.15	102.95	114.74
5⅞	66.43	72.27	78.11	83.95	89.78	101.46	113.13
6.	65.33	71.11	76.89	82.66	88.44	100.00	111.56
6¼	64.25	69.97	75.69	81.41	87.13	98.57	110.01
6½	63.19	68.85	74.51	80.18	85.84	97.17	108.50
6⅝	62.15	67.76	73.36	78.07	84.58	95.79	107.01
6⅞	61.14	66.69	72.24	77.79	83.34	94.45	105.55
6¾	60.14	65.64	71.14	76.64	82.13	93.13	104.12
6⅞	59.17	64.62	70.06	75.50	80.95	91.83	102.72
6⅞	58.22	63.61	69.00	74.39	79.78	90.57	101.35
7.	57.29	62.63	67.97	73.31	78.64	89.32	100.00

Adapted from Montgomery Rollins's "Money and Investments."

In the table on page 368 the per cents in the column at the left show the per cent of income which is realized when a bond maturing in 20 yr. and bearing interest, payable semiannually, at any one of the rates at the top of the table is bought at one of the prices shown in the seven columns at the right. All quotations are based on a par value of \$100.

EXAMPLE. What per cent of income is received when a 20-year bond of par value \$1000 on which interest at 6% is payable semiannually can be bought for \$944.50?

Solution. This price is equivalent to a quotation of 94.45 per \$100. Referring to the table, under the column headed 6%, it is found that 94.45 is opposite $6\frac{1}{2}$ in the column at the left.

Hence $6\frac{1}{2}$ % is realized on the investment.

ORAL EXERCISES

Using the bond table on page 368, state the per cent of income which is realized if 20-year bonds of par value \$1000, bearing the following rates of interest, payable semiannually, can be purchased for the amounts shown:

INTEREST RATE OF BOND		COST PRICE	INTEREST RATE OF BOND		COST PRICE
1.	5%	\$845.80	5.	$3\frac{1}{2}$ %	\$811.70
2.	7%	\$1115.60	6.	4%	\$846.40
3.	4%	\$700.60	7.	3%	\$711.10
4.	6%	\$1072.80	8.	5%	\$1025.50

Using the bond table on page 368, state the price which a man must pay for a 20-year bond of par value \$1000, bearing interest as shown, payable semiannually, in order to realize the following per cents of income upon his investment:

INTEREST RATE OF BOND		RATE OF INCOME ON INVESTMENT	INTEREST RATE OF BOND		RATE OF INCOME ON INVESTMENT
9.	5%	$6\frac{1}{4}$ %	13.	$3\frac{1}{2}$ %	$5\frac{1}{2}$ %
10.	6%	$5\frac{7}{8}$ %	14.	6%	$6\frac{7}{8}$ %
11.	4%	$5\frac{1}{2}$ %	15.	5%	$6\frac{3}{8}$ %
12.	7%	$5\frac{3}{8}$ %	16.	3%	$6\frac{1}{8}$ %

CHAPTER X

TAXES

Nature of taxes. Not only does our national government spend large sums each year for such purposes as the defense of the country, education, public health, highways, administration of the laws, and other activities which affect the welfare of each inhabitant, but the governments of the separate states and of cities and towns also spend large sums for similar activities within their borders. These expenses must be borne by those who share in the benefits, and the money to meet them is obtained by means of **taxes**. Specifically, then, a **tax** is a sum of money raised for some public purpose, and the name applied to the tax generally indicates its nature or the manner in which it is collected.

Owing to the increase in governmental activities which has taken place in recent years, the classification of the expenses of a government has become more complicated. Information as to the expenses of the local municipal or state governments may be secured and brought to class. A discussion of such material secured locally is more effective than trying to follow a general classification of expenses which applies to any type of government.

Kinds of taxes. In general, taxes are classified according to the method in which they are determined and collected.

A **property tax** is a tax levied on real estate or on personal property.

An **income tax** is a tax levied upon the incomes of individuals, partnerships, and corporations.

Sometimes the individual incomes of the partners rather than the income of the partnership are taxed.

An **estate tax** is a tax levied upon the estate of a deceased person, and an **inheritance tax** is a tax levied upon the property which is inherited from a deceased person.

A **poll tax**, or **capitation**, is a tax levied upon each inhabitant who has attained a certain age, generally that of 21 yr.

A **license fee** is another type of tax which is paid in order to obtain permission to engage in certain types of business or operations, where it is essential that supervision be carried out by the government for the protection of the public.

Where local improvements such as street paving, sidewalks, and so on are made, **special assessments** to meet part of the cost are sometimes levied on those whose property is thus improved. The assessment, which the owner of the property pays and which may be regarded as a type of tax, is merely a partial return of the increased value of his property due to the improvement.

Distribution of taxes. The expenses of the United States government are met chiefly through income taxes, estate taxes, taxes on goods imported into the country from abroad, and taxes on certain commodities such as tobacco, oleomargarine, and so on. Such taxes are collected directly by the Federal government.

Taxes on imports will be discussed later in detail.

The expenses of town, city, county, and state governments are met chiefly by property taxes, income taxes (in some states), estate and inheritance taxes, license fees, and poll taxes.

While the methods and details vary in the different states, in general, the state after preparing its estimates of expenditures determines what portion of these expenses must be raised by each county. The county, in turn, determines what portion of its state tax and of its own expenses must be raised by each city, town, or village within the county. The local municipalities then know how much they must raise for state and county purposes and how much they must raise to meet their own needs.

When taxes are collected directly by the state in the form of income taxes, the receipts are distributed to the different counties and municipal governments in varying proportions, and hence the general system is still the same as that described above. It would be obviously inconvenient if each property owner had to pay taxes separately to the city, county, and state authorities. The city in collecting the whole amount of the tax acts as the agent of the county and state.

Property taxes. Property taxes are imposed in the different states in a nearly uniform manner. Each city, town, or village elects or appoints officers called **assessors**, whose duty it is to set a value upon all property subject to taxation. The value thus fixed is known as the **assessed valuation**.

In some states the assessed valuation is only a part of the actual value of the property, while in others it is as near the actual value as can be determined. If an owner believes that his property has been valued at too high a figure, he may appeal to a board who hears the case and determines the assessed valuation at its proper figure.

The local valuations are submitted to the proper county officials and by them to the state authorities. These figures aid in determining the proper apportionment of the taxes.

After the total assessed valuation is ascertained, it is then necessary to determine the **tax rate** at such a figure as will yield an amount sufficient to meet the estimated expenditures. The tax rate may be expressed as a per cent, as so many mills on the dollar, or as so many dollars on each \$100 or on each \$1000.

Thus, if the total assessed valuation of all property in a city is \$73,683,300 and the amount to be raised by property taxes is \$1,473,666, the tax rate is determined by dividing the total amount to be raised by the total valuation. That is, $\$1,473,666 \div \$73,683,300 = 0.02$.

Hence the tax rate may be expressed in any one of the following ways: 2%; 20 mills (on \$1); \$2 per \$100; or \$20 per \$1000.

In many states the taxes for state and county purposes take from 20% to 30% of the total property tax, and the balance is for city, town, and school purposes.

Real estate and personal property which belong to religious or charitable organizations are commonly exempt from taxation. Similar exemption is sometimes granted to private educational organizations, such as colleges, technical schools, and so on.

EXAMPLES. 1. If the tax rate is 1.8%, what is the amount of the tax on property which is assessed for \$12,000?

Solution. $\$12,000 \times 0.018 = \216 , amount of tax.

2. If the tax rate is 20 mills, what amount of taxes are paid on property which has an assessed valuation of \$8000?

Solution. A rate of 20 mills on \$1 may be written as 0.020.

Hence $\$8000 \times 0.02 = \160 , amount of tax.

3. If the tax rate is \$17.50 per \$1000, what amount must be paid in taxes on property assessed for \$15,000?

Solution. The number of thousands of assessed valuation is determined by moving the decimal point three places to the left.

Then $15.000 \times \$17.50 = \262.50 , amount of tax.

4. If a city in which the total assessed valuation is \$20,760,865 has to raise \$398,610 by property taxes, what rate should be declared?

Solution. $\$398,610 \div \$20,760,865 = 0.01920 +$.

Hence the rate may be expressed as 1.92%, 19.2 mills, \$1.92 per \$100, or \$19.20 per \$1000.

ORAL EXERCISES

State the amount of the tax paid in each of the following cases:

ASSESSED VALUATION	TAX RATE	ASSESSED VALUATION	TAX RATE
1. \$5000	12 mills	6. \$9500	20 mills
2. \$7500	2%	7. \$10,000	16 mills
3. \$5000	\$18 per \$1000	8. \$10,000	\$19.25 per \$1000
4. \$8000	22 mills	9. \$10,000	\$1.95 per \$100
5. \$8000	\$2.25 per \$100	10. \$12,000	2½%

EXERCISES

Find the tax rate to the nearest 0.0001 in the following cities, and express each result (a) as a per cent, (b) as mills, and (c) as dollars on \$1000:

CITY	ASSESSED VALUATION	TOTAL LEVY
1. A	\$3,509,315	\$101,087
2. B	\$2,280,778	\$77,530
3. C	\$66,974,506	\$1,719,172
4. D	\$44,475,810	\$1,334,256
5. E	\$520,041	\$20,810
6. F	\$22,671,298	\$600,908
7. G	\$21,286,243	\$770,812
8. H	\$588,556,266	\$15,578,016

Using the results of Exs. 1-8, find in each case the tax paid by an individual who owns property assessed as follows:

	CITY	ASSESSED VALUATION		CITY	ASSESSED VALUATION
9.	A	\$8742.97	13.	E	\$54,843.29
10.	B	\$10,541.71	14.	F	\$68,472.97
11.	C	\$12,693.26	15.	G	\$108,452.31
12.	D	\$15,821.34	16.	H	\$164,523.82

In each case find the assessed valuation of property on which taxes were paid as follows:

	AMOUNT OF TAX	TAX RATE
17.	\$282.70	18 mills
18.	\$325.40	\$2.25 per \$100
19.	\$687.35	21 mills
20.	\$486.90	\$3.175 per \$100
21.	\$1248.40	\$19.75 per \$1000
22.	\$3847.60	\$24.50 per \$1000
23.	\$4723.80	35 mills
24.	\$7525	\$23 per \$1000

25. Prepare a table which shows the types of receipts and disbursements and the total of these items for a county in which the receipts and disbursements in a recent year were as follows:

Receipts: taxes, \$345,162; fees and charges, \$16,479; gifts and grants, \$89,128; departmental earnings, \$29,747.

Disbursements: county government, \$59,923; protection of person and property, \$10,548; health conservation and sanitation, \$3698; highways and bridges, \$81,418; education, \$9210; charities and institutions of correction, \$96,711; miscellaneous, \$1864.

What was the excess of the receipts over disbursements? What per cent of the total disbursements was spent for each purpose?

26. A town had to raise the following amounts by taxation: state tax, \$6872; county tax, \$12,540.80; town tax, \$128,482.75. If there were 3875 persons subject to the poll tax of \$2 each and the valuation of the town was \$9,428,750, what tax rate should be declared?

Deduct the amount of the poll tax from the total amount to be raised before computing the property tax rate.

27. In a town in which 3080 persons were subject to a poll tax of \$1 each, the assessed valuation of real estate was \$4,875,420 and that of personal property was \$580,310. If \$78,700 had to be raised by taxation, what was the tax rate?

28. In Ex. 27 what was the amount of the tax bill of James B. Ransom who owned real estate valued at \$8540 and personal property valued at \$3270?

Ransom's tax bill should also include the poll tax of \$1.

29. A city made the following appropriation for its public schools: teaching and supervision, \$48,500; care and cleaning, \$4580; fuel, \$5000; repairs, \$3000; textbooks, \$2450; supplies, \$2240; printing, \$800; contingent fund, \$975; truant officer, \$1200; evening schools, \$3305; transportation of pupils, \$900; kindergarten, \$3100; manual training, \$1700. The assessed valuation of real estate was \$8,907,899 and that of personal property was \$2,092,101. What was the tax rate for school purposes?

30. In computing its tax rate for the year a city finds that it must allow for a rate of \$1.80 per \$1000 for the state tax, of \$3.35 per \$1000 for county purposes, and that its own needs require a rate of \$17.10 per \$1000. What tax rate is declared for that year? What amount and what per cent of the total tax paid by A. M. Mott who owns property assessed at \$17,950 goes to the city? to the county? to the state?

Federal income tax. A large part of the expenses of the national government are now met through the collection of an income tax. Each year every resident of the United States who

has an income above a certain amount, and every corporation doing business within the country, regardless of the amount of the income, must file with the Federal tax collector a return, or statement, of the income which was received during the preceding year. From the facts presented in the return the amount of the tax to be paid is determined. Every partnership must also file a return, but no tax is paid by a partnership as such, since each partner's share of the net profits as shown in the return is taxed as the income of an individual.

The details of the income-tax law are constantly changing owing to new rulings and interpretations, and the method of determining the taxable income under the different provisions of the law presents a complicated problem. It is accordingly impractical to include in any high-school textbook a detailed study of the income-tax law, and further information, if desired, can be readily secured from the office of the local Collector of Internal Revenue. A few of the more important features of the law as applied to individual incomes are given below.

Taxes on individual incomes. Under the provisions of the income-tax law a return of income must be made by every individual whose net income for the previous year amounted to \$1000 if single, to \$2000 if married, or who had a gross income of \$5000 or over, regardless of the amount of the net income.

Net income is total income less deductions; **taxable income** is total income less deductions and exemptions.

The tax on incomes of individuals consists of a **normal tax** and a **surtax**, which is an additional tax on higher amounts. In connection with the normal tax only, certain credits are allowed in determining the amount of taxable income as follows:

1. A personal exemption of (a) \$1000 for a single person, (b) \$2500 for a married person whose net income is under \$5000, and (c) \$2000 for a married person whose net income is \$5000 or over.

2. An exemption of \$400 for each dependent person under 18 yr. of age or who is incapable of self-support.

3. Dividends received from corporations.

The normal tax rate is 4% upon the first \$4000 of taxable income and 8% upon the taxable income in excess of \$4000.

The surtax is calculated independently of the normal tax, and in determining that portion of the income subject to the surtax certain credits for purposes of the normal tax are not allowed. The surtax rate is based on an increasing scale and applies on all net income above \$6000, as follows:

NET INCOME	SURTAX RATE	NET INCOME	SURTAX RATE
\$6000—\$10,000	1%	\$22,000—\$24,000	9%
\$10,000—\$12,000	2%	\$24,000—\$26,000	10%
\$12,000—\$14,000	3%	\$26,000—\$28,000	11%
\$14,000—\$16,000	4%	\$28,000—\$30,000	12%
\$16,000—\$18,000	5%	\$30,000—\$32,000	13%
\$18,000—\$20,000	6%	\$32,000—\$36,000	15%
\$20,000—\$22,000	8%	\$36,000—\$38,000	16%

The surtax rates increase 1% on each additional \$2000 up to \$100,000, and above this amount the rates are as follows:

NET INCOME	SURTAX RATE
\$100,000—\$150,000	48%
\$150,000—\$200,000	49%
\$200,000 and over	50%

The computation of the tax on the income of an individual is illustrated in the following example.

EXAMPLE. If E. H. Brown's net income for last year was \$11,875 and in computing the normal tax he is entitled to credits of \$3325, what is the amount of Brown's income tax for the year?

Solution. The normal tax is computed first as follows:

$$\$11,875 - \$3325 = \$8550, \text{ income subject to normal tax.}$$

$$\$4000 \times 0.04 = \$160, \text{ normal tax at } 4\%.$$

$$\$8550 - \$4000 = \$4550, \text{ income subject to rate of } 8\%.$$

$$\$4550 \times 0.08 = \$364, \text{ normal tax at } 8\%.$$

$$\$160 + \$364 = \$524, \text{ total normal tax.}$$

The surtax is then computed on the net income from \$6000 to \$10,000 at 1% and from \$10,000 to \$11,875 at 2%.

$$\$4000 \times 0.01 = \$40, \text{ surtax at } 1\%.$$

$$\$1875 \times 0.02 = \$37.50, \text{ surtax at } 2\%.$$

$$\$40 + \$37.50 = \$77.50, \text{ total surtax.}$$

Hence $\$524 + \$77.50 = \$601.50$, total income tax.

EXERCISES

Find the normal tax on taxable incomes of the following amounts :

- | | | |
|---------------|-----------------|------------------|
| 1. \$2458.25. | 5. \$8437.60. | 9. \$14,387.18. |
| 2. \$4781.40. | 6. \$8269.20. | 10. \$15,963.24. |
| 3. \$3852.70. | 7. \$10,182.75. | 11. \$16,785.62. |
| 4. \$5281.10. | 8. \$12,237.42. | 12. \$18,289.75. |

Find the amount of the surtax on taxable incomes of the following amounts :

- | | | |
|------------------|------------------|-------------------|
| 13. \$8697.60. | 17. \$14,825.45. | 21. \$62,747.60. |
| 14. \$9234.27. | 18. \$15,180.60. | 22. \$123,482.40. |
| 15. \$7350.25. | 19. \$19,750.92. | 23. \$175,810.50. |
| 16. \$10,250.80. | 20. \$21,487.63. | 24. \$218,450.50. |

Find the income taxes (normal tax and surtax) paid by individuals who have the following incomes :

NET INCOME (SUBJECT TO SURTAX)	CREDITS ALLOWED FOR NORMAL TAX	NET INCOME (SUBJECT TO SURTAX)	CREDITS ALLOWED FOR NORMAL TAX
25. \$6110.75	\$1259.85	28. \$12,872.34	\$2181.07
26. \$8278.21	\$2477.41	29. \$16,341.82	\$4103.11
27. \$11,846.19	\$4599.85	30. \$24,867.68	\$4468.72

State income taxes. The incomes and profits of individuals, partnerships, and corporations are also taxed by some of the states. The general principle of a state income-tax law is similar

to that of the Federal law, but such taxes are assessed and collected independently of the Federal income taxes. In general, the rates for state income taxes are lower than the Federal rates.

Information about state income-tax laws may be obtained by studying these laws as enacted in Massachusetts, New York, and Wisconsin, but on account of their similarity to the Federal income-tax law no description of the provisions of such laws are given here.

State income-tax rates on individual incomes. The following table shows the graduated scale of rates which is applied in one state in determining the amount of the tax on individual taxable incomes of from \$1000 to \$20,000 a year:

TAXABLE INCOME		RATE	AMOUNT OF TAX		TOTAL INCOME TAXED	AMOUNT OF TOTAL TAX	
1st	\$1000	1%	10	00	\$1000	\$10	00
2d	1000	1 $\frac{1}{4}$ %	12	50	2000	22	50
3d	1000	1 $\frac{1}{2}$ %	15	00	3000	37	50
4th	1000	1 $\frac{3}{4}$ %	17	50	4000	55	00
5th	1000	2%	20	00	5000	75	00
6th	1000	2 $\frac{1}{2}$ %	25	00	6000	100	00
7th	1000	3%	30	00	7000	130	00
8th	1000	3 $\frac{1}{2}$ %	35	00	8000	165	00
9th	1000	4%	40	00	9000	205	00
10th	1000	4 $\frac{1}{2}$ %	45	00	10,000	250	00
11th	1000	5%	50	00	11,000	300	00
12th	1000	5 $\frac{1}{2}$ %	55	00	12,000	355	00
13th	1000	6%	60	00	13,000	415	00
15th	1000	6%	60	00	15,000	535	00
20th	1000	6%	60	00	20,000	835	00

EXAMPLE. Using the above table, determine the amount of the tax on a taxable income of \$3872.20.

Solution. The table shows that the total tax on \$3000 of taxable income is \$37.50. On the fourth \$1000 the rate is 1 $\frac{3}{4}$ %.

$$\begin{aligned}
 \text{Tax on } \$3000 &= \$37.50 \\
 \$872.20 \times 0.01\frac{3}{4} &= 15.26 \text{ } (\$15.2635) \\
 &\quad \underline{\$52.76, \text{ total tax}}
 \end{aligned}$$

EXERCISES

Using the table on page 379, determine the amount of the state income tax paid by individuals who have taxable incomes of the following amounts:

- | | | |
|---------------|-----------------|-----------------|
| 1. \$2638.45. | 3. \$5847.58. | 5. \$12,482.36. |
| 2. \$4221.82. | 4. \$10,023.67. | 6. \$11,627.39. |

Division of state income-tax receipts. The governments of the states, counties, cities, towns, and villages each receive a share of the state income taxes which are paid by their residents. The division of the tax receipts is generally determined by means of per cents.

EXERCISES

1. In one state the city or town government receives 70% of the income taxes paid by its residents, the county government receives 20%, and the state government 10%. If the total of the income taxes paid by the residents of City A is \$78,679.24, what amount goes to the city? to the county? to the state?

At the rates in Ex. 1, determine the share of the city, county, and state governments where the total income taxes paid in each of twelve cities are as follows:

- | | | |
|------------------|-----------------|-------------------|
| 2. \$28,443.43. | 6. \$3784.38. | 10. \$244,844.29. |
| 3. \$117,445.69. | 7. \$5540.87. | 11. \$562,786.47. |
| 4. \$176,092.60. | 8. \$42,417.79. | 12. \$876,321.22. |
| 5. \$122,568.40. | 9. \$11,575.76. | 13. \$906,723.68. |

Estate taxes. The Federal government also imposes taxes, called **estate taxes**, upon the estate left by a deceased person. This tax is determined by a scale of per cents which increase according to the net amount of the estate; that is, the amount after all debts and proper expenses of the estate have been met.

This tax is imposed upon the principal of the estate and is not a tax upon individual inheritances.

The following table shows the rates charged upon the different net amounts:

NET AMOUNT OF ESTATE			RATE
Under		\$50,000	1%
\$50,000	to	\$150,000	2%
\$150,000	to	\$250,000	3%
\$250,000	to	\$450,000	4%
\$450,000	to	\$750,000	6%
\$750,000	to	\$1,000,000	8%
\$1,000,000	to	\$1,500,000	10%
\$1,500,000	to	\$2,000,000	12%
\$2,000,000	to	\$3,000,000	14%
\$3,000,000	to	\$4,000,000	16%
\$4,000,000	to	\$5,000,000	18%
\$5,000,000	to	\$8,000,000	20%
\$8,000,000	to	\$10,000,000	22%
Over		\$10,000,000	25%

EXAMPLES. 1. Find the amount of the Federal estate tax which must be paid upon a net estate of \$45,000.

Solution. Referring to the above table, it is seen that this amount is taxed at 1%.

Hence $\$45,000 \times 0.01 = \450 , amount of tax.

2. Find the amount of the Federal estate tax which must be paid upon a net estate of \$325,000.

Solution. Referring to the table, it is seen that the first \$50,000 is taxed at 1%, the next \$100,000 (from \$50,000 to \$150,000) at 2%, the next \$100,000 (from \$150,000 to \$250,000) at 3%, and the balance of the estate at 4%.

$$\begin{aligned}
 &\$50,000 \times 0.01 = \$500 \\
 &\$100,000 \times 0.02 = 2000 \\
 &\$100,000 \times 0.03 = 3000 \\
 &(\$325,000 - \$250,000) \times 0.04 \\
 &= \$75,000 \times 0.04 = 3000 \\
 &\qquad\qquad\qquad \$8500, \text{ amount of tax}
 \end{aligned}$$

EXERCISES

Using the table on page 381, find the amount of the estate tax which must be paid upon estates of the following net amounts :

- | | | |
|------------------|------------------|-------------------|
| 1. \$175,000. | 5. \$426,372.12. | 9. \$589,446.87. |
| 2. \$202,587.50. | 6. \$683,428.40. | 10. \$963,618.20. |
| 3. \$487,623.75. | 7. \$768,892.70. | 11. \$257,329.56. |
| 4. \$381,528.46. | 8. \$982,387.65. | 12. \$432,760.40. |

Inheritance taxes. Several states impose taxes, called **inheritance taxes**, upon the value of property inherited from a deceased person. The rates vary according to the amount of the inheritance and the degree of relationship of the beneficiary. The following table shows the rates which apply in one state :

DEGREE OF RELATIONSHIP	FIRST \$25,000	\$25,000 TO \$50,000	\$50,000 TO \$100,000	\$100,000 TO \$500,000	OVER \$500,000
Husband, wife, children, grandchildren, etc. . . . }	1%	2%	3%	4%	5%
Brother, sister, nephew, niece, etc. }	2%	4%	6%	8%	10%
Uncle, aunt, cousin, etc. . .	3%	6%	9%	12%	15%
Brother or sister of grand- mother or grandfather, and their descendants . }	4%	8%	12%	16%*	20%*
More distant relationships, strangers, corporations, etc. }	5%	10%	15%	20%*	25%*

* But not to exceed 15% of the property transferred to any beneficiary.

Before the tax is determined a husband or wife is allowed an exemption of \$10,000 ; a child, \$2000 ; a brother, sister, etc., \$500 ; an uncle, aunt, cousin, etc., \$250 ; a brother or sister of grandmother or grandfather, \$150 ; more distant relationships, \$100.

EXAMPLE. Under the terms of A's will, his property was to be divided as follows: \$45,000 to his widow, \$10,000 each to his two children, and \$2000 to his cousin. What is the amount of the inheritance tax paid by each beneficiary?

Solution. The tax upon the widow's inheritance is computed as follows:

$$\$45,000 - \$10,000 = \$35,000, \text{ amount taxed.}$$

$$\$25,000 \times 0.01 = \$250, \text{ tax at } 1\%.$$

$$\$10,000 \times 0.02 = \$200, \text{ tax at } 2\%.$$

$$\$250 + \$200 = \$450, \text{ amount of widow's tax.}$$

The tax upon the inheritance of each child is computed as follows:

$$\$10,000 - \$2000 = \$8000, \text{ amount taxed.}$$

$$\$8000 \times 0.01 = \$80, \text{ amount of each child's tax.}$$

The tax upon the cousin's inheritance is determined as follows:

$$\$2000 - \$250 = \$1750, \text{ amount taxed.}$$

$$\$1750 \times 0.03 = \$52.50, \text{ amount of cousin's tax.}$$

EXERCISES

Using the rates and exemptions given on page 382, determine in each case the amount of the inheritance tax to be paid by beneficiaries who receive inheritances as follows:

1. Widow, \$20,000; daughter, \$5000; sister, \$1000.

2. Widow, \$55,000; brother, sister, uncle, aunt, \$5000 each.

In this case the tax upon the widow's inheritance must be computed at two rates, as in the above example.

3. Three children who share equally in a bequest of \$85,000.

4. Widow, \$65,000; two children, \$20,000 each; friend, \$4000; housekeeper, \$4000.

5. Widow, \$480,000; sister, \$75,000.

6. Widow, \$625,000; three children, \$125,000 each; brother and sister of grandmother, \$5000 each.

Duties. The national government also imposes and collects taxes upon goods which are brought into this country from foreign countries. Such taxes are known as **duties** or **customs**.

Thus, if a New York merchant receives a shipment of cloth from England, he will have to pay duty upon the shipment before the goods are delivered to him.

These duties are collected at **customhouses**, which are established by the government for the collection of duties and the regulation of shipping at different ports. A port at which a customhouse is established is known as a **port of entry**.

For example, New York, Boston, Philadelphia, Charleston, and many other cities are ports of entry. Customhouses are also established upon the Canadian and Mexican borders.

The rates at which duties are assessed are set forth in the **tariff**, or schedule of rates, as enacted by Congress. The schedules of the tariff laws specify in detail the rates which are to be charged upon all sorts of goods and commodities.

Typical rates are shown in the table on page 387.

Certain duties are levied as a per cent of the net cost of the goods imported. A duty of this type is known as an **ad valorem duty**, the expression *ad valorem* meaning "according to value." Ad valorem duties are not computed on fractions of \$1, since 50¢ or more is considered as the next higher \$1, and less than 50¢ is disregarded.

Thus on \$428.50 and \$795.80 the duty is reckoned on \$429 and \$796 respectively; on \$546.48 it is reckoned on \$546.

Other duties are levied as fixed sums upon each article, or upon each pound, ton, yard, or other standard measure, without regard to the cost of the goods. Such a duty is called a **specific duty**. In computing specific duties, fractions of a unit are not reckoned. A fraction of half a unit or greater is counted as the next whole unit, and a fraction less than half a unit is disregarded.

Thus $568\frac{1}{2}$ bu. is reckoned as 569 bu., $623\frac{3}{4}$ lb. as 624 lb., and $723\frac{1}{4}$ yd. as 723 yd.

Some articles are subject to both ad valorem and specific duties.

Still other articles may be imported without paying any duty at all. Such articles are said to be **duty free**, or to be upon the **free list**.

In computing the duty on articles on which specific duty is charged by weight, an allowance for the weight of the box or container is made. This allowance is called **tare**.

Thus, if the total weight of a crate of chinaware is 650 lb. and the contents weigh 582 lb., the tare is 68 lb.

ORAL EXERCISES

State the amount of duty which must be paid upon each of the following shipments, and also state whether the duty is an ad valorem duty or a specific duty:

NATURE OF SHIPMENT	RATE
1. Cloth valued at \$550	30%
2. Barley, 700 bu.	30¢ per bushel
3. Carpets valued at \$800	40%
4. Books valued at \$400	25%
5. Vanilla beans, 160 lb.	30¢ per pound
6. Cutlery valued at \$200	50%
7. Hay, 62½ T.	\$4 per ton
8. Wheat, 6000 bu.	30¢ per bushel
9. Rugs valued at \$2500	50%
10. Butter, 1200 lb.	8¢ per pound

Values of foreign money. When invoices are priced in foreign money or the foreign value of goods is considered, it is necessary to change the prices to United States money before computing the duty. The Treasury Department proclaims periodically the values of foreign monetary units in terms of United States money, and these values must be used in determining the duty unless the foreign currency is depreciated. In such cases a consular certificate which shows the amount of depreciation must be attached to the invoice.

The subject of foreign money was treated on page 319, and may be reviewed in this connection.

The table below shows the par values of foreign monetary units which are to be used in the exercises which follow :

COUNTRY	MONETARY UNIT	VALUE IN U. S. GOLD
Brazil	Milreis	\$0.546
Denmark, Norway, Sweden .	Crown	.268
France, Belgium, Switzerland	Franc	.193
Germany	Mark	.238
Great Britain	Pound sterling	4.8665
Italy	Lira	.193
Japan	Yen	.498
Mexico	Peso	.498
Netherlands	Florin	.402
Philippine Islands	Peso	.50
Russia	Ruble	.515
Spain	Peseta	.193

As explained on pages 318-319, these values are the par values, and since the World War the monetary units of many foreign countries have greatly depreciated. On account of the lack of stable conditions in foreign exchange, it is preferable to use the par values in the exercises.

American tariff schedule. The table on the opposite page shows the rates of duty which are charged on certain articles.

ORAL EXERCISES

Using the schedule on page 387, state the amount of duty paid on each of the following :

1. A shipment of butter weighing 1256 lb. ; tare, 56 lb.
2. A shipment of hay weighing 38,400 lb.
3. A shipment of castor beans weighing 1650 lb.
4. A shipment of wheat weighing 18,000 lb.
5. A shipment of 750 bu. of barley.
6. A shipment of books valued at \$256.
7. A shipment of cotton tablecloths valued at \$256.
8. A shipment of silk clothing valued at \$1210.
9. A shipment of ingrain carpets valued at \$385.

ARTICLES	DUTY	
	Specific	Ad Valorem
Automobile robes (value not over 50¢ per pound)	18¢ per pound	30%
Axminster rugs		40%
Barley (48 lb. to the bushel) . .	20¢ per bushel	
Barley malt	40¢ per 100 lb.	
Books		25%
Brussels carpet		40%
Butter	8¢ per pound	
Castile soap		15%
Castor beans	$\frac{1}{2}$ ¢ per pound	
China and porcelain, undecorated		60%
Cider	5¢ per gallon	
Clocks and clock movements . .		45%
Cotton tablecloths		30%
Hay	\$4 per ton	
Ingrain carpets		25%
Knit woollens (value not over \$1 per pound)	30¢ per pound	40%
Knitting machines		40%
Leather goods		30%
Microscopes		45%
Optical instruments		45%
Pocket knives (value not over 40¢ per dozen)	1¢ each	50%
Silk clothing		60%
Toilet soap		30%
Wheat (60 lb. to the bushel) . .	30¢ per bushel	

EXERCISES

Using the above schedule, find the total amount of duty paid on each of the following shipments:

1. Axminster rugs valued at £235; clocks valued at 1500 fr.; cotton tablecloths valued at £52; Brussels carpet valued at 2500 fr.

2. Barley, 480,000 lb. ; castor beans, 30,000 lb. ; barley malt, 120,000 lb. ; wheat, 66,000 lb.

3. Box of knit woolens, weight 270 lb., tare 25 lb., valued at £50 ; 40 books valued at \$2.50 each ; 50 doz. pocketknives valued at 40¢ per dozen ; 60 clocks valued at \$7.75 each.

4. Leather goods valued at 10,500 florins.

5. Automobile robes, weight 450 lb., valued at \$212.50.

6. Silk clothing, 4200 m., valued at 4.5 M. per meter.

7. Porcelain goods valued at 5280 crowns from Denmark.

8. Ingrain carpets valued at £185.

9. Toilet soap valued at 1125 M.

10. An importer ordered optical instruments valued at 12,540 fr. shipped to him from Paris. What was the amount of the duty on the shipment ?

11. The transportation and other charges on the shipment in Ex. 10 amounted to \$105. If exchange was quoted at 19¢ when the importer made his remittance, what was the total cost of the shipment in United States money ?

12. A New York merchant purchases 3620 yd. of Brussels carpet at 5 fr. per yard. If the transportation and other charges amount to \$165, what is the total cost of the shipment, including duty ?

13. If the merchant in Ex. 12 sells the carpet at an advance of 40% above the total cost, what selling price per yard should he place on it ?

14. A Chicago importer bought Dresden china valued at 5000 M. If the transportation and other charges in addition to the duty amounted to \$350, what was the total cost of the shipment ?

15. For how much would the importer in Ex. 14 have to sell the china in order to make a gross profit of 45% of the cost ?

16. A Philadelphia importer receives a shipment of cotton tablecloths from Manchester, England, valued at £550 10 s. How much duty does he have to pay on the shipment? If the prepaid transportation charges amounted to £18, and exchange is \$4.865, what is the cost of the draft which the importer sends to Manchester?

17. An importer bought 250 bx. of castile soap at 28¢ per pound. Each box weighed 80 lb., and an allowance of 5% of the gross weight was made for tare. What was the total cost of the shipment and the cost per pound, including duty?

18. If the importer in Ex. 17 sold the soap so as to realize a gross profit of 50% based on the selling price, what was the amount of the gross profit?

CHAPTER XI

LIFE INSURANCE

Nature of life insurance. When a man desires to make provision for his family so that in the event of his death they may not suffer financial hardship through the loss of his earning power, he often does so by means of **life insurance**. The protection afforded by fire insurance was discussed on page 233, and the same general principles apply to life insurance with the exception that life insurance is a protection for others in the event of the death of the insured.

The terms *policy*, *premium*, *insured*, and so on are used in life insurance with the same meanings as defined on pages 233-234.

Thus, in life insurance the insured pays premiums regularly to a company which agrees to pay a definite sum to the person designated in the policy upon the death of the insured. The person to whom the policy is to be paid is known as the **beneficiary**, and while quite generally it is a member of the insured's family, any person may be named as a beneficiary.

Life-insurance companies. Companies which do business in life insurance are corporations which are either stock companies or mutual companies.

In a stock company the capital is furnished by the stockholders, who share in the profits and are liable for losses in excess of the company's reserve resources according to the amount of stock which they own.

In a mutual company each policyholder shares in the profits, and his liability for the losses of the company is limited to the amounts which he has paid for insurance.

There are also fraternal benefit associations or orders who issue life insurance to their members. Such associations generally

depend upon monthly assessments on their members to meet death claims. These assessments may be increased when it is necessary to meet claims.

The business of life-insurance companies is supervised by many of the states which have passed laws governing the conditions as to reserve funds and other details which the regular life-insurance companies must observe. These laws are not strictly applied to fraternal organizations.

Rates. The rates at which life insurance is issued are quoted at so much for each \$1000 of insurance per year and vary with the age of the insured at the time the policy is written. In determining the different rates there are three elements to consider: (*a*) the amount necessary to pay the death claims, (*b*) the interest which can be earned on the premiums paid by the insured and invested by the company, and (*c*) the cost of doing business.

The amount necessary to meet the death claims can be very accurately foretold through the use of mortality tables. These tables have been compiled from the observation of large groups of individuals, and show how many persons out of a group of 100,000 are expected to die during a given year and how many are expected to live. Since such tables cover so large a group, they give results very close to actual experience. Thus, the first element in determining the rates for life insurance can be found by dividing the estimated amount of the death claims by the number of persons insured.

The second element is dependent upon the rates at which money can be invested by the companies. In all calculations an assumed rate of 3% or $3\frac{1}{2}\%$ is used, since it is certain that at least this rate of interest can be obtained. The interest earned reduces the amount required to pay death claims.

The third element is determined from investigation or from the company's experience, since its records show how much it costs to secure new business, to maintain its offices, and so on. An amount, called *loading*, sufficient to cover the expenses is added to the rate as determined by the first two elements.

Life-insurance premiums are payable annually in advance, but may be paid in semiannual or quarterly advance installments. The amount of the premium to be paid each year is fixed at the time the policy is issued and cannot be increased.

ORAL EXERCISES

1. Describe the nature of life insurance.
2. What is the difference between a stock company and a mutual company?
3. What three elements are considered in determining the rates for life insurance?
4. How are life-insurance rates quoted? How may premiums be paid?

Types of policies. Of the different types of policies issued, the ordinary life, limited-payment life, endowment, and term policies are the leading types.

The chief features of the **ordinary life policy** are that the insured is to pay premiums annually during his lifetime, and that upon due proof of his death the company pays the amount of the policy to the beneficiary. With the exception of term insurance, this type is issued at the lowest rates, since on the average policy the premiums will be paid for a large number of years.

In a **limited-payment life policy** the premiums are paid only for a stated number of years, often twenty, and upon due proof of the death of the insured the company pays the amount to the beneficiary. In such a case if the insured lives longer than the fixed term, he is still insured for the amount of the policy, but has no further premiums to pay. Since the premiums are paid for only a limited number of years, the rates for such insurance must be higher than on ordinary life policies.

Under an **endowment policy** the insurance is similar to that offered by the limited-payment life policy, with the addition that the company further agrees that if the insured lives until a certain date the amount of the policy will then be paid to him.

Thus, a man might take out an endowment policy when he was 30 yr. old which would be payable to his beneficiary if his death occurred before the insured reached the age of 50, but which would be payable to him if he lived until that time. Policies of this type are of advantage in making provision for support in old age, when a man's earning power has decreased.

Since in an endowment policy the full value of the policy must have accumulated by the time the policy is payable to the insured, such policies are issued at the highest rates of any type.

A **term policy** insures a person for only a limited number of years, generally five or ten, and in the event of his death within that time the company pays the amount of the policy to the beneficiary. Such policies cannot be renewed, but may be changed to one of the other types of policies by complying with the conditions prescribed by the company. Since the term of the company's liability to pay death claims is limited, such policies can be issued at the lowest rates of any type.

Term policies are useful when protection at low cost is desired until such time as the insured feels that he can afford to change the policy to a standard form. No credit is allowed the term policyholder for any premiums paid.

The following table shows typical rates for different ages on each of the four types of policies mentioned above:

ANNUAL PREMIUM RATES							
PER \$1000 OF INSURANCE							
AGE	ORDINARY LIFE		20-PAYMENT LIFE		20-YEAR ENDOWMENT		5-YEAR TERM
25	20	55	30	63	48	86	10 61
30	23	31	33	44	49	57	11 15
35	26	88	36	85	50	64	11 96
40	31	56	41	10	52	33	13 28
45	46	36	53	86	59	87	19 95

Reserve. Under the provisions of the insurance laws a company is obliged to set aside part of the premiums which are paid on its policies as a fund to be used toward the payment of the policies when they become due. Such a fund is allowed to accumulate at 3% or $3\frac{1}{2}\%$, as required by law, and is called the **reserve**. By this provision each policy has back of it a share in the company's reserve fund.

Thus, one company which had insurance amounting to \$2,196,673,032 in force at the end of a recent year had to hold \$415,323,325 (over 18%) as a reserve on its policies. This reserve consisted of investments in mortgages, bonds, and other securities.

Surplus. As prescribed by law, the excess of the assets of an insurance company over its liabilities is called the **surplus**.

A surplus may be accumulated when the company is able to invest its money at a rate better than 3% or $3\frac{1}{2}\%$, which is the rate on which the calculations for the premium rates are made; when the death claims are fewer than was expected; or when savings can be made in the cost of doing business.

Except as otherwise provided in the insurance laws, the surplus may be used for such purposes as the company deems best.

After retaining a part of the surplus sufficient to provide for contingencies and unexpected losses, a mutual life-insurance company distributes the remainder of the surplus among its policyholders, each policyholder receiving the share to which his policy entitles him.

Dividends. The distribution of the surplus is commonly called the payment of a **dividend**. Dividends are payable in cash, but they are often used to reduce the amount of the next year's premium or to purchase additional insurance.

Options. If a policyholder wishes to discontinue the payment of premiums on a policy, he is entitled to receive some return on the premiums which have been paid. Under such circumstances the insured generally has several choices, called **options**, by which he may receive this return. Thus, (a) he may take the cash value, or practically all the reserve value of his policy; (b) he may take a paid-up policy for such an amount as its reserve value will purchase, and thus be insured for a less amount without the payment of further premiums; or (c) he may take a term policy for the face of his present policy for as many years and days as its reserve value will purchase.

The illustration on page 395 shows the first page of an ordinary life policy.

THE PROTECTIVE MUTUAL LIFE
INSURANCE COMPANY

OF CHICAGO, ILLINOIS

No. 242,215

Age 35

In Consideration of the payment of

Two hundred sixty-eight and 80 Dollars,
100

the receipt of which is hereby acknowledged, and of the payment of
a like sum on or before the first day of February

in every year during the lifetime of

JOHN WILLIAMS

of MILWAUKEE, WISCONSIN (hereinafter called the Insured),
and immediately upon receipt of this Policy and due proof of
the death of the Insured, THE PROTECTIVE MUTUAL
LIFE INSURANCE COMPANY (hereinafter called the Com-
pany) promises to pay at its Home Office the sum of

TEN THOUSAND Dollars,

less any unpaid premium or premiums to the end of the current
policy year and any other indebtedness on account of this Policy, to

his wife, Jane Williams, Beneficiary,

with reservation to the Insured of the right of revocation
and change of Beneficiary.

This Policy is issued by the Company and accepted by the parties
in interest subject to the provisions stated on the consecutively num-
bered pages hereof which are hereby made a part of this contract.

IN WITNESS WHEREOF the Company has by its President
and Secretary executed this contract this first day of
February, 19.

E. L. Merrill

Secretary

J. B. Hart

President

ORDINARY LIFE POLICY

On the second page of the policy shown on page 395 appears the following table, which shows the options which the insured may elect in case he desires to discontinue the payment of premiums:

NONFORFEITURE AND LOAN VALUES					
BASED UPON A POLICY OF \$1000 FREE FROM INDEBTEDNESS AND WITHOUT DIVIDEND ADDITIONS					
AT END OF POLICY YEAR	LOAN OR CASH VALUE		PAID-UP INSURANCE	EXTENDED TERM INSURANCE	
				Years	Days
2	16	13	37	1	297
3	34	76	78	3	333
4	53	77	119	6	6
5	68	16	148	7	181
6	82	94	177	8	325
7	98	11	206	10	56
8	113	68	234	11	99
9	129	65	262	12	86
10	146	01	289	13	20
11	162	76	316	13	269
12	179	87	343	14	107
13	197	35	369	14	270
14	215	16	395	15	32
15	233	28	420	15	127
16	251	68	445	15	195
17	270	34	469	15	238
18	289	22	492	15	258
19	308	32	515	15	260
20	327	58	537	15	245
21	347	00	559	15	214
22	366	52	579	15	171

Thus, if John Williams decides at the end of the tenth policy year that he does not wish to pay premiums any longer on this policy, he may make one of the following choices: (a) he may take the cash value of his policy (which is tentimes the amount shown in the first column, since these figures are quoted for each \$1000 of insurance) and thus receive \$1460.10 in cash; (b) he may take a paid-up policy for \$2890 ($\289×10); or (c) he may take a term insurance policy for \$10,000, which will insure him for a period of 13 yr. 20 da.

The cash value is also the amount that the insured may borrow from the company on his policy. When such a loan is made the company holds the policy as security. If the borrower should die before the loan is paid, the company would deduct the amount loaned from the payment to the beneficiary.

ORAL EXERCISES

1. Describe in your own words the principal features which distinguish the four leading types of life-insurance policies.

2. For how many years do premiums have to be paid on each of the following types of policies: 20-year endowment; 20-year limited-payment life; ordinary life; 5-year term?

3. In your own words discuss the meaning of the terms "reserve," "surplus," and "dividend," as used in connection with life insurance.

4. What kind of policy is the one shown on page 395? Who is the beneficiary? the insured? What is the amount of the annual premium? If the insured should die, how much would the beneficiary receive?

5. A man aged 35 takes out an ordinary life policy for \$5000 in the same company as the one which issued the policy shown on page 395. If he decides to discontinue paying premiums after the fifth annual premium is paid, how much paid-up insurance can he obtain?

The student should consult the table of nonforfeiture and loan values on page 396.

6. If the insured in Ex. 5 elected to take extended term insurance, for how long a period would he be insured? What amount would the beneficiary receive if he died within this period?

7. In Ex. 5 how large a sum may the insured borrow on his policy after fifteen annual premiums have been paid?

EXERCISES

1. If the insured in the policy on page 395 had taken out a 20-payment policy for the same amount at the rates shown in the table on page 393 (age 35), how much more would his insurance have cost him in 10 yr., exclusive of dividends and interest?

2. If the insured in Ex. 1 should die just before the twelfth annual premium on his 20-payment policy was due, how much more would his beneficiary receive than had been paid for the insurance?

3. At the rates shown in the table on page 393, what annual premium must a man 45 yr. old pay on an ordinary life policy for \$15,000?

4. At the rates shown in the table on page 393, a man on his 25th birthday took out an ordinary life policy for \$5000; on his 35th birthday, a 20-year endowment policy for \$6000; and on his 40th birthday a 20-payment life policy for \$10,000. If he died when he was $43\frac{1}{2}$ yr. old, how much more, exclusive of dividends and interest, did his heirs receive than he had paid the company?

5. At the rates shown in the table on page 393, a man took out a 20-payment life policy for \$5000 on his 25th birthday, and died just before the 20th payment came due. During this period the company paid him \$87.50 per \$1000 of insurance in dividends, and these were used to reduce the premiums. How much more did his heirs receive than was paid in premiums?

Using the table of rates given on page 393, find the annual premium on each of the following policies:

	AGE	TYPE OF POLICY	AMOUNT OF INSURANCE
6.	25	20-year endowment	\$2000
7.	40	20-payment life	\$10,000
8.	30	Ordinary life	\$5000
9.	45	5-year term	\$8000
10.	35	20-payment life	\$5000
11.	30	20-year endowment	\$3000
12.	25	Ordinary life	\$10,000
13.	35	20-year endowment	\$5000
14.	40	Ordinary life	\$10,000
15.	35	Ordinary life	\$5000

SUPPLEMENTARY PROBLEMS

INVOICING

NOTE. The differences in prices within a problem are due to differences in quality and style.

1. Elliott & Company buy of Sampson & Son the following goods: 3 doz. Pajamas @ \$17.50; $1\frac{1}{2}$ doz. @ \$24; $\frac{1}{4}$ doz. @ \$36; $1\frac{1}{4}$ doz. @ \$22.50; $2\frac{1}{4}$ doz. @ \$26; $\frac{3}{4}$ doz. @ \$27; $2\frac{1}{2}$ doz. @ \$23.50; 3 doz. @ \$18.75; $2\frac{1}{2}$ doz. @ \$24.50; $1\frac{1}{4}$ doz. @ \$19.50. Find the amount of the bill.

2. John S. Wilson buys of Field & Company the following goods: $\frac{6}{12}$ doz. Men's Trousers @ \$36; $\frac{5}{12}$ doz. @ \$48; $\frac{6}{12}$ doz. @ \$42; $\frac{5}{12}$ doz. @ \$45; $\frac{3}{12}$ doz. @ \$46; $\frac{5}{12}$ doz. @ \$50; $\frac{6}{12}$ doz. @ \$43.50; $\frac{3}{12}$ doz. @ \$49; $\frac{6}{12}$ doz. @ \$47; $\frac{5}{12}$ doz. @ \$39. What is the amount of the check which John Wilson writes for the bill?

3. Amos Teller buys of Grant & Company the following goods: $\frac{9}{12}$ doz. Union Suits @ \$11.50; $\frac{9}{12}$ doz. @ \$13.50; $\frac{6}{12}$ doz. @ \$9.25; $\frac{3}{12}$ doz. @ \$12.50; $\frac{6}{12}$ doz. @ \$13.75; $1\frac{6}{12}$ doz. @ \$18.75; $\frac{9}{12}$ doz. @ \$22.50; $\frac{6}{12}$ doz. @ \$14.50; 2 doz. @ \$18.75; $1\frac{3}{12}$ doz. @ \$16.25; 3 doz. @ \$21.50; $2\frac{6}{12}$ doz. @ \$21.75. Find the amount of the bill.

4. A. R. Thompson buys of Gross & Company the following invoice of merchandise: 54 pc. Gingham, 41 yd. in each piece, @ $6\frac{1}{4}\text{¢}$; 20 pc. Prints, 43 yd. in each piece, @ 9¢ ; 11 pc. Suiting, 32 yd. in each piece, @ $93\frac{1}{3}\text{¢}$; 12 pc. Prints, 55 yd. in each piece, @ 8¢ ; 6 pc. Ticking, 58 yd. in each piece, @ $7\frac{1}{2}\text{¢}$; 42 pc. Flannel, 31 yd. in each piece, @ 23¢ ; 13 pc. Sheeting, 56 yd. in each piece, @ 9¢ ; 9 pc. Gingham, 43 yd. in each piece, @ $8\frac{1}{3}\text{¢}$. What is the amount of the bill?

5. William Arnold buys of Stern & Company the following goods: $3\frac{1}{2}$ doz. Girls' Suits, @ \$10.50; $2\frac{1}{2}$ doz. @ \$10.65; $\frac{1}{3}$ doz. @ \$7.65; $\frac{1}{6}$ doz. @ \$9.50; $\frac{1}{4}$ doz. @ \$10.50; $\frac{1}{3}$ doz. @ \$12.25; $\frac{5}{12}$ doz. @ \$14.40; $\frac{1}{3}$ doz. @ \$13.75; $\frac{1}{3}$ doz. @ \$14; $\frac{1}{2}$ doz. @

\$12.75; $\frac{5}{12}$ doz. @ \$18; $\frac{1}{3}$ doz. Boys' Suits @ \$13.50; $\frac{1}{8}$ doz. @ \$15.50; $\frac{1}{3}$ doz. @ \$18.50; $\frac{1}{6}$ doz. @ \$17.50; $\frac{2}{3}$ doz. Children's Sleeping Suits @ \$9.50; $\frac{1}{2}$ doz. @ \$10.25; $1\frac{1}{4}$ doz. @ \$11; $\frac{1}{3}$ doz. @ \$8.75; $\frac{2}{3}$ doz. @ \$13. Find the amount of the bill.

6. James Miller buys of Gough & Company the following goods: 2 pc. Drapery, $30\frac{1}{4}$, $32\frac{1}{2}$, @ $67\frac{1}{2}\epsilon$; 6 pc. Wedgemere Chintz, 24, $32\frac{1}{4}$, 31, $26\frac{1}{2}$, $20\frac{1}{2}$, $25\frac{1}{2}$, @ $32\frac{1}{2}\epsilon$; 10 pc. Fenway Cretonne, 20, $22\frac{3}{4}$, $32\frac{1}{2}$, $30\frac{1}{2}$, $24\frac{1}{4}$, 31, $20\frac{3}{4}$, $23\frac{1}{2}$, 32, 31, @ $32\frac{1}{4}\epsilon$; 6 pc. Ardmore, $19\frac{3}{4}$, $21\frac{1}{2}$, 25, 20, 22, $23\frac{1}{2}$, @ $27\frac{1}{2}\epsilon$; 9 pc. D. F. Ardmore, 25, 21, 20, $26\frac{1}{2}$, 27, 25, 20, 23, $22\frac{3}{4}$, @ 28ϵ ; 7 pc. Eureka Cretonne, $26\frac{1}{4}$, 30, 27, $30\frac{3}{4}$, $31\frac{1}{2}$, $32\frac{3}{4}$, 28, @ 21ϵ . Find the amount of the bill.

7. David Jones buys of Wells and Company the following goods: $\frac{3}{4}$ doz. Artificial Flowers @ \$4.50; $1\frac{1}{3}$ doz. @ \$4; $\frac{1}{2}$ doz. @ \$3.75; $\frac{3}{4}$ doz. @ \$3.85; $\frac{1}{2}$ doz. @ \$4.55; $\frac{2}{3}$ doz. @ \$6.40; $\frac{3}{4}$ doz. @ \$9; $\frac{1}{2}$ doz. @ \$4.25; $\frac{1}{6}$ doz. @ \$8.50; $\frac{1}{3}$ doz. @ \$8.25; 2 gro. Wire #7 @ \$1.25; $\frac{1}{2}$ gro. #10 @ \$4.35; $\frac{1}{2}$ gro. #8 @ \$3.85; 3 gro. #1 @ 45ϵ ; $1\frac{5}{12}$ doz. Favors @ \$3.40; $1\frac{1}{2}$ doz. @ \$5.40. Goods returned amounted to \$7.35. Find the amount of the bill.

8. Harold Emerson buys of John S. Carson the following goods: 2 doz. Hose @ \$12.25; $\frac{3}{4}$ doz. @ \$15.50; $1\frac{1}{2}$ doz. @ \$12.75; $2\frac{1}{2}$ doz. @ \$19.50; $1\frac{1}{2}$ doz. @ \$14.75; $\frac{3}{4}$ doz. @ \$7.75; $3\frac{1}{2}$ doz. @ $\$7.62\frac{1}{2}$; $1\frac{3}{4}$ doz. @ \$4.25; $\frac{1}{2}$ doz. @ \$8.75; $3\frac{1}{2}$ doz. @ \$6.25; $1\frac{3}{4}$ doz. @ \$4.75; $2\frac{1}{2}$ doz. @ \$5.25; $1\frac{3}{4}$ doz. @ \$5.65; $\frac{1}{2}$ doz. @ \$3.85; $2\frac{1}{2}$ doz. @ \$2.55; $3\frac{1}{2}$ doz. @ \$3.75; $\frac{3}{4}$ doz. @ \$4.50; $2\frac{3}{4}$ doz. Vests @ \$4.65; $\frac{1}{2}$ doz. @ \$5.35; $2\frac{1}{4}$ doz. @ \$4.95; $\frac{3}{4}$ doz. Suits @ \$14.75; $1\frac{1}{4}$ doz. @ \$13.50. Goods returned amount to \$4.50. Find the amount of the bill.

9. John S. Bird buys of Clement & Co. the following goods: 7 pc. H. Jeans, 40, 41, 43, 45, 42, $43\frac{1}{2}$, $40\frac{1}{2}$, @ $33\frac{1}{3}\epsilon$; 10 pc. Flannel, 41, 43, 40, $42\frac{1}{2}$, 41, 44, $42\frac{1}{2}$, 38, 41, 44, @ 36ϵ ; 6 pc. Plaids, 38, 41, $39\frac{1}{2}$, 37, $41\frac{1}{2}$, 40, @ $13\frac{1}{2}\epsilon$; 8 pc. Flannel, 42, 38, 38, 41, 43, 40, 42, 44, @ $49\frac{1}{2}\epsilon$; 12 pc. Cambric, 41, 29, $37\frac{1}{2}$, 33, $41\frac{3}{4}$, 42, 41, $40\frac{1}{2}$, 44, 43, 42, 45, @ $11\frac{1}{2}\epsilon$; 6 pc. Cotton, 43, 45, 40, 44, $43\frac{3}{4}$, $41\frac{1}{2}$, @ $8\frac{1}{3}\epsilon$; 37 doz. Buttons @ $\$1.12\frac{1}{2}$. Find the amount of the bill.

10. James Durant buys of Haller & Co. the following goods: 12 pc. Shirting, 43, $47\frac{1}{2}$, 54, $48\frac{1}{2}$, 37, $39\frac{1}{2}$, 41, $48\frac{3}{4}$, $51\frac{1}{2}$, 46, 47, $43\frac{3}{4}$, @ $12\frac{1}{2}\epsilon$; 5 pc. Cashmere, 48, $53\frac{1}{2}$, 41, $49\frac{3}{4}$, $40\frac{1}{2}$, @ $\$1.33\frac{1}{3}$. Find the amount of the bill.

11. Find the amount of the bill for the following fancy ribbons:
34 yd. @ 75¢; 77 yd. @ 28¢; 83 yd. @ $87\frac{1}{2}$ ¢; 67 yd. @ $8\frac{1}{3}$ ¢;
77 yd. @ 99¢; 133 yd. @ $33\frac{1}{3}$ ¢; 73 yd. @ \$1.12 $\frac{1}{2}$; 64 yd. @ 78¢;
52 yd. @ $83\frac{1}{3}$ ¢; 67 yd. @ 42¢; 154 yd. @ $42\frac{1}{2}$ ¢; $65\frac{1}{2}$ yd. @ $12\frac{1}{2}$ ¢.

12. John Wilbur buys of Ford & Company the following goods:
3 Waffle Irons @ \$9.75; 2 Thernax Toasters @ $\$2.37\frac{1}{2}$; 3 Hot
Point Irons @ \$3.25; 5 Electric Stoves @ \$1.35; 4 Universal
Irons @ \$4.75; 5 Waffle Irons, B, @ \$4.75; 3 Percolators @ \$11.67;
3 Universal Table Stoves @ $\$8.97\frac{1}{2}$; 3 Universal Heating Pads @
\$5.65; 6 Electric Wavers @ \$2.45; 7 Electric Curling Irons @
 $\$0.37\frac{1}{2}$; 4 Electric Fans @ \$8.25; 6 Tumbler Holders @ \$0.63;
24 Soap Dishes @ \$0.29; 18 Tooth Brush Holders @ \$0.48; 15
Thermometers @ \$0.87; 5 Copper Thermometers @ \$0.98; 8
Chair Fasteners @ \$0.57; 6 Foot Bolts @ \$0.68; 5 Window Hold-
ers @ \$0.53; 6 Door Stops @ \$0.84. Find the amount of the bill.

13. James Wesson buys of Henry Peterson the following goods:
 $1\frac{1}{2}$ doz. Bath Robes @ \$47.75; $\frac{1}{8}$ doz. @ \$63.25; $\frac{1}{2}$ doz. @ \$37.75;
 $\frac{3}{4}$ doz. @ \$45.25; $\frac{1}{3}$ doz. @ \$61.50; $2\frac{1}{2}$ doz. @ \$39.25; $\frac{1}{4}$ doz. @
\$49.55; 5 doz. Jackets @ $\$2.37\frac{1}{2}$; $\frac{1}{4}$ doz. @ \$4.65; $\frac{1}{3}$ doz. Sweat-
ers @ \$42.50; $\frac{1}{6}$ doz. @ \$31.50; $\frac{1}{4}$ doz. @ \$29.25. Find the amount
of the bill.

14. James Rainey buys of Sampson & Sons the following goods:
9 Oak Bedsteads @ \$24.75; 8 Brass Bedsteads @ \$19.75; 24
Dining Chairs @ \$3.35; 15 Library Rockers @ \$14.75; 8 Corner
Chairs @ \$22.75; 6 Toilet Tables @ \$34.65; 12 Chiffoniers @
\$15.75; 12 Dressing Cabinets @ \$28.75; 6 Parlor Tables @ \$15.25;
8 Hall Stands @ \$41.75; 3 Sideboards @ \$95.40; 6 Writing Desks
@ \$11.25; 4 Bedroom Sets @ \$33.60; 9 Parlor Lamps @ \$12.50;
6 Iron Bedsteads @ \$27.75; 4 Card Tables @ \$9.75; 4 Dining-
room Sets @ \$95.25; 3 Fancy Rockers @ \$9.75; 2 Chiffoniers @
\$22.75; 6 Music Cabinets @ \$8.65. Find the amount of the bill.

15. Miss Alice Homer purchases from a lace dealer the following
kinds and amounts of lace: $7\frac{1}{2}$ yd. Filet @ \$0.89; $8\frac{3}{4}$ yd. Manlies
@ \$8.78; 12 yd. Brussels Point @ \$12.35; $11\frac{1}{2}$ yd. Irish Crochet
@ \$1.58; $12\frac{1}{4}$ yd. Valenciennes @ \$2.35; $8\frac{3}{4}$ yd. Point de Paris @
\$3.47; $15\frac{1}{2}$ yd. Point de Paris @ $\$3.12\frac{1}{2}$. Find the amount of
the bill.

PRACTICAL MEASUREMENTS

1. A gas tank of a certain gas company has the shape of a cylinder 15 yd. in diameter and 20 yd. high. How many cubic yards does it contain?

2. One of the California big trees is 25 ft. in diameter and 300 ft. tall. How many tons does it weigh? Lumber is 5% lighter than water, which weighs $62\frac{1}{2}$ lb. per cubic foot. (Assume the tree to have the shape of a cylinder.)

3. A spherical balloon is 50 ft. in diameter. What is the volume of the balloon in cubic yards? ($V = \frac{4}{3} \times \frac{2}{7} \times \text{radius}^3$.)

4. A bowl in the form of a half sphere is 1' 2'' in diameter. How many cubic inches does it contain?

5. Lead is 11.4 times as heavy as water, and 1 cu. ft. of water weighs 1000 oz. Find, to the nearest ounce, the weight of a lead ball $2\frac{1}{4}$ " in diameter.

6. Gold is 19.2 times as heavy as water. Find the weight of a brick of gold $2'' \times 4'' \times 8''$.

7. Compute the value of the gold brick in problem 6 above, gold being worth \$16 per ounce.

8. What effect does doubling the edge of a cube have on the volume? Test your statement on the following three pairs of cubes whose edges are: 1' and 2'; 2'' and 4''; 5' and 10'.

9. A farmer finds that to store the feed for a herd of 10 cows for 6 months he needs a cylindrical silo 10 ft. in diameter and 25 ft. high. How high must another silo be made if it is 18 ft. in diameter and must supply 16 cows for 8 months?

10. A swimming tank 50 ft. long and 25 ft. wide is 3 ft. deep at one end and 7 ft. at the other. How many gallons does it contain if there are $7\frac{1}{2}$ gallons per cubic foot?

11. How many cubic inches are there in a cylindrical rod 6 ft. 4 in. long and $1\frac{2}{3}$ in. in diameter?

12. How many square feet of sheet iron will it take to make a pipe 8 in. square and 9 ft. long, allowing $\frac{1}{2}$ in. for the fold?

13. The dimensions of a brick are $\frac{2}{3}$ ft. by $\frac{1}{3}$ ft. by $\frac{1}{6}$ ft. How many bricks are there to the cubic yard?

14. A rectangular granite block is 5 ft. high and has a base $7\frac{1}{2}$ ft. by $2\frac{1}{2}$ ft. Find the cost of polishing the sides and the top at 5 cents per square foot.

15. What fraction of a square foot is covered by a square 4" by 4"?

16. What is the area of the largest circular piece of tin that can be cut from a sheet 5 ft. 7 in. square? What fractional part of the original sheet is left?

17. One of the rooms in a factory is heated by 240 ft. of steam pipe 2 in. in diameter. Find the radiating surface (the area of the curved surface which gives out the heat).

18. In making metal bedsteads a factory uses iron rods 0.8 in. in diameter covered with thin rolled brass. How many square feet of rolled brass will be needed for 3200 ft. of iron-rod material?

19. How many bars of soap each 4 in. long, $2\frac{3}{4}$ in. wide, and $1\frac{1}{2}$ in. thick can be packed in a box 16 in. long, $13\frac{3}{4}$ in. wide, and 1 ft. deep?

20. Express in stricken bushels the capacity of a grain bin which is 6 ft. by 8 ft. by 4 ft. 3 in. (1 stricken bu. = $1\frac{1}{4}$ cu. ft.)

21. In California the orange box is sometimes $11\frac{1}{4}$ in. by $26\frac{1}{2}$ in. by $11\frac{1}{4}$ in. and sometimes $7\frac{3}{4}$ in. by 22 in. by $17\frac{1}{2}$ in. Which box holds the more, and how many cubic inches does it hold?

22. A freight car 8 ft. by $26\frac{3}{4}$ ft. is filled with corn to a depth of 6 ft. Find the value of the corn at 75 cents per stricken bushel.

23. The bottom of an ordinary farm-wagon box is usually 3 ft. by 10 ft. and the depth of the box is 25 in. Find the capacity in cubic feet for a depth of 23 in.

24. A state highway 12 mi. long and 45 ft. wide was covered with a layer of crushed rock to an average depth of 6 in. What was the cost of the crushed rock at \$1.50 per cubic yard?

25. Express in heaped bushels the capacity of a fruit bin which is 5 ft. by 7 ft. by 5 ft. 6 in.

26. A bushel cranberry box in New Jersey is $8\frac{3}{4}$ in. by 12 in. by 22 in. How does the volume of this box differ from that of the standard bushel? (1 standard bu. = 1848 cu. in.)

27. A mason plans to build a wall 32 ft. long, 6 ft. high, and 1 ft. thick. How many bricks will be needed if 22 bricks with the mortar fill one cubic foot?

28. A coal bin is 21 ft. long, 12 ft. wide, and 7 ft. deep. How many tons of coal will it hold if the volume of a ton of coal is 38 cu. ft.?

29. A hay mow is 23 ft. long, 12 ft. wide, and 8 ft. high. How many tons of hay will it hold if a ton of hay may be taken as 500 cu. ft.?

30. A cistern is 5 ft. square and $6\frac{2}{3}$ ft. deep. How many barrels of water will it hold?

31. Milk is sold in France by the liter. If it costs the equivalent of 8 cents per liter, how much does it cost per gallon?

32. A French boy is 5 ft. $4\frac{1}{2}$ in. tall. What would he say his height was (based on meters and centimeters)?

33. A certain city building lot measures 46 ft. by 95 ft. Express its area in acres. How much is it worth at 80¢ per square foot?

34. It is desired that a desk drawer shall contain 2464 cu. in. If it is 32 in. long and 14 in. wide, how deep must it be?

35. A certain yacht club flag is a triangle in shape, with sides measuring 12'', 27'', and 27''. How many square inches of material does it contain?

36. Mrs. Johnson wishes to cover her cylindrical wastebasket with cretonne. The basket is 25 in. high and the bottom has a diameter of 14 in. How long a strip of cretonne 39 in. wide will she have to buy to glue around the basket, and what will be the area of the strip of cretonne that is not used? Can a circle to glue on the bottom of the basket be cut from the remaining strip, and, if so, how much material will be left after it is cut out? (Do not allow for overlapping or turning under the material.)

37. A dining-room floor measures 16 ft. by 12 ft. 6 in. How many board feet of flooring 4 in. wide will be required, allowing $\frac{1}{4}$ for waste? How much will the lumber cost at \$95 per M?

38. A gardener is planting a circular garden which is 14 ft. in diameter. Around the extreme outer edge he is to plant rose-bushes at 12 in. intervals. How many will he need?

PERCENTAGE

1. The state of Colorado contains 103,900 sq. mi. The area of California is 52.6% greater. Find the area of California.

2. The population of the United States increased from 91,972,000 in 1910 to 105,711,000 in 1920. Compute the percentage of increase.

3. In a certain school there are 427 boys and 583 girls. What per cent of the total enrollment are boys? What per cent are girls?

4. A merchant built an addition to his store which increased the floor space 36%. After the addition was completed, the floor space was 2320 sq. ft. What was the number of square feet of floor space in the original store, and what was the amount of the addition?

5. Mr. Brown's salary was increased \$50. He then received \$280 per month. What was the per cent of increase?

6. How much should a salesman sell in a year to yield him a commission of \$4000 at $3\frac{1}{2}\%$ on goods sold?

7. Data of a recent year from the Bureau of Labor Statistics show the following number of families provided for in new dwellings in 257 cities: in one-family dwellings, 188,074; in two-family dwellings, 64,298; and in three-or-more-family dwellings, 209,842. Compute the per cent of these families provided for in each of the three types of new dwellings.

8. Mr. Allen invested \$15,000 in a business, Mr. Bates \$8000, and Mr. Hood his services. Out of the profits of \$9000 Mr. Hood was allowed \$1500 a year and each of the others 12% of the sum invested by him. The remainder of the profits was divided equally among the three; find the share of each.

9. A dealer was obliged to sell a stock of damaged goods at 18% below cost, thereby losing \$450. What was the cost of the goods?

10. Merchandise must be sold with 15% loss at the price of \$270.80. Find the cost price.

11. After increasing the price of merchandise 10%, a dealer found that he could not sell it; he then decreased it 20% below the previous price. How much was this price below the first marked price?

12. What per cent is made in a class test if 124 points are obtained out of a possible 156?

13. A certain basket-ball player can shoot on an average 80% of his free throws. If there are 15 fouls called on his opponents in a game, how many points should he make on fouls? (1 point per free throw; 1 free throw per foul.)

14. The length of the Mississippi River is 3160 miles; the length of the Ohio River is 950 miles. The length of the Ohio is what per cent of the length of the Mississippi?

15. The lead production of the United States in a recent year in thousands of short tons shows the following figures:

Colorado	31.8	Oklahoma	78.5
Idaho	123.4	Utah	166.8
Missouri	208.5	Other states	77.3

What per cent of the lead production has each state?

16. In a certain year the coal production in the United States was 1320 millions of metric tons; in a following year it was 1353 millions. What was the per cent of increase?

17. The fastest sailing ship, the *Flying Cloud*, made $433\frac{1}{2}$ nautical miles in one day; the steamship *Leviathan* has an average speed of 24.67 knots per hour. (1 knot = 1 nautical mile.) Express the rate of the *Leviathan* as a per cent of the maximum rate of the *Flying Cloud*.

18. The registered tonnage of the United States steamship *Leviathan* is 59,957; that of the British steamship *Majestic* is 56,551. By what per cent is the *Leviathan* greater than the *Majestic*; the *Majestic* smaller than the *Leviathan*?

19. The number of manufacturing establishments of the several sections of Greater New York in a recent year are given below:

Bronx	839
Brooklyn	4,293
Manhattan	17,138
Queens	1,203
Richmond	141

What per cent of the total is the share of each section?

20. The cane sugar production in the United States in 1927 was 94,460,800 pounds; that of beet sugar was 1,699,532,000 pounds. What per cent of the total was the production of each kind of sugar? Express the beet sugar production as a per cent of the cane sugar production.

21. The tobacco production in a recent year in the United States in thousands of pounds was 1,268,566; in Brazil, 158,850; in Japan, 138,206. Express the tobacco production of Brazil and Japan as per cents of the production of the United States.

22. The length of the telephone wire in the United States in thousands of miles is approximately 52,200; that of the telegraph wire is 2035. Express the length of the telephone wire as a certain per cent of the length of the telegraph wire.

23. The Great Northern Telegraph Co. has 110 cables with a total length of 53,962 miles; the Western Telegraph Co. has 43 cables with 28,883 miles; the Western Union Telegraph Co. has 40 cables with 28,620 miles; and the All American Cables Inc. has 46 cables with 24,887 miles. Calculate the per cent each of these companies has of the total length of the cables.

24. The water-power development in North America in thousands of H. P. is 16,800; in South America, 677; in Europe, 12,800; in Asia, 2000; in Africa, 14; in Oceania, 225. What per cent of the total has each area?

25. A man bought a house for \$8000 as an investment. The average yearly expenses are as follows: insurance, \$6; taxes, \$142; water, \$18; upkeep, \$150. What monthly rental must he charge in order to receive 10% net on his investment?

26. The owner of an apartment house receives in rentals \$4000 a year. His yearly expenses are: insurance, \$17; taxes, \$400; water, \$25; upkeep, \$350. At what price should he offer it for sale as an investment which will net 12% to the purchaser?

27. A landowner plans to increase his cultivated land each year by 10%. If he has 400 acres of cultivated land at present, how many acres will he have under cultivation at the end of 5 years?

28. Mr. White paid \$9000 for a house. He rented it for \$75 a month. In a certain year he paid \$183 for taxes, \$7 for insurance, and \$80 for repairs. What per cent did he receive on his investment?

29. Mrs. Ryan received a letter suggesting that she put \$20,000 in a moving-picture concern which promised dividends of at least 15%. From another source she was advised to invest the same sum in a building lot in a section where lots were said to increase in value 12% per year. What would be the difference in returns from these investments during the first three years if they were as promised?

30. A syndicate purchases a 100-acre tract of land at \$1000 per acre. The tract is to be subdivided into 500 fifty-foot lots and fully improved, each lot to sell at an average of \$1500. What is the gross return?

The estimate of total expenses for the subdivision of the tract is as follows:

Subdivision: Planning, surveying, staking, recording	\$4,350
Improvement: Cleaning, grading, water connections, sidewalks, gas and electricity installation, etc.	270,500
Sales Expense: Office help, literature, advertising, automobiles, etc.	19,500
Title Recording: Trustee, deeds, revenue, abstracts, attorney, etc.	3,550
General Overhead	37,500
Sales Commission: Sales manager and salesmen	112,500
Total	<u>\$447,900</u>

What is the net return to the syndicate? What per cent of the expenditure was used in actual improvement of the property, and what per cent was used in each of the other items involved in the promotion project?

31. The syndicate mentioned above consisted of five shareholders, who invested as follows: Mr. Ash and Mr. Bean, \$10,000 each; Mr. Cork, \$15,000; Mr. Davis, \$25,000; and Mr. Ells, \$40,000. During the first four years a need for more capital arose from increased materials and labor costs, and a special assessment of 170% of his investment was levied on each shareholder. The following year, however, 85% of the syndicate's total investment was returned to the shareholders. How much did each one receive?

GRAPHS

1. Draw a bar graph to express the population statistics for the following cities, census of 1920 :

CITIES	POPULATION
Boston, Mass.	748,000
Chicago, Ill.	2,702,000
Detroit, Mich.	994,000
Philadelphia, Pa.	1,825,000
St. Louis, Mo.	773,000
San Francisco, Cal.	508,000

Use the scale 1" = 500,000 people.

2. The consumption of electric current in a certain district shows for 2-hour intervals the following records expressed in thousands of kilowatt hours (K.W.H.) :

	A. M.						P. M.					
Hour	2	4	6	8	10	12	2	4	6	8	10	12
K. W. H. in thousands	6.1	5.8	6.1	8.7	9.7	11.4	13.1	13.3	22	12.6	11.1	8.4

Draw a broken-line graph showing the trend in the consumption of electric current during the day.

3. The values of cotton products manufactured in a given year in several states are, in millions of dollars, Massachusetts, 605; North Carolina, 318; South Carolina, 228; Georgia, 192; Rhode Island, 177. Prepare a circle graph showing the percentage of cotton manufacturing done in each state.

4. In 1925 the wheat production in millions of bushels was as follows: the United States, 669; Russia, 482; Canada, 417. Draw a circle graph showing the percentage of each country.

5. The development of the United States railroads is shown by the following figures in thousands of miles: 1870, 53; 1880, 93; 1890, 159; 1900, 193; 1910, 239; 1920, 253. Draw a broken-line graph to show the increase in railroad mileage.

6. The monthly rainfall of the city of New York in a certain year was as follows :

MONTH	INCHES	MONTH	INCHES
Jan.	5.97	July	4.68
Feb.	2.33	Aug.	1.12
March	4.08	Sept.	2.34
April	2.07	Oct.	4.65
May	1.58	Nov.	2.26
June	1.86	Dec.	3.78

Draw a broken-line graph showing the year's rainfall by months. Use as scale $1'' = 1''$.

7. The main sheep-producing countries are : Australia, 83 millions ; Russia, 63 millions ; the United States, 39 millions. Draw a bar graph (vertical) to show the share of sheep production of each of these countries.

8. The copper production of the United States in thousands of tons has been : 1870, 13 ; 1880, 27 ; 1890, 116 ; 1900, 270 ; 1910, 482 ; 1920, 540 ; 1925, 854. Draw a broken-line graph to show the trend of copper production in the United States.

9. The following table shows the savings deposits and depositors in all banks of the United States for a period of years :

YEAR	TOTAL SAVINGS DEPOSITS	TOTAL NUMBER OF SAVINGS DEPOSITORS	U. S. PER CAPITA SAVINGS DEPOSITS
1919	\$13,040,393,000	18,221,453	\$124
1920	15,314,061,000	22,415,148	144
1921	16,500,663,000	27,792,948	153
1922	17,578,920,000	30,544,738	161
1923	19,726,534,000	35,878,758	178
1924	21,188,734,000	38,741,634	185
1925	23,134,052,000	43,850,127	204
1926	24,696,192,000	46,762,240	211

Draw three broken-line graphs for the above data : (a) to show the trend in the total savings deposits, using the scale $\frac{1}{4}'' = 1$ billion dollars ; (b) to show the trend in total number of savings depositors, using the scale $1'' = 10,000,000$ depositors ; (c) to show the trend in per capita savings deposits, using a convenient scale.

TRADE DISCOUNT

1. An implement dealer purchased repair parts for a gasoline engine at list \$12, less 25%, and binder repairs at list \$20, less 35%. What was the total net invoice cost?

2. Find the total list price of a shipment of electric-light bulbs if the trade discount of 20% amounts to \$17.

3. The discount series on articles listed at \$2000 is 40%, 10%, and 10%. What is the invoice cost? What equivalent single discount applied to the list price will give the same invoice cost?

4. The invoice cost of a radio, after deducting a trade discount of 40%, was \$210. What was the list price?

5. A wholesale jeweler offered a dealer a trade discount of 40% and 10% on merchandise listed at \$120. The dealer, in turn, offered a customer a trade discount of $33\frac{1}{3}\%$ from list price. What was the invoice cost to the dealer? to the customer? What was the dealer's gross profit on the transaction?

6. Dresses listed at \$48 are invoiced at \$32. What is the trade discount percentage?

7. A dealer quoted merchandise at \$1500, less 40% and 20%. He later sold the merchandise for \$36 less than the original quotation. What was the new discount series?

8. The invoice cost of an article upon which a trade discount series of 50%, 20%, and 5% is allowed is \$798. What is the list price?

9. A certain dealer is allowed a trade discount of 50% on list price. On merchandise listed at \$800, what is his gross profit if he offers a discount of 40%? of $33\frac{1}{3}\%$ and 10%? of 25% and 10%?

10. A wholesaler wished to increase the selling price of merchandise listed at \$150 to a net of \$112.50. The original net selling price was 65% of list. What was the original discount percentage? the new discount percentage?

11. A jobber purchases merchandise at list \$1200, less 40% and 10%. He wishes to sell at a gross profit of $33\frac{1}{3}\%$ on sales. If he quotes a trade discount of 50% and 20% to his customers, at what price must he list the merchandise to yield him the desired gross profit?

12. The prices listed by the Adams Co. for No. 6 Dry Cells are in four groups as follows :

1. Less than 12	Each \$0.45
2. 12 or less than 50	" .39
3. 50 packed in wire-bound boxes	" .35 $\frac{1}{2}$
4. Barrels, lots of 125 or multiples	" .35

If the net invoice price for groups 1 and 2 is 35¢ each, and for groups 3 and 4 is \$30.50 per C, what is the trade discount percentage allowed on each group?

13. The Brown Co. order the following flashlights from the Auto Supply Co. The list price per flashlight is also given.

3 No. 12	@ \$1.35
4 22	1.75
6 23A	2.25
1 MA12	1.75
6 MA22	2.35
1 22FS	3.00
3 22S	3.20

The trade discount is 33 $\frac{1}{3}$ % in lots of less than six, but 45% in lots of six or more. The terms are 2/10, n/30. What is the net amount to be paid by the Brown Co. during the discount period?

14. The Jackson Garage orders the following Balloon Tire Chains from the Auto Supply Co. at the list prices quoted :

50 pr. 32 × 5.77	@ \$7.50 per pair
6 pr. 32 × 6.20	8.50
18 pr. 33 × 6.20	9.00
12 pr. 33 × 6.60	9.50

The trade discount allowed on lots of less than 12 is 30%; lots of 12 and up to 50, 35%; lots of 50, 40%. The terms are 2/10, n/30. What is the net amount to be paid by the Jackson Garage during the discount period?

15. If the catalogue price of a library table is \$90 and the trade discount is 50%, 33 $\frac{1}{3}$ %, and 10%, what is the invoice cost?

16. Find the total cost of the following items, with trade discounts as indicated : loose-leaf supplies listed at \$155 less 40%; automatic pencils at \$72 less 30% and 10%; erasers at \$24 less 25% and 15%.

17. A. L. Dunn buys 10 dining-room sets for which he pays a total of \$1938. If this is the net price after a trade discount of 15% and 5% has been deducted, what was the list price per set?

18. A typewriter dealer lists a certain model at \$180. In January he offers a trade discount of 25% and 10% and sells 400 typewriters. In February he changes the discount to 30% and sells 390 typewriters. During which month are his gross sales greater?

19. A hardware dealer has been offering a trade discount of 20% and 10% on hammers listed at \$60 a gross and has been selling an average of 6300 gross per year. He now decides to offer a further discount of 5%. How many gross must he sell per year to make the same total volume of sales as before?

20. A wholesaler offers a trade discount of 25% on merchandise listed at \$240. If he wishes to lower the net price by $6\frac{2}{3}\%$, what must his new discount rate be?

21. A storekeeper buys from a wholesaler the following list of goods, all subject to a trade discount of 25% and 10%, terms being 3/10, n/30. Find the total of the bill if paid six days after date:

1 gross Hammers	@ \$26.75
6/12 doz. Meat Choppers	41.00
3 gross No. 11 screws	1.35
50 lb. Wire Nails	.45

22. Mr. Downes, a wholesaler, offers the following discounts to retailers: 40%, 30%, and 10% on merchandise in Catalogue M-2; 40%, 25%, and 10% on merchandise in Catalogue M-4; and 50%, 30%, and 20% on merchandise in Catalogue X-3. What three single rates could he offer that would net the same prices?

23. A dealer has listed articles of merchandise at the following prices: \$1.40, \$2.30, \$5.60, \$3.50. He wishes to sell them at the following net prices, respectively: \$1.12, \$2.07, \$3.36, \$3.00. What trade discounts must he offer to yield these prices to the buyer?

24. Mr. Barnes buys a series of goods listed at a total of \$435, less a trade discount of 25% and 10%, and obtains a further cash discount of $2\frac{1}{2}\%$ by paying the bill promptly. At a later date he buys the same quantity and kind of goods from a different dealer, at a list price of \$429, less a trade discount of 20% and 15% and receives no cash discount. For which purchase did he pay more?

BUYING AND SELLING

1. The Mason Department Store ordinarily sells men's handkerchiefs of a certain quality at 20¢ each. The average monthly sale is 1000. During December they plan to offer them at 6 for \$1.

(a) How many must be sold to maintain the same volume of sales?

(b) If the handkerchiefs cost 15¢ each, what is the amount of gross profit on each one? the percentage of gross profit?

2. A rug dealer reduced his price on 9×12 Worsted Wilton Rugs from \$120 to \$95. What was the percentage of discount?

3. The Elite Department Store decided to hold a three-day pre-inventory sale and offer the following discounts: Department 1, $33\frac{1}{3}\%$; Department 2, 25%; Department 3, 15%. The sales during the three days at reduced selling prices were: Department 1, \$2700; Department 2, \$1350; Department 3, \$3570. The goods were originally marked to yield the following gross profits: Department 1, 50%; Department 2, 40%; Department 3, $33\frac{1}{3}\%$. What was the total volume of sales at the original selling prices? What was the amount of gross profit in each department? in the three departments?

4. A merchant has on hand 1260 pairs of girls' stockings which cost him \$5.85 per dozen pairs. He estimates that his expenses for selling them will be 20%, and he proposes to mark them at a price which will yield him a net profit of 15%. At what price per pair must he sell them?

5. The average figures or percentages for 254 jewelry firms reporting in a recent year were:

Total salaries and wages	17.7%
Rent	4.8%
Advertising	3.1%
Other expenses	12.7%
Gross profit	39.5%
Net profit	1.2%

(a) Compute the amounts which a typical jeweler with sales amounting to \$30,000 would spend for each of the above items.

(b) If sales increased to \$40,000 and the above percentages remained the same, show the increase in amount for each of the above items.

6. The following are the percentages of average experience for 1284 retail hardware stores in 1926:

Gross profit	25.62%
Operating expenses	23.23%
Net operating profit	2.39%

(a) If net operating profits of a certain store were \$3585, use the above percentages to construct a complete statement of its operating conditions, including the establishment of the original sales price.

(b) Construct a similar operating statement for the next year if the amount of the gross profit increases 15% and the percentages remain the same.

7. The budget for a certain firm allowed 2% of the total sales for advertising. The total sales for the year were \$150,000 and the advertising was apportioned as follows:

Newspaper and periodical space	\$1800
Street-car space	420
Signs and posters	600
Miscellaneous advertising expense	180

It is estimated that the sales for the second year will increase 20%.

(a) Using the proportions given above, find how much would be spent the second year for each type of advertising.

(b) If it is assumed that sales are directly proportional to advertising, what should the third year's sales be if the advertising appropriation is \$4500?

8. A dealer in silverware buys olive spoons at \$2.27 each, less a trade discount of 50%. He sells them at \$1.75 each. What is his percentage of gross profit on the sale?

9. A merchant made a gross profit of \$16,000 on sales amounting to \$32,000. His net profit was \$4000. What was his percentage of gross profit? of operating expense? of net profit?

10. A dealer purchased a radio for \$300, less 40%. He sold it for \$280, of which \$200 was paid in cash and \$80 allowed on an old phonograph. The phonograph was sold for \$40. What was the profit or loss on the transaction?

11. Mr. Coates, a dealer in automobiles, sold a car at the list price of \$1050. The trade discount to him had been 20%. If he accepts a truck as part of the payment, allowing \$75 on it, and later sells it for \$100, what is his total gross profit on the transaction?

12. Eight desks listed at \$215 each, less 40%, are sold at \$175 each. What is the gross profit on the transaction? The percentage of gross profit?

13. A manufacturer finds that certain machines cost him \$75 to manufacture. If he lists them at \$250, less 50%, he will sell 50 machines, while if he lists them at \$250, less 40% and 10%, he will sell only 45 machines. Which discount should he offer and why? Show computations.

14. A manufacturer found that his costs were as follows: Material, 48%; labor and factory expense, 27%; operating expenses, 12%. If his material cost was \$10,800, what were his total sales? labor and factory expense? operating expense? net profit?

15. Mr. Bates wishes a return of 7% upon his investment of \$24,000. What must be his volume of sales to yield him the desired return if the average net profit is 12%?

16. A grocery dealer suffered a net loss of \$400 on sales amounting to \$8000. What was his percentage of net loss? His operating expenses were 20% of sales. What were his operating expenses? his gross profit? his cost of goods sold?

17. The total sales of a business were \$35,000, cost of sales \$21,875, and operating expenses \$8750. Find the percentage of gross profit; of operating expenses; of net profit.

18. From the following available figures of a business, find total sales, gross profit, net profit, percentage of net profit:

Cost of goods sold, \$27,000

Gross profit, 40% of sales

Operating expenses, \$11,250

19. A real-estate dealer wishes to sell a parcel of land to yield him a profit of 15%. If the land cost him \$6150, what must his selling price be?

20. A dealer in toys had sales during the month of December amounting to \$7500. These same toys had been purchased from the manufacturer for \$9000, less 45%. Salaries and other expenses amounted to \$1750. What was his percentage of gross profit? of operating expenses? of net profit?

21. A manufacturing concern found that its operating loss at the end of a year was \$33,000. The gross profit of \$24,000 averaged 20% of sales. What were the sales for the period? the operating expenses? the percentage of operating loss?

22. If the total investment in the above firm was \$150,000, what was the percentage of loss on investment?

23. It has been reported that for the year 1926 the percentages for the various groups of operating expenses in retail grocery stores in one state were as follows: selling, 6.29%; delivery, 2.43%; management, 3.94%; fixed expenses, 3.61%; miscellaneous, .92%.

(a) Use these percentages to find the expenses in each of the groups for a grocer whose sales were \$88,000.

(b) If the cost of goods sold was \$71,764, what was his per cent of net profit or loss?

24. The percentages for the various groups of operating expenses in the retail grocery business of John Herbert were as follows: selling, 6.54%; delivery, 2.61%; management, 4.20%; fixed expenses, 4.20%; miscellaneous, 0.81%.

The total sales for the year were \$72,000, and the percentage of gross profit was 16.21.

(a) What was the amount of each of the groups?

(b) What was the net profit or loss made by John Herbert for the year?

25. The average percentage spent for salaries and wages in several hundred department stores during 1926 was 15%. The distribution of these salaries and wages among the various groups of operating expenses in one store is as follows: administrative and general expenses, 4%; occupancy expenses, 0.5%; publicity, 0.5%; buying, receiving, etc., 1.4%; general selling and delivery, 8.6%. If the total sales in this store were \$310,000, what was the total amount spent for salaries and wages and what was the amount spent within each group of operating expenses?

26. In one city there were 225 grocery stores in business on Jan. 1, and during the year 74 went out of business. Of the latter number 50 were failures, and of these but 20 had any previous experience in this line of merchandising.

(a) What was the percentage of stores which went out of business?

(b) What was the percentage of failures to total stores?

(c) What was the percentage of failures to those which went out of business?

(d) What per cent of those failing had no experience?

27. The sales by clerks in Department D of the Farwell Hardware Store on March 20 are given herewith. Rule a daily sales summary by clerks, similar to the one on page 208, and enter the data in the proper spaces. Determine the per cent of the total sales and the per cent of the total gross profits made by each clerk.

CLERK NO.	SALES	GROSS PROFITS
46	\$178.20	\$59.40
47	246.80	61.75
48	95.40	36.20
49	123.25	39.50

28. The daily sales for three departments of the Acme Dry Goods Store for a week are given herewith. Rule a form similar to the one on page 209 and enter the data in the proper spaces. Determine the per cent of the total weekly sales made on each day, and the per cent of the week's sales made by each department.

	DEPT. A	DEPT. B	DEPT. C
Monday	\$621.75	\$401.25	\$507.70
Tuesday	542.80	382.45	581.25
Wednesday	931.50	287.60	627.50
Thursday	427.30	357.50	413.20
Friday	682.25	475.50	528.40
Saturday	987.40	567.25	639.80

29. A merchant sold a lot of merchandise which cost \$625 at a profit of 60%; he sold a second lot which cost \$420 at a loss of 25%; and he sold a third lot which cost \$255 at a profit of 20%. What per cent of gross profit did he realize on the total sales?

COMMISSION AND BROKERAGE

1. If Mr. Parker's commission was 2% and he remitted to his principal \$555.17, what was the total sale?

2. If Mr. Parker's sales were \$684 and his net remittance to his principal was \$666.90, what was the rate of commission he received?

3. If Mr. Parker's commission was 3%, additional charges amounted to \$7.50, and he remitted to his principal \$768.50, what were his total sales? How much commission did he receive?

4. A cotton broker sold 105 bales of cotton, of an average weight of 490 lb. The selling price was $14\frac{1}{2}\text{¢}$ per pound; the commission was 15¢ per bale; the additional charges were \$123.40. Find the net remittance to the principal.

5. William Elton's broker bought for him 12,000 bu. of wheat at \$1.30. Five days later he sold it at $\$1.37\frac{1}{2}$ per bushel. If the brokerage was $\frac{1}{8}\%$ for buying and the same for selling, what was the total brokerage, and the net profit to Mr. Elton?

6. Harold Knight, a commission merchant, remitted to Frank Ellis, for whom he acted as agent, \$1487.64 as the proceeds of the sale of a quantity of wheat. If the rate of commission was 2%, and the selling price of the wheat was \$1.65 per bushel, how many bushels of wheat were sold?

7. A live-stock broker sold for Ellis & Son 28 head of cattle of an average weight of 1525 lb., at \$11.50 per hundredweight. The charges were: freight, \$118.20; yardage, \$12.75; feed, \$8.50; commission, 4%. What amount did the broker remit to Ellis & Son?

8. Upon orders from Carlton Allis, his broker bought 1240 bbl. of apples at \$5.10 per barrel; the rate of commission was 3%. Later the broker sold the entire lot of apples for \$6.20 per barrel, the rate of commission being $2\frac{1}{2}\%$. The freight charges were \$64.25; cartage, \$23.40. What was Mr. Allis's profit on the transaction?

9. Frank Hartley, as agent, sold a small farm for Henry Long on a commission of $3\frac{1}{2}\%$. If he remitted to Mr. Long \$8241.10, what was the selling price of the property?

10. A commission merchant sold for Elton Harris the following: 40 bbl. Apples @ \$4.25; 30 crates Onions @ \$0.85; 60 bu. Potatoes

@ \$1.80. The commission merchant paid freight of \$3.50 and charged a commission of 4%. What sum did he remit to Mr. Harris?

11. Albert Lewis, commission merchant, remitted to Frederick Ames \$778 as the net proceeds of a consignment of oranges. The following were the charges: freight, \$9.40; drayage, \$6.10; insurance, \$4.50; commission, \$42. What was the rate of commission?

12. Albert Lewis, as agent, sold 180 bbl. of apples at \$4.75 per barrel, on a commission of 4%. He paid freight amounting to \$5.80. By direction of Ellery Davis, for whom he was acting, he invested the net proceeds in potatoes, charging 3% commission. What was the total commission he received?

13. Harold Knight received \$52.50 for selling a quantity of wheat on a 3% commission. If the selling price of the wheat was \$1.40 per bushel, how many bushels did he sell?

14. Fuller and Company, commission merchants, sell for Hiram Brown 142 bbl. of potatoes, as follows: 80 bbl. @ \$6.25; 62 bbl. @ \$5.15. Charges amounted to: storage, \$11.50; freight, \$61.45; cartage, \$21.55. The commission was 6%. Find the amount of commission and the net proceeds.

15. Fuller and Company, on a commission of 5%, received \$78. The other charges were: freight, \$28.90; storage, \$7.20; cartage, \$11.40; insurance, \$3.50. Find the gross proceeds and the net proceeds.

16. Albert Clark has in storage for Mason & Gray a quantity of corn. They instruct him to sell a sufficient number of bushels to yield them the net sum of \$2700. He sells at \$1.05 per bushel. The charges are: commission, $3\frac{1}{2}\%$; cartage, \$5; freight, \$10.51. Find the number of bushels sold and Mr. Clark's commission.

17. A real-estate firm which charges a commission of 5% on the first \$5000 and $2\frac{1}{2}\%$ on the balance sold for a client a piece of property at \$7650. What was the net return to the owner on the sale?

18. A broker sold a piece of property for \$18,500. He received 5% on the first \$6000, $2\frac{1}{2}\%$ on the second \$6000, and 1% on the balance of the sum obtained for the house. How much did he receive?

INVENTORIES

1. A hardware dealer had the following hinges on hand at the end of a year. Prepare an inventory form, fill in the proper data, and compute the value of this merchandise on December 31. The prices given are per pair.

STOCK NO.

472	21 pr, 10'' T hinges	\$0.45
473	19 " 8'' T "	.33
474	24 " 6'' T "	.18
476	11 " 3'' T "	.07
561	2 " 10'' strap hinges	.30
563	6 " 6'' " "	.15
565	8 " 4'' " "	.09
566	18 " 3'' " "	.07
308	2 " 8'' gate hinges	.30

2. What is the value of the following composition books which a stationer held in stock at the end of a year?

NUMBER	PRICE PER THOUSAND
3000 Open side, regular ruling, 24 leaves, $7 \times 8\frac{1}{2}$	\$28.89
500 Open side, regular ruling, 52 leaves, $7 \times 8\frac{1}{2}$	47.22
1125 Open side, marginal ruling, 46 leaves, $7 \times 8\frac{1}{2}$	47.22
750 Open side, marginal ruling, 16 leaves, 8×10	29.00
250 Open end, ruled, 80 leaves, $3\frac{1}{4} \times 4\frac{3}{4}$	25.00
1250 Open end, ruled, 72 leaves, $3\frac{1}{2} \times 6$	26.50
500 Open end, ruled, 54 leaves, $4\frac{1}{8} \times 6\frac{7}{8}$	27.78
2000 Open end, ruled, 26 leaves, 6×9	28.35

3. A dealer in men's furnishings inventoried his stock of overcoats at selling price. Following the form on page 252, compute the amount of his inventory at both cost and selling prices if the percentage of gross profit is $33\frac{1}{3}\%$ of selling price.

5 #732 Overcoats	@ \$45
3 #871 "	60
10 #453 "	35
8 #248 "	40
4 #564 "	65
12 #380 "	35

4. The following 20-rod rolls of wire fencing were inventoried by a dealer. At the prices given per rod, compute the value of the inventory.

6 rolls	7-26-6	@ \$0.40
4 "	10-47-6	.60
8 "	8-32-6	.50
10 "	10-47-12	.43
5 "	20-53-6	.48
6 "	8-32-12	.34

5. A retail furniture dealer received the following shipment at prices given, less 50%: 5 #1048 Roll-top Desks @ \$90; 7 #846 Bookcases @ \$97.

Freight on the above shipment amounted to \$22.50, of which \$12 referred to the desks. At what prices should the items be included in the inventory?

6. The following inventory of radio cabinets is found in the store of J. R. Dawes. He desires to have the inventory extended at both cost and selling prices. The prices given herewith are the retail prices, and the cost prices are 40% less. Draw up a suitable form and complete the inventory as requested.

1 Model	5-75	@ \$75.00
2 "	5-50	50.00
3 "	4-29	29.00
2 "	5-38	38.00
4 Type	CF 7 × 18 Panel	26.00
5 "	S 7 × 28 "	16.00
3 "	EJ 7 × 30 "	18.80

7. A small garage, taking inventory of its tubes, finds the following to be on hand:

6	31 × 4 Tubes	@ \$2.45
6	32 × 4 Tubes	2.55
8	33 × 4 Tubes	2.70
12	34 × 4 Tubes	2.80
12	32 × 4½ Tubes	3.40
9	33 × 4½ Tubes	3.70

These are the cost prices; the selling prices are 35% higher. Complete the inventory, showing both cost and selling prices.

BUSINESS STATEMENTS

1. From the following facts taken from the books of J. Holmes, a wholesale grocer, prepare a profit and loss statement :

Sales, \$3396.60 ; Inventory, Jan. 1, \$3000 ; Purchases, \$3318.50 ; Inventory, Dec. 31, \$3999.60.

Operating Expenses : Rent, \$100 ; Salaries, \$480 ; Advertising, \$54.20 ; Heat and Light, \$20.42 ; Sundry General Expense, \$47.30.

2. Prepare a financial statement for Mr. J. Holmes if his investment on Jan. 1 was \$7000, and his assets and liabilities on Dec. 31 of the same year were as follows :

Assets : Cash, \$1334 ; Notes Receivable, \$339.98 ; H. M. Black, \$450 ; J. A. Bates, \$547.20 ; Merchandise Inventory, \$3999.60 ; Furniture and Fixtures, \$610 ; Auto Truck, \$1000.

Liabilities : Notes Payable, \$495 ; A. W. Shaw, \$211.25 ; F. J. Main, \$198.75. •

3. The following facts are taken from the books of E. M. Barr. Using the figures given, prepare a profit and loss statement and a financial statement on Dec. 31.

Sales, \$4395 ; Inventory, Jan. 1, \$3421 ; Purchases, \$4270 ; Inventory, Dec. 31, \$3589.

Operating Expenses : Rent, \$90 ; Salaries, \$150 ; Delivery Expense, \$45 ; Heat and Light, \$46 ; Office Supplies, \$30.

Investment, Jan. 1, \$7500.

Assets, Dec. 31 : Cash, \$976 ; Notes Receivable, \$1765 ; Accounts Receivable, \$1177 ; Merchandise Inventory, given above ; Furniture and Furnishings, \$1500.

Liabilities, Dec. 31 : Notes Pay., \$800 ; Accounts Pay., \$775.

4. Prepare a profit and loss statement for Mr. H. O. Young if his books yielded the following figures :

Sales, \$1975 ; Inventory, \$1058 ; Purchases, \$2760.

Operating Expenses : Salaries, \$125 ; Rent, \$85 ; Supplies, \$32.50.

5. Prepare Mr. Arthur Dean's balance sheet if his books on Jan. 1 showed the following totals :

Assets : Cash, \$575 ; A. Rollins, \$335 ; Lloyd & Co., \$476 ; Merchandise Inventory, \$564 ; Real Estate, \$15,500 ; Furniture and Furnishings, \$462.85.

Liabilities : A. D. Wall, \$365 ; Notes Payable, \$245.

6. The following figures are from the books of John & John Company. Construct the profit and loss statement for the period and the balance sheet as of June 30, the end of the financial period.

Sales, \$3885; Inventory, Jan. 1, \$854; Purchases, \$3189; Inventory, June 30, \$1286.

Operating Expenses: Rent, \$105; Salaries, \$225; Heat and Light, \$32; Office Supplies, \$30.

Investment, Jan. 1, \$5100.

Assets, June 30: Cash, \$710; R. Haines, \$202; Notes Receivable, \$456; Merchandise Inventory, given above; Furniture and Furnishings, \$532; Real Estate, \$3000.

Liabilities, June 30: Notes Payable, \$253; A. T. Jones, \$46.50; A. P. West, \$52.50.

7. Prepare a profit and loss statement to summarize the following facts about the business of A. L. Jenney:

Sales, \$250,000; Inventory, Jan. 1, \$83,500; Inventory, Dec. 31, \$26,345; Purchases, \$139,600.

Operating Expenses: Rent, \$8500; Light and Heat, \$2300; Insurance, \$1625; Wages, \$8924; Advertising, \$2367; Sundry Expense, \$5436.

8. Draw up Mr. R. L. Thatcher's balance sheet as of June 30, 19—, basing it upon the following figures which his books furnish:

Assets: Cash, \$774.10; Notes Receivable, \$960; Accounts Receivable, \$530; Merchandise Inventory, \$4234; Furniture and Furnishings, \$975; Fuel Inventory, \$30; Office Supplies Inventory, \$25.

Liabilities: Notes Payable, \$795; Accounts Payable, \$670.

9. Draw up Mr. Thatcher's profit and loss statement for the period ending June 30, given the following figures in addition to those in Ex. 8:

Sales, \$11,060; Purchases, \$12,870.

Operating Expenses: Rent, \$450; Salaries, \$600; Fuel, \$90; Office Supplies, \$65; Interest Paid, \$24.50.

10. Compute the net gain from the following items:

Gains: \$19,477.06.

Operating Expenses: Insurance, \$1123; Heat and Light, \$1927.18; Rent, \$1518; Wages, \$8104.36; Traveling Expenses, \$962.75; Advertising, \$1091.21; Sundry Expenses, \$2608.06.

INTEREST

1. The face of an interest-bearing note is \$3679.83; the rate of interest is $5\frac{1}{2}\%$; the time is 5 mo. What is the value of the note at maturity?

2. John Simmons held two interest-bearing notes. The one was for \$1256.90, dated May 24, with interest at $5\frac{1}{2}\%$, and was paid in full on August 12. The other was for \$893.88, dated May 28, with interest at 6%, and was paid in full on August 12. How much did he receive?

3. John Healy deposited \$263.12 in a savings bank which credited interest twice a year, at $4\frac{1}{2}\%$. How much would his savings account total at the end of one year?

4. Find the total interest on the following: \$429.60, for 6 mo. 23 da. at 5%; \$4500 for 2 mo. 4 da. at 5%; \$6000 for 4 mo. 12 da. at $5\frac{1}{2}\%$; \$460.65 for 3 mo. 27 da. at 6%; \$4550 for 6 mo. 1 da. at 5%.

5. Find the total interest on the following: \$345.25 for 5 mo. 11 da. at 5%; \$1450 for 7 mo. 10 da. at 6%; \$2300 for 9 mo. 14 da. at $5\frac{1}{2}\%$; \$4486.10 for 2 mo. 12 da. at 6%; \$327.40 for 8 mo. 28 da. at 5%.

6. A bank collected the following notes, with the interest due on them: a note for \$654.50 at $5\frac{1}{2}\%$, for 2 mo. 2 da.; a note for \$553.45 at 6%, for 8 mo. 4 da.; a note for \$126.55 at $5\frac{1}{2}\%$, for 5 mo. 2 da.; a note for \$52.75 at 5%, for 9 mo.; a note for \$125.75 at 5%, for 2 mo. 13 da. Find the total interest and the total proceeds of the notes collected.

7. Paul Harris borrowed from his local bank \$1335.50 for 4 mo. 20 da. at 5%. At maturity he paid the interest and \$750 cash to apply on the face of the note. The old note was canceled and a new note was given for the amount still due the bank. What was the amount of the new note? How much interest was paid?

8. On the first business day of the year Jesse Paul made a deposit in the savings bank of \$250. The bank paid an annual interest of 4%, interest being added on July 1 and Jan. 2. If a withdrawal of \$50 was made on Oct. 1, what amount would stand to his credit on Nov. 1?

9. Mr. William Harris receives an invoice of merchandise amounting to \$1340, less trade discounts of 10% and 10%. At the end of 30 days he gives a 60-day note for the net amount of the bill, with interest at 6%. The holder of the note discounts it at the end of 30 days, at 5%. Find the discount and the net proceeds of the note.

10. On March 1 Mercer & Company owed Albert Clark, on account, \$1250. They settled it by giving cash \$200, a 60-day note for \$600, with interest at $5\frac{1}{2}\%$, and a 90-day note for \$450, with interest at 6%. How much did Mercer & Company pay in all?

11. Find the total interest on the following notes: \$212.50 for 5 mo. 15 da., at 6%; \$5000 for 6 mo. 17 da., at 6%; \$1225 for 4 mo. 29 da., at $5\frac{1}{2}\%$; \$423.75 for 3 mo. 4 da., at 5%; \$25.80 for 2 mo. 5 da., at 6%; \$367.40 for 4 mo. 9 da., at $5\frac{1}{2}\%$; \$429.20 for 5 mo. 19 da., at 5%.

12. John Lane settled for an invoice of \$1729.50 by giving three interest-bearing notes, as follows: a 30-day note for \$550, with interest at $5\frac{1}{2}\%$; a 45-day note for \$625, with interest at $5\frac{1}{2}\%$; and a 60-day note for the balance, with interest at $5\frac{1}{2}\%$. If the notes are all paid when due, what will be the total interest?

13. Find the interest on each of the following notes, at 6%: a note that amounts to \$646.40 in 60 da.; a note that amounts to \$548.10 in 90 da.; a note that amounts to \$362.70 in 45 da.; a note that amounts to \$888.42 in 30 da.; a note that amounts to \$253.75 in 90 da.; a note that amounts to \$457.50 in 100 da. Prove the correctness of your work.

14. A certain piece of suburban property was sold at the beginning of each of the following years, at the prices stated below:

1902	\$20,000
1912	65,000
1920	300,000
1927	500,000

Which of the owners realized the highest return on his investment? (Use simple interest; do not consider taxes etc.)

15. A man buys household furniture valued at \$500. He pays \$200 down and \$100, with interest at 7% on the unpaid balance.

at the end of each year until the debt is paid. Find the total payment of principal and interest.

16. Mr. Aikens buys a house worth \$12,000. He pays \$4000 in cash and gives a mortgage at 6% for the remainder. Twice each year until the house is paid for he pays \$500 on the principal, together with the accrued interest on the mortgage. What is the total amount paid?

17. Mr. Young bought a house for \$12,000 and rented it for \$80 a month. He paid annually \$270 for taxes, \$9 for insurance, and \$85 for repairs. Five years later he sold the property for \$13,500. How much more, or less, did he gain than he would have gained by investing the \$12,000 for 5 years at 6% compound interest, compounded semiannually?

18. A farm is purchased by paying \$10,000 cash and \$4000 and accrued interest at the end of each of the next 3 years. The value of the farm was \$25,000. How much was the last payment at the end of the fourth year, at 5% on unpaid balances?

19. A man bought a house for \$5600 and placed on it a mortgage of \$2500, on which he has to pay 6% interest. The expenses for taxes, insurance, and repairs amount to \$170. The house brings \$40 a month rent. On the \$3100 invested he could have obtained 7% interest from good securities. Does he lose or gain by his purchase, and how much?

20. Find the value on the date of maturity of a 60-day note for \$346 which draws interest at $5\frac{1}{2}\%$.

21. A note for \$456, with interest at 6%, is due 4 mo. after date. How much will the holder of the note receive at maturity?

22. A man bought a radio priced at \$250, paying \$75 in cash and giving a 30-day note for \$100 and a 60-day note for the remainder, both notes with interest at 6%. How much did he actually pay for the radio?

23. A. L. Mack borrows \$860 on March 1, with the understanding that he may pay it back when he likes, with $6\frac{1}{2}\%$ interest from the day he borrows it until the day he repays it. If he pays it back on June 17, how much interest will he have to pay?

24. John Black borrows \$4500 at $6\frac{1}{2}\%$ interest and pays it back 1 year and 5 days later. How much interest does he pay?

BANK DISCOUNT

1. On June 1 Hugh Fleming finds that his bank balance has dropped to \$167.20. Accordingly he discounts the following notes, at 6%, and has the proceeds credited to his account: a note for \$483.75, due July 21; a note for \$375.28, due July 25. He then draws a check for \$318.45. What is his bank balance after he has drawn the check?

2. On Jan. 14 James Warren's bank balance was \$235.70. On that day he discounted the following notes, at $5\frac{1}{2}\%$: a note for \$320.50, due March 1; a note for \$98.75, due March 5; a note for \$212.25, due Feb. 28. What was his bank balance after the proceeds of the notes were credited to his account?

3. On Feb. 1 Moore & Son discounted the following notes at 5%: a note for \$1100, due March 10; a note for \$146.50, due March 15; a note for \$181.33, due April 1; a note for \$525, due April 5. Find the total discount and the total net proceeds.

4. On May 1 Paul Morton discounted the following notes at 6%: a 60-day note for \$365.70, dated April 20; a 90-day note for \$635.40, dated April 1; a 50-day note for \$296.87, dated May 1. Find the total discount and the total net proceeds.

5. On May 11 Johnson & Company's bank balance was \$1285.67. On that day they discounted the following notes at 6% and had the proceeds credited to their account: a note for \$856.45, due June 25; a note for \$406.70, due July 2. They then drew two checks, one for \$1060.43, the other for \$671.24. Find the total discount and their bank balance after the checks were drawn.

6. William Ellis opens a bank account on March 1 by depositing \$256.50 in cash. He then discounts the following notes at $5\frac{1}{2}\%$: a 90-day note for \$345.60, dated Jan. 2; a 60-day note for \$163.45, dated Feb. 11. Find the total discount and his bank balance after the proceeds of the notes were credited to his account.

7. Allen and Company's bank balance on April 21 is \$853.46. On that day they discount at 5%, and have credited to their account, the following notes: a note for \$286.75, due in 53 days; a note for \$200.50, due in 47 days; a note for \$279.30, due in 38 days. They then draw checks for the following amounts: \$86.45,

\$231.12, and \$175.73. Find the total discount on the notes, and their bank balance after the checks are drawn.

8. Find the total net proceeds on the following interest-bearing notes, discounted by French and Foster: a 90-day note for \$1250, with interest at 6%, dated April 1, discounted May 1 at 6%; a 3-months note for \$1000, with interest at $5\frac{1}{2}\%$, dated April 17, discounted June 3 at 5%; a 2-months note for \$456.60 with interest at 6%, dated May 3, discounted June 1 at 6%.

9. On May 1 Albert Hubbard has a bank balance of \$865.45. On that day he discounts, at 5%, the following interest-bearing notes: a 60-day note for \$356.50, with interest at $5\frac{1}{2}\%$, dated April 1; a 50-day note for \$600, with interest at $5\frac{1}{2}\%$, dated May 1. Find the total discount and Mr. Hubbard's bank balance after the proceeds of the notes are credited to his account.

10. On March 1 Gary Bassett's bank balance is \$236.80. On that day he discounts a 55-day note for \$720, with interest at 6%, dated Feb. 3. On March 2 he discounts a 60-day note for \$500, dated Feb. 28. On March 5 he discounts a 3-months note for \$1000, dated March 2. The discount rate is 6% in each case. Find the total discounts, the total proceeds of the notes, and Mr. Bassett's bank balance on March 6.

11. Martin Fiske's bank balance is \$313.15. He wishes to bring it up to \$500 and decides to do so by drawing a 60-day note and presenting it for discount, the rate being 6%. For what amount must he draw the note?

12. On Sept. 14 John Melrose's bank balance was \$538.50. In his morning mail he received an invoice of merchandise amounting to \$1456.50, dated Sept. 11, subject to a cash discount of 4% if paid within 10 days. He at once discounted the following notes at 6%: a 60-day note for \$458.50, dated Aug. 15; a 50-day note for \$594.50, dated Sept. 10. He then issued a check in payment of the invoice. Find the amount of the check, and his bank balance after all proceeds were credited to his account and the check was issued.

13. On May 1 Thomas Lacey received two invoices of merchandise, one amounting to \$412.50, dated April 28, terms 3/10, n/30, the other amounting to \$548.75, dated April 25, terms 2/10, n/60.

His bank balance was only \$326.54, so he at once discounted the following notes at 6%: a 90-day note for \$600, with interest at 6%, dated April 25; a 60-day note for \$850, dated April 30. He then issued checks in payment of both the invoices, after deducting the discount. Find the total bank discount and his bank balance after the checks were issued.

14. The Farwell Bank discounts three notes all due in 40 days at 6%, taking as discount \$45.16, \$12.67, and \$54.80. What were the face values of the notes?

15. Mr. Miller wishes to discount three notes which will yield him respectively \$350, \$275, and \$185, and which will fall due in 30, 60, and 90 days. For what sums must he draw the notes if the discount rate is 6%?

16. Mr. Thomas Wright owes a bill for merchandise amounting to \$2376. He wishes to acquire cash to pay it by discounting his 60-day note at the bank. If the discount rate is 6%, for what sum must he draw the note?

17. Find the total amount of discount paid on the following notes:

FACE OF NOTE	DATE OF NOTE	TIME OF NOTE	DATE OF DISCOUNT	RATE OF DISCOUNT
\$525.24	June 12	3 mo.	June 12	6%
\$571.03	June 15	60 da.	June 18	6%
\$309.40	June 21	60 da.	June 23	5%
\$4000.00	June 25	90 da.	June 25	5½%

18. Mr. William Willis holds a 90-day note for \$500, with interest at 5½%, dated June 4, and a 3-month note for \$650, with interest at 6%, dated June 10. On July 1 he offers them for discount at his bank, where the discount rate for the day is 6%. How much does he receive from the two notes?

19. How much would Mr. Willis have received from the two notes mentioned in Exercise 18 if he had discounted them on Aug. 1 at 6%?

20. On Sept. 10 Mr. Knowles discounts his 60-day note for \$4000 at 6%. On Sept. 12 his bank is offering a discount rate of 5½%. How much more would he have received had he waited to draw his note and present it for discount on Sept. 12?

APPENDIX

TABLES OF MEASURES

LENGTH

12 inches (in.) = 1 foot (ft.)

3 feet = 1 yard (yd.)

$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet = 1 rod (rd.)

320 rods, or 5280 feet = 1 mile (mi.)

City lots are generally measured in feet and decimals of a foot; farm land is often measured in rods.

For the tables of measures in the metric system, see pages 137-138.

SQUARE MEASURE

144 square inches (sq. in.) = 1 square foot (sq. ft.)

9 square feet = 1 square yard (sq. yd.)

$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)

160 square rods = 1 acre (A.)

640 acres = 1 square mile (sq. mi.)

In government surveys a square mile is generally called a **section**.

CUBIC MEASURE

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)

27 cubic feet = 1 cubic yard (cu. yd.)

128 cubic feet = 1 cord (cd.)

A load of earth, gravel, sand, and so on is equivalent to 1 cu. yd.

The **perch**, used in stonework and masonry, varies in different parts of the country, but an average equivalent is $24\frac{3}{4}$ cu. ft.

WEIGHT

16 ounces (oz.) = 1 pound (lb.)

100 pounds = 1 hundredweight (cwt.)

2000 pounds = 1 ton (T.)

The **long ton** of 2240 lb. is used by the government in computing duties upon imports, and it is also used in wholesale transactions in certain mining products.

LIQUID MEASURE

$$4 \text{ gills (gi.)} = 1 \text{ pint (pt.)}$$

$$2 \text{ pints} = 1 \text{ quart (qt.)}$$

$$4 \text{ quarts} = 1 \text{ gallon (gal.)}$$

A gallon contains 231 cu. in. or 0.134 cu. ft. A cubic foot of water weighs approximately $62\frac{1}{2}$ lb., and contains about $7\frac{1}{2}$ gal. The **liquid quart** contains 57.75 cu. in.

Barrels and **hogsheads** vary in size, but a barrel is generally taken as equivalent to $31\frac{1}{2}$ gal. and a hogshead to 63 gal.

DRY MEASURE

$$2 \text{ pints (pt.)} = 1 \text{ quart (qt.)}$$

$$8 \text{ quarts} = 1 \text{ peck (pk.)}$$

$$4 \text{ pecks} = 1 \text{ bushel (bu.)}$$

The **stricken bushel** used in measuring grains and the like contains 2150.42 cu. in., or approximately $1\frac{1}{4}$ cu. ft. The **heaped bushel** used in measuring large fruits, potatoes, and so on contains 2747.71 cu. in. The **dry quart** contains 67.2 cu. in.

METRIC EQUIVALENTS

The following table shows the approximate equivalents of the common metric measures:

$$1 \text{ meter (m.)} = 39.37 \text{ in.}$$

$$= 3.28 \text{ ft.}$$

$$1 \text{ centimeter (cm.)} = 0.39 \text{ in.}$$

$$1 \text{ millimeter (mm.)} = 0.04 \text{ in.}$$

$$1 \text{ kilometer (Km.)} = 0.62 \text{ mi.}$$

$$1 \text{ inch (in.)} = 2.54 \text{ cm.}$$

$$1 \text{ foot (ft.)} = 30.48 \text{ cm.}$$

$$1 \text{ mile (mi.)} = 1.61 \text{ Km.}$$

$$1 \text{ liter (l.)} = 1 \text{ qt. (liquid)}$$

$$1 \text{ gram (g.)} = 15.5 \text{ grains}$$

$$= 0.035 \text{ oz.}$$

$$= 0.002 \text{ lb.}$$

$$1 \text{ kilogram (Kg.)} = 2.2 \text{ lb.}$$

$$1 \text{ metric ton (T.)} = 2200 \text{ lb.}$$

$$1 \text{ ounce (oz.)} = 28.35 \text{ g.}$$

$$1 \text{ pound (lb.)} = 0.45 \text{ Kg.}$$

$$1 \text{ ton (T.)} = 0.91 \text{ T. (metric).}$$

ANGULAR MEASURE

60 seconds (60'') = 1 minute (1')

60 minutes = 1 degree (1°)

There are 90° in a right angle and 360° in a circle

TIME

60 seconds (sec.) = 1 minute (min.)

60 minutes = 1 hour (hr.)

24 hours = 1 day (da.)

7 days = 1 week (wk.)

12 months (mo.) = 1 year (yr.)

The **common year** has 365 da., the **leap year** has 366 da., and the **commercial year** is reckoned as 360 da. The **century** is 100 yr. Centennial years divisible by 400 and other years divisible by 4 are leap years.

The months of April, June, September, and November have 30 da. each, and all the other months, except February, have 31 da. each. February has 28 da. in a common year and 29 da. in a leap year.

In running trains across such a broad stretch of country as the United States, it is important to have a uniform time over considerable territory. For this reason a system of **standard time** in which the United States is divided into four time belts is adopted. Each belt covers approximately 15° of longitude, and the time within each belt is the same. **Eastern time**, the first belt, takes its time from the 75th meridian; **central time** from the 90th; **mountain time** from the 105th; and **Pacific time** from the 120th. There is a difference in time of 1 hr. between adjacent time belts, and hence when it is 11 A.M. by eastern time it is 10 A.M. by central time, 9 A.M. by mountain time, and 8 A.M. by Pacific time. In order that railroads may change time at important stations or terminals, the boundaries of the time belts are irregular.

COUNTING

12 units = 1 dozen (doz.)

12 dozen = 1 gross (gro.)

12 gross = 1 great gross

PAPER MEASURE

24 sheets = 1 quire (qr.)

20 quires = 1 ream (rm.)

In folded paper the quire of 24 sheets and the ream of 480 sheets are generally used. In other papers the quire of 25 sheets and the ream of 500 sheets are more often used on account of the greater convenience in counting. Paper is also sold by the pound.

UNITED STATES MONEY

10 mills = 1 cent (¢ or ct.)

100 cents = 1 dollar (\$)

The mill is not coined but is frequently used in calculations. The 10-cent piece is called a **dime**, the 25-cent piece a **quarter** (of a dollar), and the 50-cent piece a **half**. The 10-dollar gold piece is called an **eagle**.

For tables of foreign money see page 319.

BUSINESS SYMBOLS AND ABBREVIATIONS

The same abbreviations are used for both singular and plural forms. Thus, *in.* means inch or inches according to the connection in which it is used, and similarly for the other abbreviations.

A. . . . acre	Coll. . . . collection
a/c account	Cr. credit; creditor
a/s account sales	cs. case
+ addition; plus	cu. cubic (as in cubic
() aggregation	feet, cubic meter,
& and	etc.)
... and so on	cwt. hundredweight
@ at	d. pence
Apr. . . . April	° degree
Aug. . . . August	÷ division; divided by
bbl. barrel	\$ dollar
bd. ft. . . . board foot	da. day
bdl. bundle	Dec. December
bg. bag	dm. decimeter
bkt. basket	doz. dozen
bl. bale	Dr. debit; debtor; doctor
bu. bushel	= equality; is equal to
bx. box	etc. and so forth
C hundred	/ fraction (as in 5/6)
c. centime	' foot; minute
c/o care of	Feb. February
¢, ct. . . . cent	f.o.b. . . . free on board
✓ check mark	fr. franc
cd. cord	ft. foot
cm. centimeter	g. gram
Co. company; county	gal. gallon
C.O.D. . . . collect on delivery	gi. gill

gr. . . . grain	$1^1, 1^2, 1^3$. one and one fourth;
gro. . . . gross	one and one half;
hhd. . . . hogshead	one and three fourths
hr. . . . hour	Oct. . . . October
" inch; second	oz. . . . ounce
in. . . . inch	p. . . . page
Jan. . . . January	% per cent; hundredths
Kg. . . . kilogram	£ pound sterling
kg. . . . keg	pc. . . . piece
Km. . . . kilometer	per by the; by
K.W.H. . kilowatt hour	pf. . . . preferred; pfennig
l. . . . liter	pk. . . . peck
lb. . . . pound	pkg. . . . package
M thousand	pp. . . . pages
M. . . . mark (German mone- tary unit)	pr. . . . pair
m. . . . meter	pt. . . . pint
× multiplication; times	Q. . . . quintal
Mar. . . . March	qr. . . . quire
mdse. . . . merchandise	qt. . . . quart
Messrs. . Gentlemen; Sirs	rd. . . . rod
mi. . . . mile	rm. . . . ream
min. . . . minute	s. . . . shilling
mm. . . . millimeter	√ square root of
mo. . . . month	— subtraction; minus
Mr. . . . Mister	sec. . . . second
Mrs. . . . Mistress	Sept. . . . September
n/30 . . . net in 30 da.	sq. . . . square (as in square feet, etc.)
# number (when placed before a figure); pounds (when after a figure)	T. . . . ton
No. . . . number	tb. . . . tub
Nov. . . . November	wk. . . . week
	wt. . . . weight
	yd. . . . yard
	yr. . . . year

CALCULATING MACHINES

Machines or mechanical devices for performing arithmetical operations are now widely used in stores, offices, banks, factories, and so on. Where there is much calculation to be done such machines are practically indispensable, since in the hands of a trained operator a machine will list and add figures more rapidly and accurately than any person can hope to do with pencil and paper.

There are many types of these machines on the market today. Some merely add figures; others list and add them, and print the items and the totals; and still others are bookkeeping machines that do the work of posting and balancing accounts, making statements, and so on. The illustration at the right shows a listing machine. There are also types of calculating machines which subtract, multiply, and divide as readily as they add. Some machines are operated by simply depressing the keys, but to operate others the numbers to be added are set up on the keyboard and a handle is then pulled to add them. Machines are often equipped with electric motors to lessen the work of operating them.



LISTING MACHINE

There are so many different types of machines and so many uses to which they may be put that a detailed description is impractical in this connection. Furthermore, a description of the operation of a machine is of but little value unless the student has at hand a machine upon which he can experiment as he reads the description. The companies which manufacture these machines maintain agencies in almost every city and detailed information about their machines can be readily secured, if desired, for class discussion. The catalogues published by these companies generally show illustrations of the machines and how to operate them.

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